



Random vibration fatigue analysis based on IGA model in LS-DYNA®

Yun Huang¹, Stefan Hartmann², David J. Benson¹

¹Livermore Software Technology, an Ansys company

²DYNAMore GmbH

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2. Introduction: random vibration fatigue analysis
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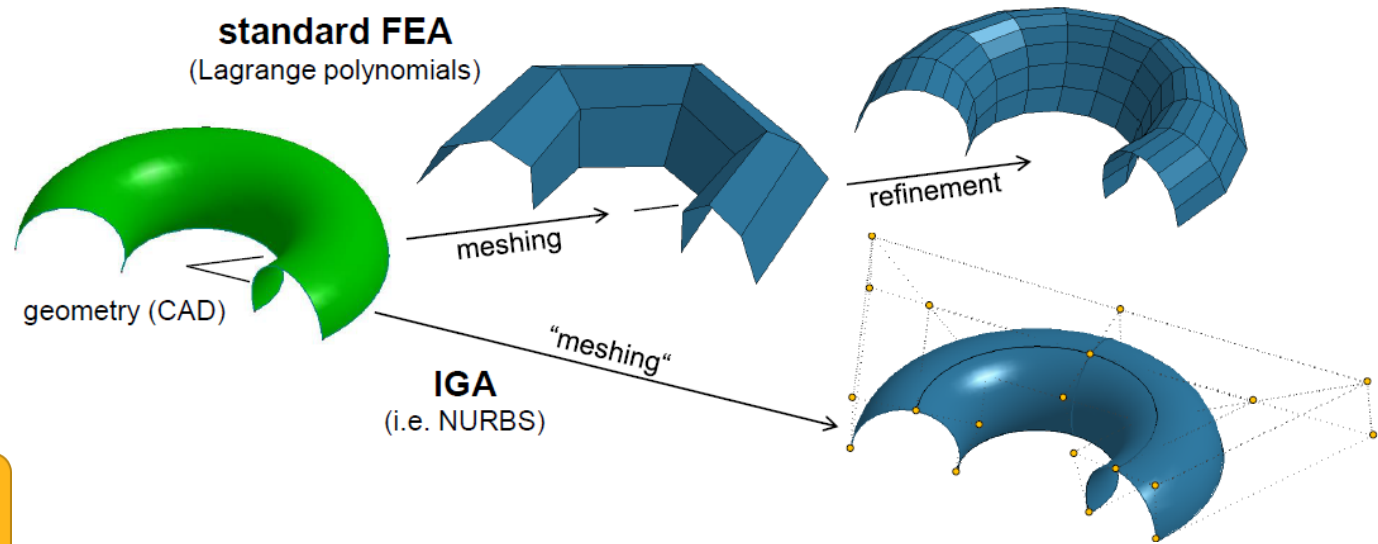
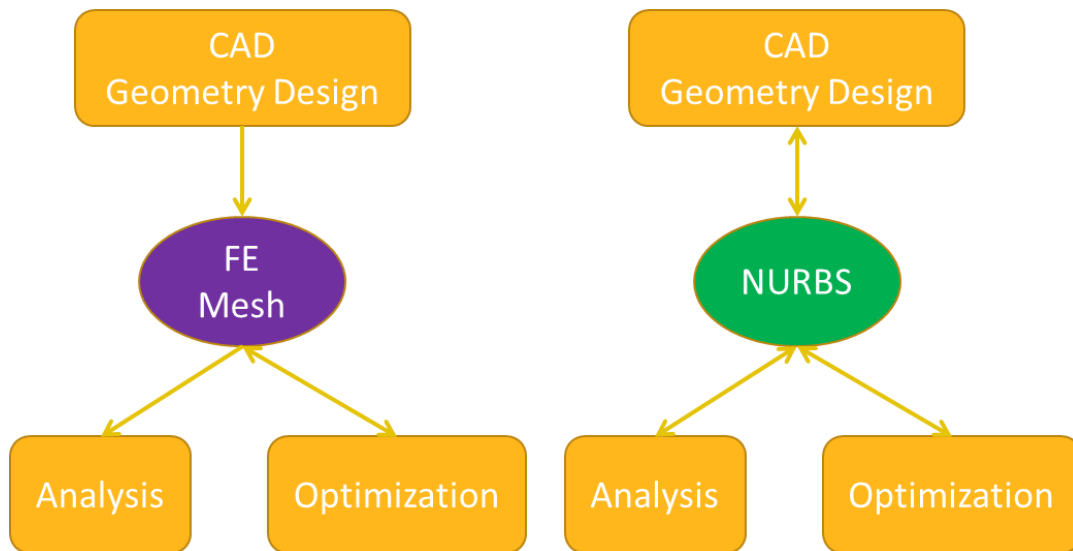
1. Introduction: IGA

What is IGA

Same description of the geometry in the design (CAD) and the analysis

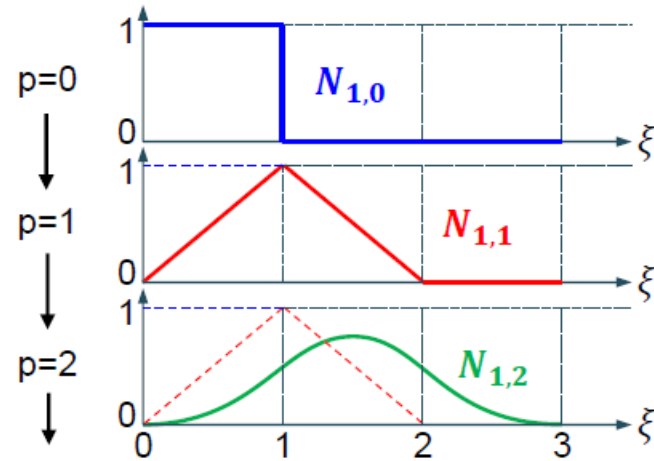
Why IGA

Reduce effort of geometry conversion from CAD into a suitable mesh for FEA



■ B-Spline Basis Functions

■ Are constructed recursively

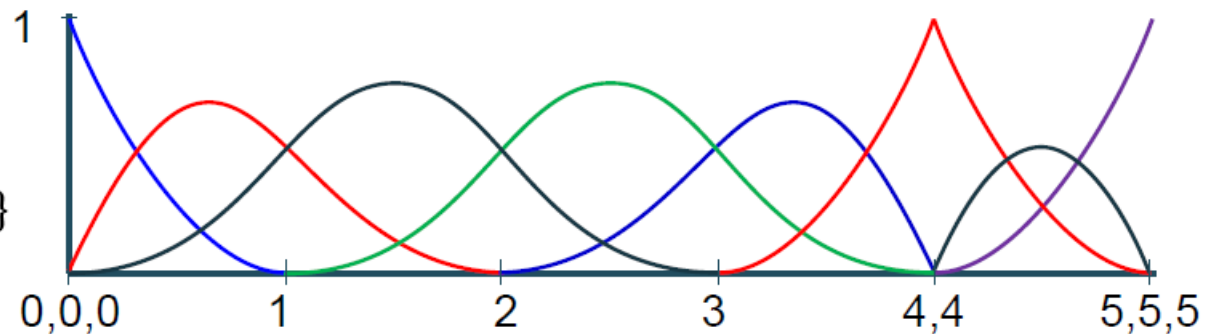


$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \leq \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$

open, non-uniform
knot vector

$$\Xi = \{0,0,0,1,2,3,4,4,5,5,5\}$$

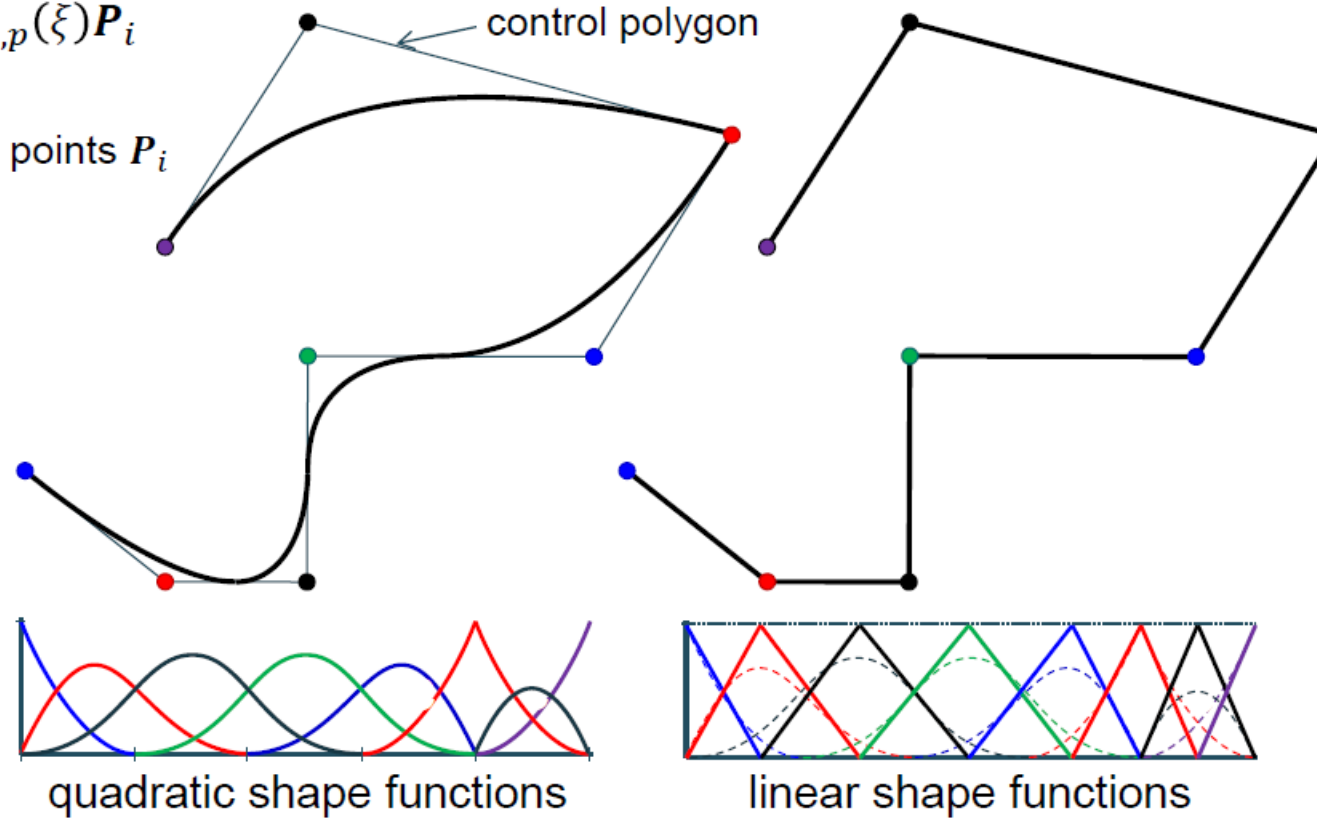


■ B-Spline - Curves

- Linear combination of B-Spline basis functions, just like in standard FEA

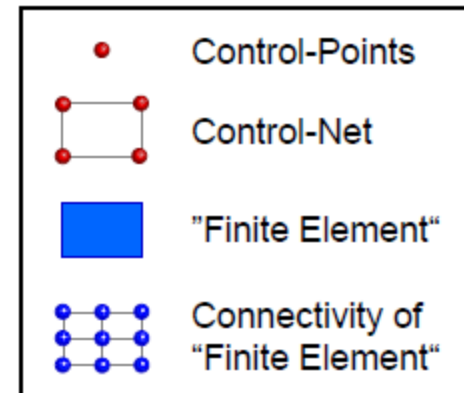
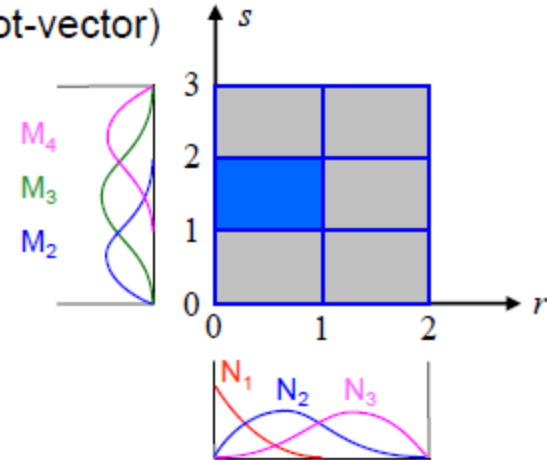
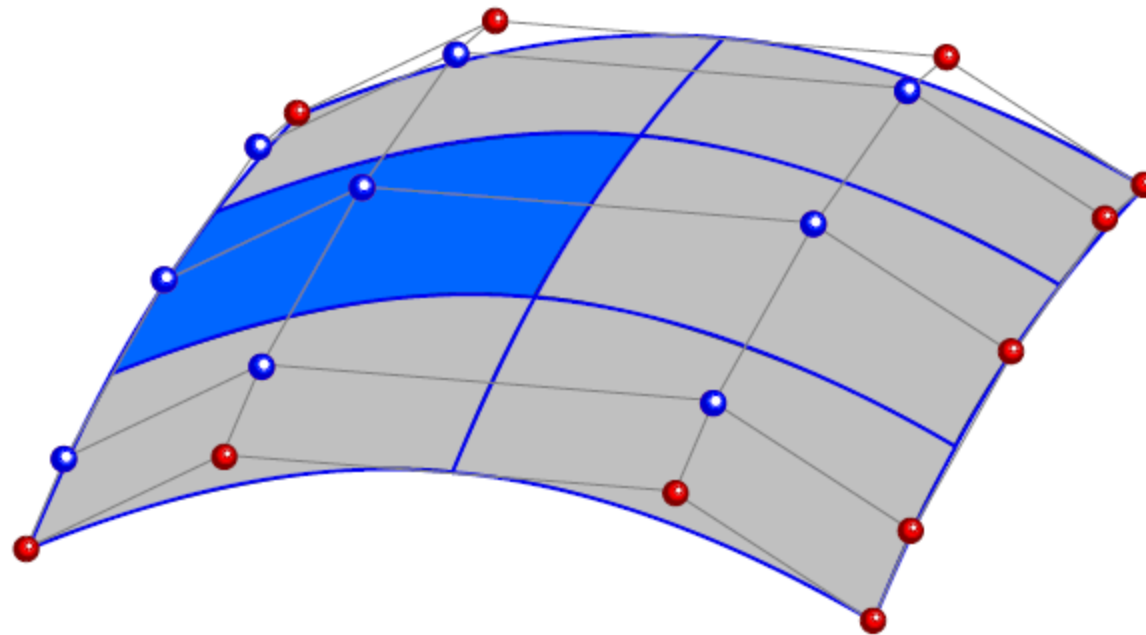
$$\mathbf{C}(\xi) = \sum_{i=1}^n N_{i,p}(\xi) \mathbf{P}_i$$

- With the control points \mathbf{P}_i



Finite Element Analysis with NURBS

- A typical NURBS-Patch – Connectivity of elements
 - Possible "overlaps" (→ higher continuity!)
 - Size of "overlap" depends on polynomial order (and on knot-vector)



■ LS-DYNA keyword for NURBS-shells – 3

■ *ELEMENT_SHELL_NURBS_PATCH

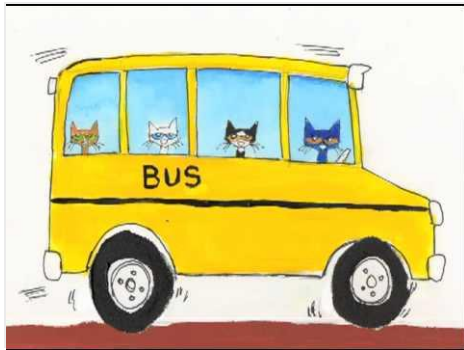
	1	2	3	4	5	6	7	8
Card 1	NPID	PID	NPR	PR	NPS	PS		
Card 2	WFL	FORM	INT	NISR	NISS	IMASS		IDFNE
Card A	RK1	RK2	RK3	RK4	RK5	RK6	RK7	RK8
Card B	SK1	SK2	SK3	SK4	SK5	SK6	SK7	SK8
Card C	N1	N2	N3	N4	N5	N6	N7	N8
Card D	W1	W2	W3	W4	W5	W6	W7	W8

- RKi: values of univariate knot-vector in local r-direction
→ #of Card A: $\text{ceil}[(NPR + PR + 1)/8]$
- SKi: values of univariate knot-vector in local s-direction
→ #of Card B: $\text{ceil}[(NPS + PS + 1)/8]$
- Ni: control point IDs (defined via *NODE) to define the control-net
→ two dimensional table of NPS rows and NPR columns
→ per column requires #of Card C: $\text{ceil}[(NPR)/8]$
→ total #of Card C: $NPS \times \text{ceil}[(NPR)/8]$
- Wi: weights at control points (optional, only if WFL=1)
→ identical order as Ni (Card C)

2. Introduction: random vibration fatigue analysis

Why we need to run random vibration analysis?

- The loading on a structure is not known in a definite sense
- Vibration environments are not related to a specific driving frequency (input from multiple sources)
- To be precise, all vibration is random (due to all kinds of disturbance)
- Examples:
 - Wind load on wind-turbine
 - Air flow over a wing or past a car body
 - Earthquake ground motion
 - Tires running over a rough road
 - Ocean wave loads on offshore platforms



What is fatigue?

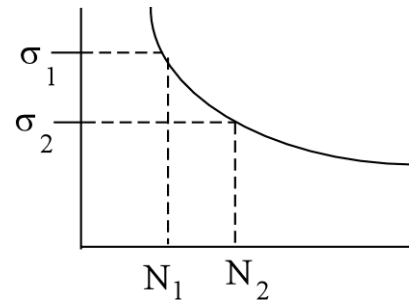
- Fatigue is a process in which damage accumulates due to repetitive application of loads that may be well below the yield point.
- Fatigue is a complex process involving many steps, but it can be broken down into initiation and propagation of fatigue cracks.
- For many years, fatigue has been a significant and challenging problem for engineers, especially for those who design structures such as aircrafts, railroad vehicles, automotive, bridges, pressure vessels, and cranes.
- Fatigue analysis can be performed in time domain and frequency domain.



Overview of random vibration fatigue analysis in LS-DYNA

- Keyword `*FREQUENCY_DOMAIN_RANDOM_VIBRATION_FATIGUE`
- Calculate fatigue life of structures under random vibration
- Based on S-N fatigue curve
- Based on probability distribution & Miner's Rule of Cumulative Damage Ratio

$$R = \sum_i \frac{n_i}{N_i}$$



- Schemes:
 - ✓ *Steinberg's Three-band technique considering the number of stress cycles at the 1σ , 2σ , and 3σ levels.*
 - ✓ *Dirlik method based on the 4 Moments of PSD.*
 - ✓ *Narrow band method*
 - ✓ *Wirsching method*
 - ✓ ...

Typical SN (or Wöhler) curve

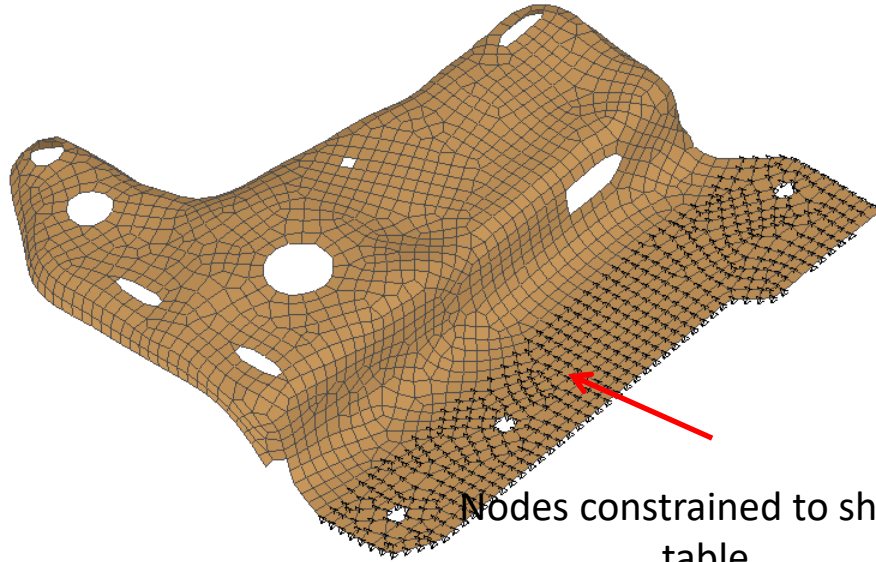
PDF
(probability density function)

Keywords to run random vibration fatigue analysis

```
*FREQUENCY_DOMAIN_RANDOM_VIBRATION_FATIGUE
$#  mdmin      mdmax      fnmin      fnmax      restrt      unused      restrm
      1          0          0.0          0.0          0          0
$#  dampf      lcdam      lctyp      dmpmas      dmpstf      dmptyp
      0.035        0          0          0.0          0.0          0
$#  vaflag      method      unit      umlt      vapsd      varms      napsd      ncpsd
      1          1          0          0.0          0          0          1          0
$#  ldtyp      ipanelu      ipanelv      temper      unused      ldflag      icoarse      bcoarse
      0          0          0          0.0          0          0          0          0.1
$#  sid      stype      dof      ldpsd      ldvel      ldflw      ldspn      cid
      0          0          3          1          0          0          0          0
$#  mftg      nftg      sntype      texpos      strsf      inftg
      2          1          0          12.0          2.0          0
$#  pid      lcid      ptype      ltype      a      b      sthres      snlimt
      1          2          0          1          0.0          0.0          0.0          0
```

```
*DATABASE_FREQUENCY_BINARY_D3FTG
$#  binary
      1
*DATABASE_FREQUENCY_BINARY_D3PSD
$#  binary
      1
$#  fmin      fmax      nfreq      fspace      lcfreq
      0.01      400.0      3999          0          0
*DATABASE_FREQUENCY_BINARY_D3RMS
$#  binary
      1
```

Example: an aluminum bracket



Aluminum 2014 T6

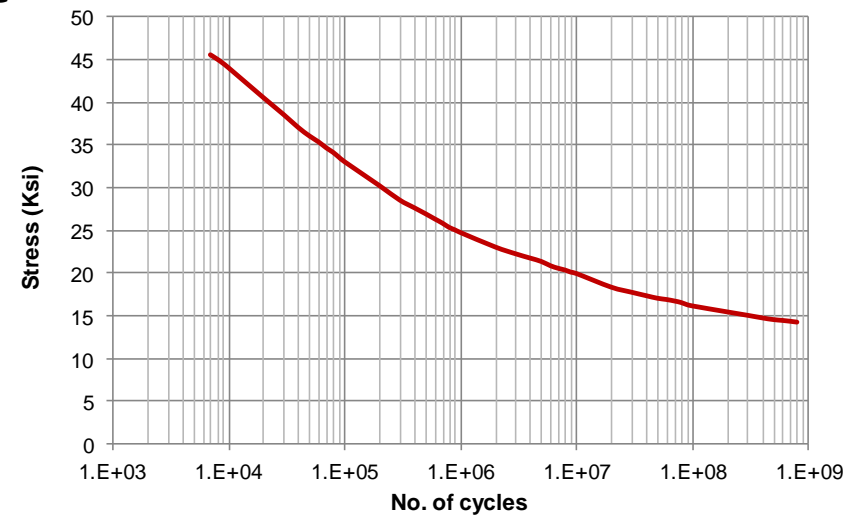
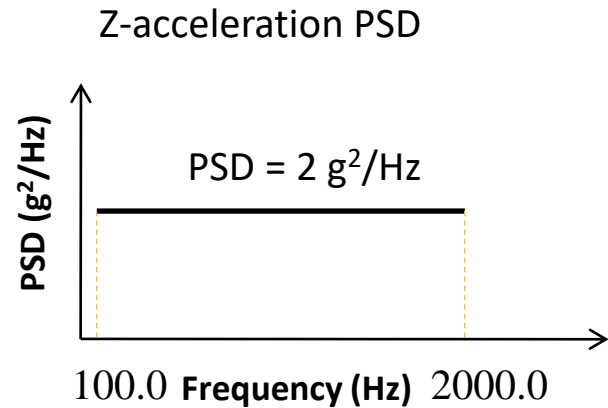
$$\rho = 2800 \text{ kg/m}^3$$

$$E = 72,400 \text{ MPa}$$

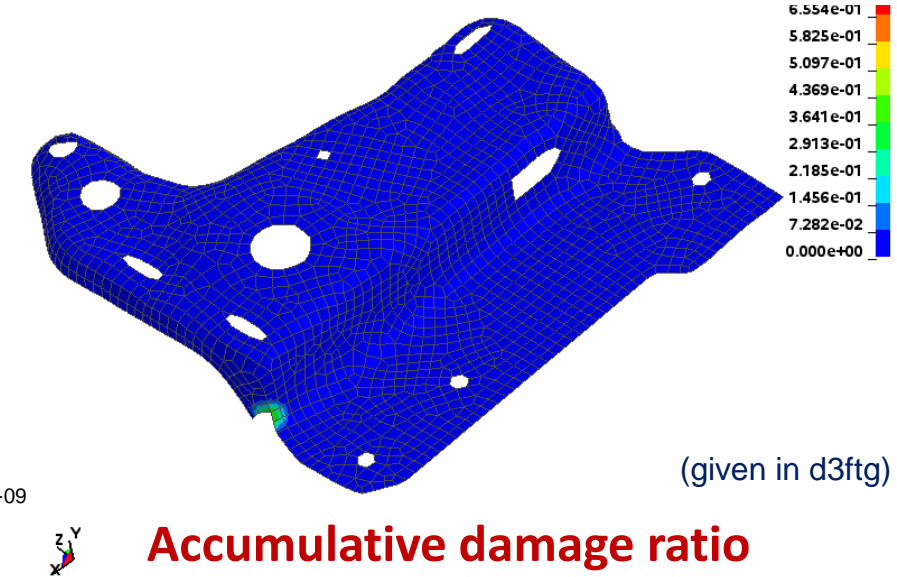
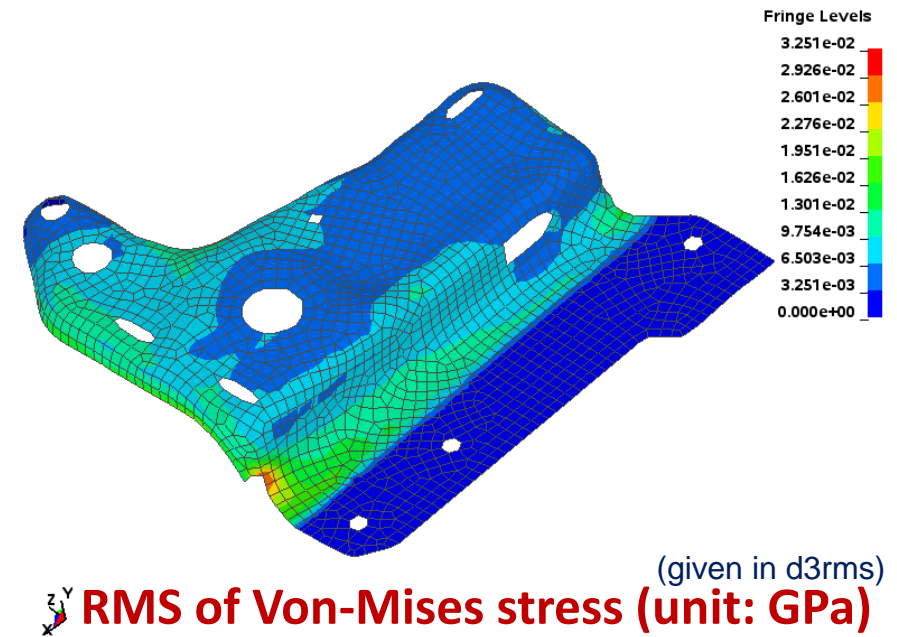
$$\nu = 0.33$$

Time of exposure: 4 hours

Method: Steinberg's method

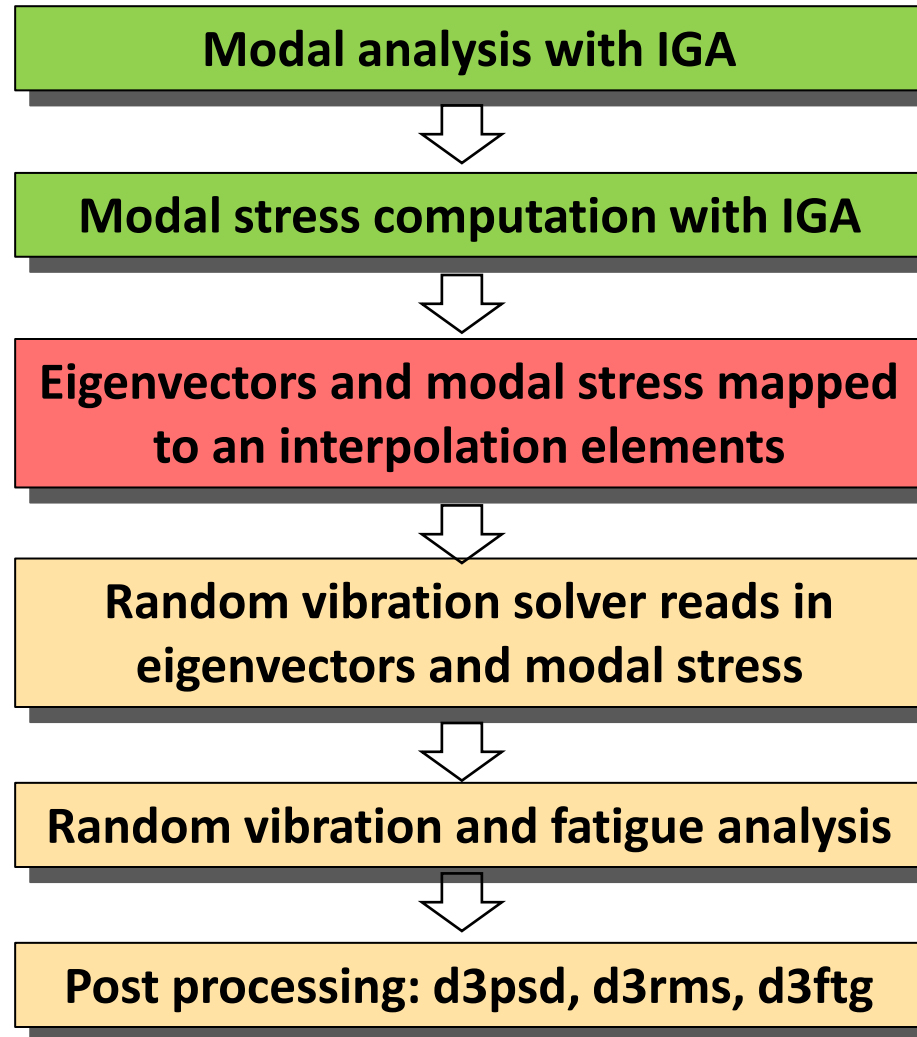


S-N fatigue curve



3. Random vibration fatigue analysis based on IGA model

Flow chart for random vibration fatigue analysis using IGA model



4. Example and preliminary results

FEM information

No. of nodes: 2601

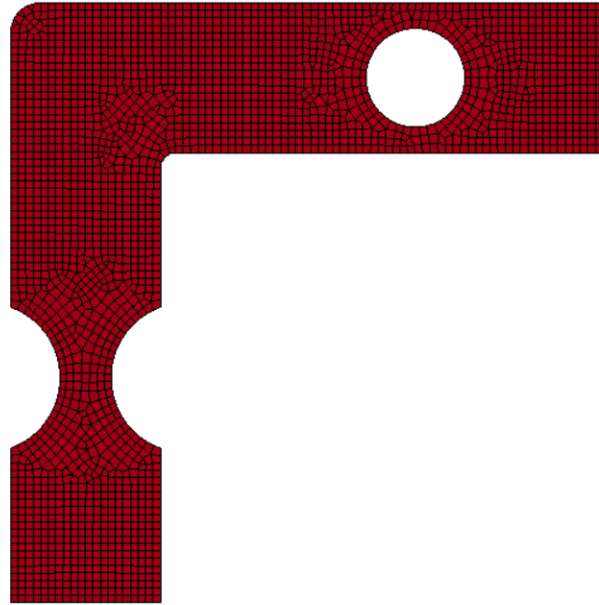
No. of shell elements: 2427

Material properties

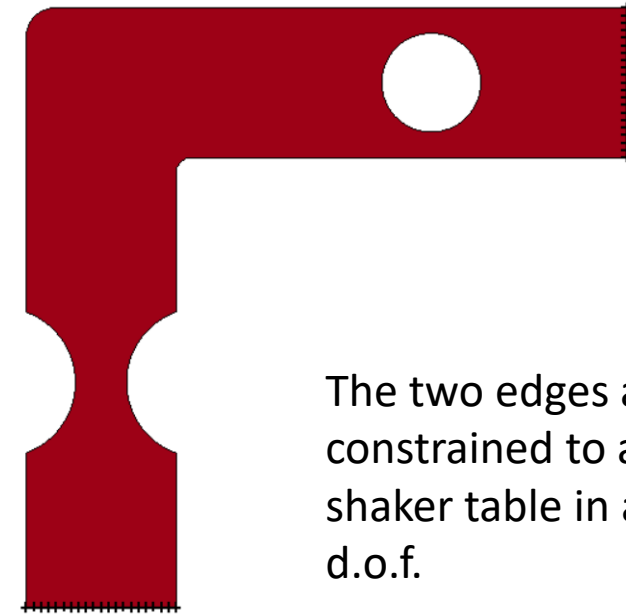
Mass density: 3810 kg/m³)

Young's modulus: 3.1e+11 Pa

Poisson ratio: 0.33



Courtesy of Shubiao Wang, INSA Rouen, France



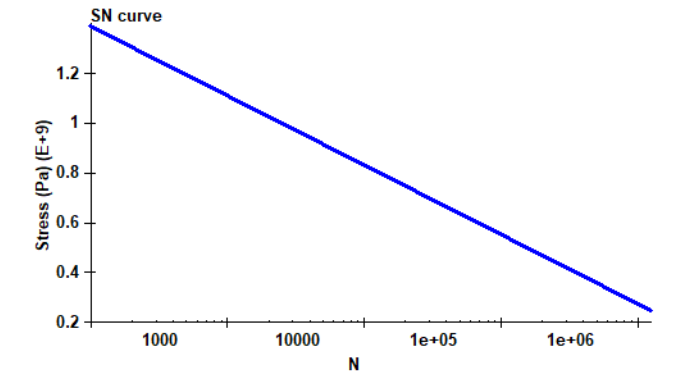
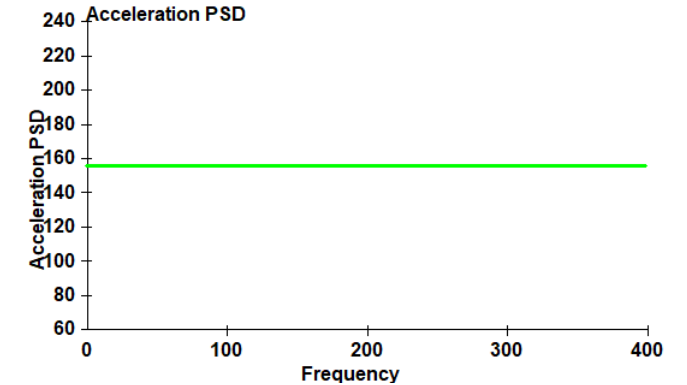
The two edges are constrained to a shaker table in all d.o.f.

A metal bracket (originally modelled by IGA) subject to base acceleration PSD

Keyword and loading condition

*FREQUENCY_DOMAIN_RANDOM_VIBRATION_FATIGUE

\$#	mdmin	mdmax	fnmin	fnmax	restrt	unused	restrm	
	1	0	0.0	0.0	0		0	
\$#	dampf	lcdam	lctyp	dmpmas	dmpstf	dmptyp		
	0.035	0	0	0.0	0.0	0		
\$#	vaflag	method	unit	umlt	vapsd	varms	napsd	ncpsd
	1	1	0	0.0	0	0	1	0
\$#	ldtyp	ipanelu	ipanelv	temper	unused	ldflag	icoarse	bcoarse
	0	0	0	0.0		0	0	0.1
\$#	sid	stype	dof	ldpsd	ldvel	ldflw	ldspn	cid
	0	0	3	1	0	0	0	0
\$#	mftg	nftg	sntype	texpos	strsf	inftg		
	2	1	0	12.0	2.0	0		
\$#	pid	lcid	pctype	ltype	a	b	sthres	snlimt
	1	2	0	1	0.0	0.0	0.0	0



Loading and other computation conditions

1. Base acceleration PSD (constant for whole range of frequency 1-400 Hz)
2. Use mode 1 only
3. Constant modal damping ratio 0.035
4. Dirlik method is used for fatigue analysis (mftg = 2)

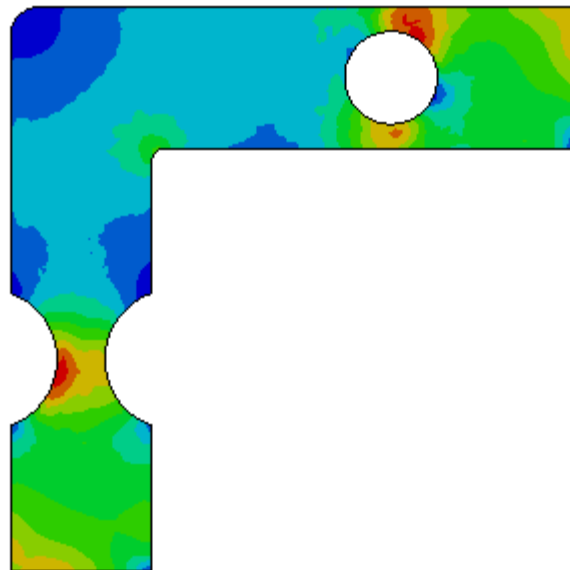
Stress results from random vibration analysis

FEM

fatigue_analysis_fem
Contours of Effective Stress (v-m)
max IP. value
min=6.55771e+06, at elem# 279
max=1.11258e+08, at elem# 518

Effective Stress (v-m)

1.113e+08
1.008e+08
9.032e+07
7.985e+07
6.938e+07
5.891e+07
4.844e+07
3.797e+07
2.750e+07
1.703e+07
6.558e+06

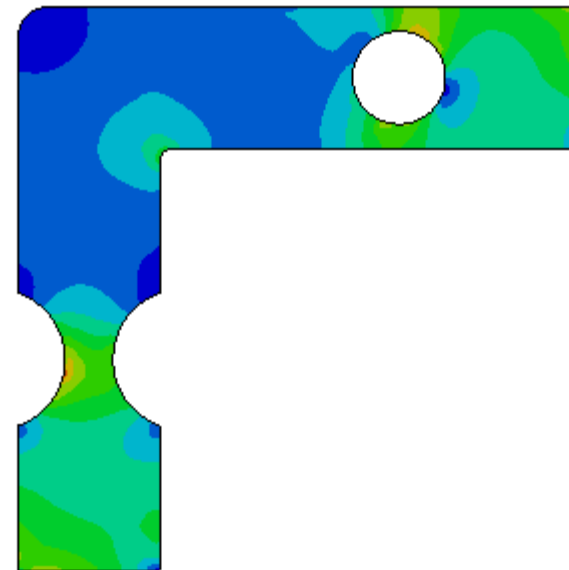


IGA

fatigue_analysis_IGA
Contours of Effective Stress (v-m)
max IP. value
min=1.45162e+07, at elem# 9855
max=2.78761e+08, at elem# 2057

Effective Stress (v-m)

2.788e+08
2.523e+08
2.259e+08
1.995e+08
1.731e+08
1.466e+08
1.202e+08
9.379e+07
6.737e+07
4.094e+07
1.452e+07



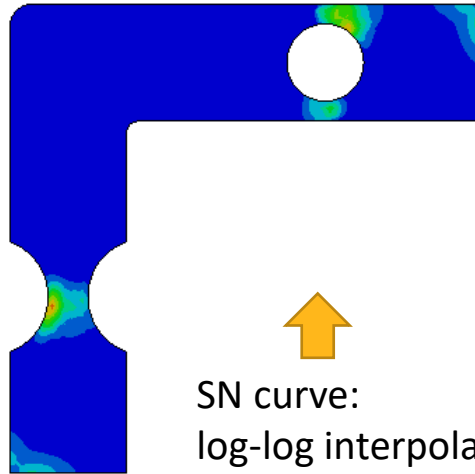
Fatigue analysis results

FEM

fatigue_fem_loglog
Contours of Cumulative damage ratio
max IP. value
max=8.4341e-05, at elem# 518

Cumulative damage ratio

8.434e-05
7.591e-05
6.747e-05
5.904e-05
5.060e-05
4.217e-05
3.374e-05
2.530e-05
1.687e-05
8.434e-06
1.396e-11

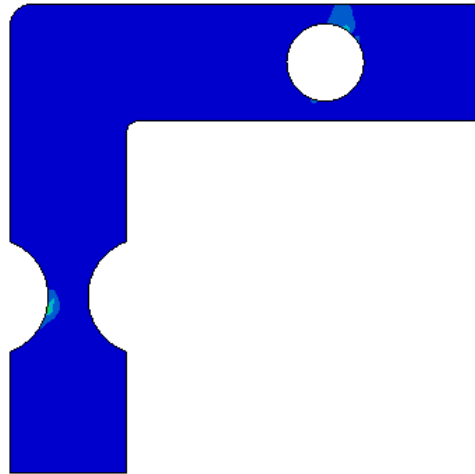


SN curve:
log-log interpolation

fatigue_IGA_loglog
Contours of Cumulative damage ratio
max IP. value
max=0.0136266, at elem# 2057

Cumulative damage ratio

1.363e-02
1.226e-02
1.090e-02
9.539e-03
8.176e-03
6.813e-03
5.451e-03
4.088e-03
2.725e-03
1.363e-03
1.139e-09

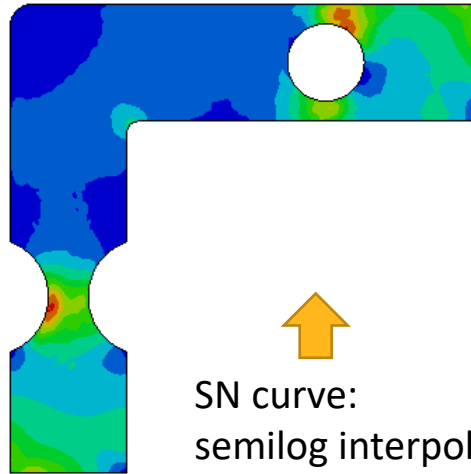


IGA

fatigue_fem_semiolog
Contours of Cumulative damage ratio
max IP. value
max=0.000136442, at elem# 518

Cumulative damage ratio

1.364e-04
1.272e-04
1.180e-04
1.088e-04
9.956e-05
9.034e-05
8.112e-05
7.190e-05
6.268e-05
5.346e-05
4.424e-05

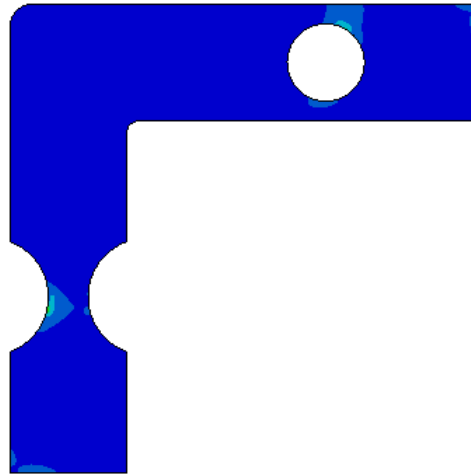


SN curve:
semi-log interpolation

fatigue_IGA_semiolog
Contours of Cumulative damage ratio
max IP. value
max=0.00261761, at elem# 2057

Cumulative damage ratio

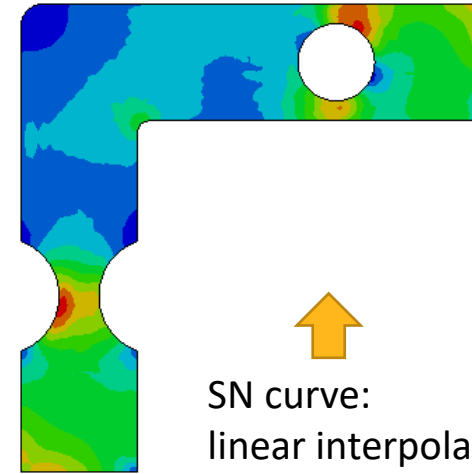
2.618e-03
2.361e-03
2.104e-03
1.847e-03
1.590e-03
1.333e-03
1.076e-03
8.190e-04
5.621e-04
3.051e-04
4.818e-05



fatigue_fem_linear
Contours of Cumulative damage ratio
max IP. value
max=0.00029886, at elem# 518

Cumulative damage ratio

2.989e-04
2.965e-04
2.941e-04
2.917e-04
2.893e-04
2.868e-04
2.844e-04
2.820e-04
2.796e-04
2.772e-04
2.748e-04

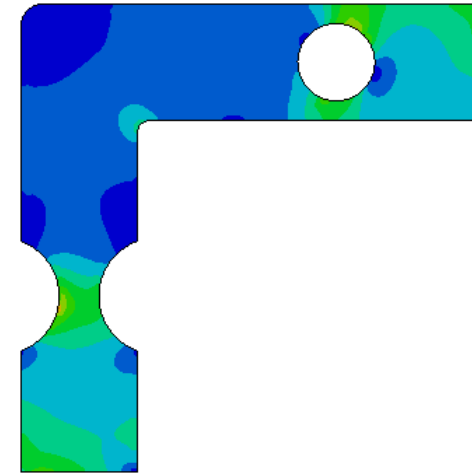


SN curve:
linear interpolation

fatigue_IGA_linear
Contours of Cumulative damage ratio
max IP. value
max=0.000363321, at elem# 2057

Cumulative damage ratio

3.633e-04
3.551e-04
3.468e-04
3.385e-04
3.303e-04
3.220e-04
3.137e-04
3.055e-04
2.972e-04
2.890e-04
2.807e-04



Max values from FEM & IGA

		FEM	IGA
Von Mises Stress (Pa)		1.113e+8	2.788e+8
Cumulative damage ratio	log-log	8.434e-5	1.363e-2
	semilog	1.364e-4	2.618e-3
	linear	2.989e-4	3.633e-4

- There are some difference in stress and fatigue damage results (actually, it's big difference for cumulative damage ratio results, when using log-log interpolation on SN curves);
- The stress results from IGA are higher than those given by FEM, as IGA can essentially capture the stress concentration better;
- Consequently the cumulative damage ratios from IGA are higher;
- The location of max stress / cumulative damage ratio is consistent for FEM and IGA.

5. Future work

- Refine the FEM mesh to see if a better match in stress and cumulative damage ratio by IGA model and FEM model can be reached;
- For random vibration module, get the modal stress from IGA integration points directly, without using the interpolation elements;
- Run random vibration and fatigue analysis on integration points of IGA directly;
- Update the post-processing tool to show vibration and fatigue results on integration points of IGA directly.
- Time domain fatigue analysis based on IGA.
- Suggestions from users ...

Acknowledgement

We would like to thank Shubiao Wang of INSA Rouen of France for permitting us to use his model as an example.



 **Ansys**

