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Using LS-DYNA for Heat Transfer & Coupled Thermal-Stress Problems

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ftp://ftp.lstc.com/outgoing/shapiro/heat_transfer_class.zip

How to obtain files from LSTC

LSTC

To obtain:
- distribution versions of LS-Dyna
- manuals, tutorials, papers
- example problems

ftp://user:computer@ftp.lstc.com

To obtain:
- development versions of LS-Dyna, LS-PrePost
- other files of interest

ftp://beta:keyboard@ftp.lstc.com

Development version not QA'd

Only use the development version if it has a feature you need.

To get files from LSTC → outgoing
To send files to LSTC → incoming

ftp:anonymous@ftp.lstc.com

Philosophy

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"Forty-two!" yelled Loonquawl. "Is that all you've got to show for seven and a half million years' work?"

"I checked it very thoroughly," said the computer, "and that quite definitely is the answer. I think the problem, to be quite honest with you, is that you've never actually known what the question is."

The Hitchhiker's Guide to the Galaxy
Douglas Adams

When FEA is someone's life, that person uses FEA on everything. The technology gets the nod even when hand calculations or physical testing would be faster, less expensive, and more accurate.

P. Kurowski, "When good engineers deliver bad FEA", Machine Design, November 1995.

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Chapter 1 – getting started

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Units – This is the 1st difficulty encountered by novice users in trying to model coupled thermal-stress problems. One of 2 things happen

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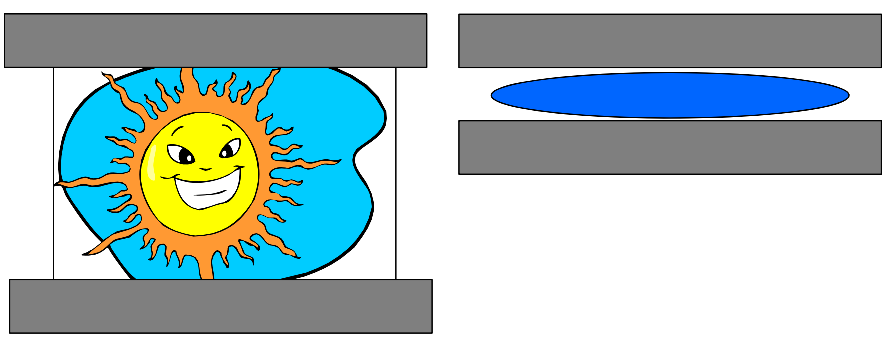
1 I have small deformation, but I have reached the temperature of the sun.

2 I have turned the part into a pancake and there is no temperature change.



A consistent set of units must be used in performing a coupled thermal-stress analysis. Problems arise due to a mismatch between the mechanical unit for work and the thermal unit for energy.

Units

Steel property values in different unit sets

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mass	Length	Time	ρ	E	Cp	k	h
kg	m	sec	7.8e+03	2.1e+11	4.6e+02	7.1e+01	1
kg	mm	sec	7.8e-06	2.1e+08	4.6e+08	7.1e+04	1
kg	mm	msec	7.8e-06	2.1e+02	4.6e+02	7.1e-05	1.e-09
ton	mm	sec	7.8e-09	2.1e+05	4.6e+08	7.1e+01	1.e-03
g	mm	msec	7.8e-03	2.1e+05	4.6e+02	7.1e-02	1.e-06
g	mm	sec	7.8e-03	2.1e+11	4.6e+08	7.1e+07	1.e+03

Properties in SI

Density	**ρ**	**[kg/m^3]**
Elastic modulus	**E**	**[Pa]**
Heat capacity	**Cp**	**[J/kg C]**
Thermal conductivity	**k**	**[W/m C]**
Convection coefficient	**h**	**[W/m^2 C]**

Units

Mechanical equivalent of heat → EQHEAT

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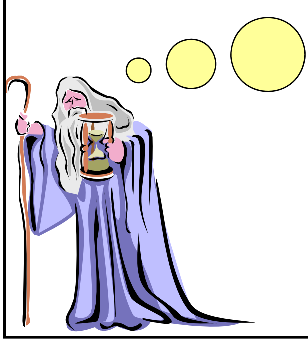

1982 → Coupled NIKE2D & TOPAZ2D

We used the English Engineering system of units
• force → lbf
• energy → Btu

1st Law of Thermodynamics
work = thermal energy
778 ft lbf = 1 Btu

Units

Mechanical equivalent of heat → EQHEAT

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***CONTROL_THERMAL_SOLVER**

ATYPE PTYPE SOLVER CGTOL GPT **EQHEAT** FWORK

Nothing to worry about if using SI units
1 N m = 1 J

Units

EQHEAT user e-mails

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I don't know why this units thing confounds me so. After you read this, you're either going to laugh or cry. Do I understand correctly that EQHEAT allows you to have two systems of units if you desire... one for thermal input and one for mechanical input?

YES! But, no one in their right mind would do this.

Units

How to evaluate EQHEAT for the unit set ton, mm, sec
Objective: keep J and W for all heat transfer parameters

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quantity	unit
mass	ton
length	mm
time	sec
energy	Joule

$$work = Fd = (ma)d \left[(ton)\left(\frac{mm}{s^2}\right)(mm) \right]$$

EQHEAT is the conversion factor (i.e., multiplier) to convert (*ton*)(*mm*^{2}) / (*s*^{2}) to SI units of N m = J.

$$\frac{(ton)(mm^2)}{(\sec^2)} * \underbrace{\left[\frac{10^3 kg}{ton} * \frac{m^2}{10^6 mm^2} \right]}_{\text{EQHEAT}} = 10^{-3} \frac{(kg)(m)}{(\sec^2)}(m) = 10^{-3}(N)(m) = 10^{-3}(J)$$

$$(eqheat)w = \rho c V \Delta T$$

Interpreting temperature contours

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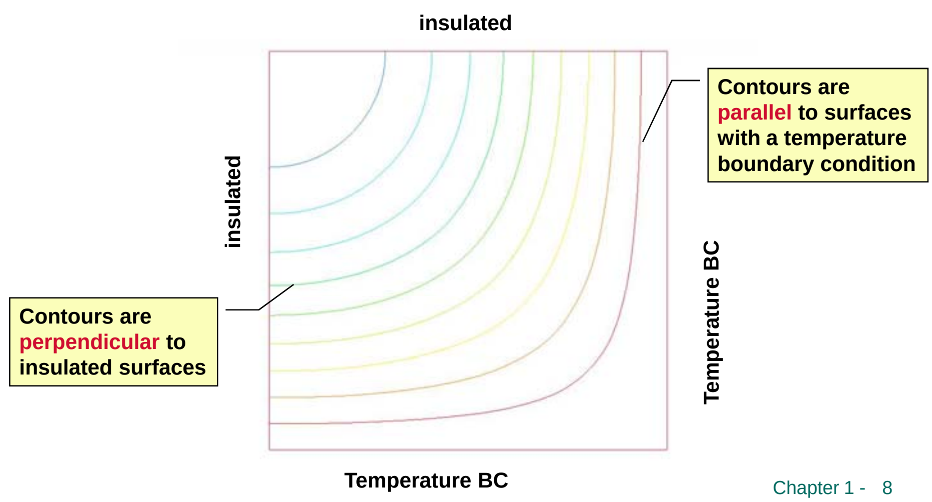
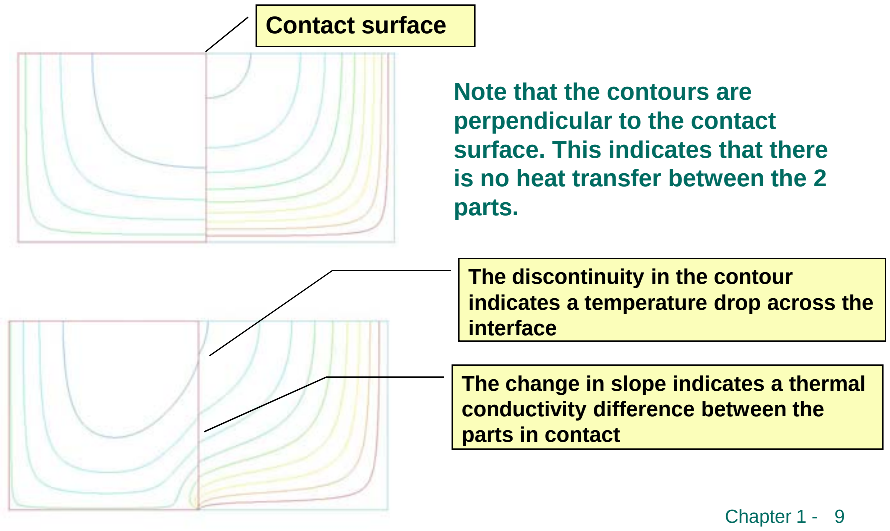


Interpreting temperature contours

LSTC



Note that the contours are perpendicular to the contact surface. This indicates that there is no heat transfer between the 2 parts.

Interpreting temperature contours

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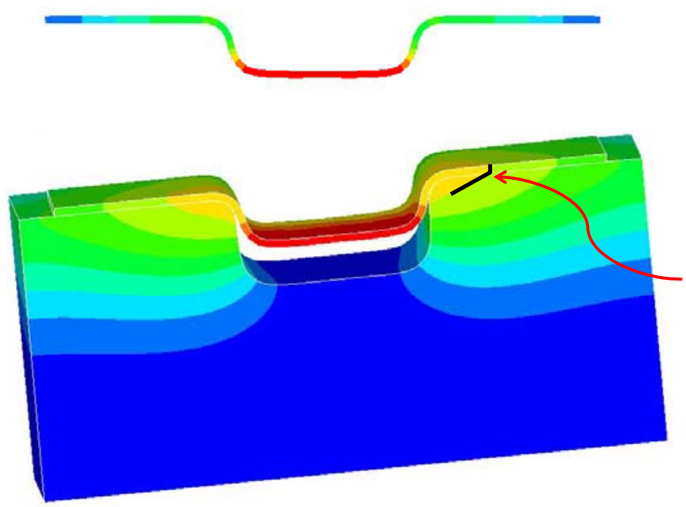

Contours are perpendicular through the thin shell (no temperature gradient through thickness) but when considering the interface, we see that the T contour is not perpendicular on the tool side of the interface.

Problem cp01.k – coupled thermal-stress

Problem definition

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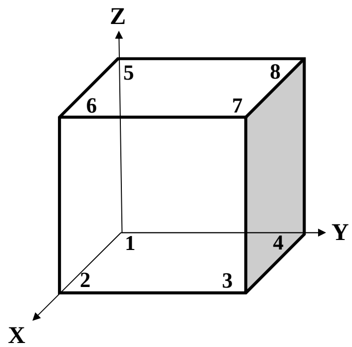

Aluminum 1100

Density	2700 kg/m^3
modulus of elasticity	70.e+09 Pa
Poisson Ratio	0.3
coeff. of expansion	23.6e-06 m/m K
heat capacity	900 J/kg K
thermal conductivity	220 W/m K
heat generation	2.43e+07 W/m^3

Problem cp01.k – coupled thermal-stress

Keyword input

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The KEYWORD input provides a flexible and logically organized database. For example, under the keyword *ELEMENT, are included solid, shell, and beam elements. The keywords can be entered in an arbitrary order. However, for clarity, we will conform to the following general block structure and enter the appropriate keywords in each block

1. define solution control & output parameters
2. define model geometry & material parameters
3. define initial & boundary conditions

*KEYWORD

The first line of the input file must begin with *KEYWORD. This identifies the file as containing the "keyword" format instead of the "structured" format which can also be used.

Problem cp01.k – coupled thermal-stress

Keyword input – solution control

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```
*CONTROL_SOLUTION
$    soln
        2

*CONTROL_TIMESTEP
$ dtinit
      0.

*CONTROL_THERMAL_TIMESTEP
$     tsc       tip       its
        0        1.        .1

*CONTROL_TERMINATION
$ endtim
      1.

*CONTROL_THERMAL_SOLVER
$   atype     ptype    solver
        1         0         3
```

0 = mechanical
1 = thermal
2 = thermal-stress

Mechanical time step

dtinit = 0, calculate default explicit time step

tsc = 0 fixed time step
1 variable time step

tip = 1.0 fully implicit
0.5 Crank Nicolson

its = 0.1 initial time step

solution termination time

atype = 1 → transient
ptype = 0 → linear
solver=3 → iterative solver

Chapter 1 - 13

Problem cp01.k – coupled thermal-stress

Keyword input – material properties

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```
*PART
aluminum block
$       pid       sid       mid      tmid
          1         1         1         1

*SECTION_SOLID
$       sid
          1

*MAT_ELASTIC_PLASTIC_THERMAL
          1     2700.
         0.      100.
   70.e+09   70.e+09
         .3        .3
  23.6e-06  23.6e-06

*MAT_THERMAL_ISOTROPIC
          1     2700.
       904.      222.
```

Mechanical material models 4 and 106 calculate thermal strains (expansion)

The keyword

*MAT_ADD_THERMAL_EXPANSION

Can be used with other material models to calculate thermal strains

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Problem cp01.k – coupled thermal-stress

Keyword input – geometry

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The keyword ***PART** contains data that points to other attributes of this part, which in turn point elsewhere to additional attribute definitions.

```
*PART              pid    sid

*SECTION_SOLID     sid    elform

*ELEMENT_SOLID     eid    pid    nid1   nid2 . . .

*NODE              nid    x      y      z
```

elform
1 → 1pt solid
2 → fully integrated
10 → 1pt ALE

Problem cp01.k – coupled thermal-stress

Keyword input – geometry

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```
*PART
aluminum block
      1       1

*SECTION_SOLID
         1

*ELEMENT_SOLID
    1    1    1    2    3    4    5    6    7    8

*NODE
    1            0.          0.          0.     7    7
    2            1.          0.          0.     5    0
    3            1.          1.          0.     3    0
    4            0.          1.          0.     6    0
    5            0.          0.          1.     4    0
    6            1.          0.          1.     2    0
    7            1.          1.          1.     0    0
    8            0.          1.          1.     1    0

*END
```

This keyword signifies the end of the input

Problem cp01.k – coupled thermal-stress

See Appendix F – problem cp01.k

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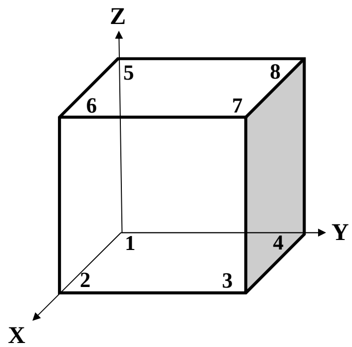

Calculate the expansion due to heating

1. analytical solution
2. numerical solution
3. why are there wiggles in a node displacement vs time plot
4. adjust input to attenuate wiggles
5. solve with implicit mechanics

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LSTC

Chapter 1 - 20

Chapter 2 – mathematical theory

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Heat Diffusion Equation

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change in internal energy = conduction in & out + source & sink

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot K \nabla T + \dot{q}'''$$

density

heat capacity

time

heat generation

temperature

thermal conductivity

For steady state, we solve $0 = \nabla \cdot K \nabla T + \dot{q}'''$

Mathematical preliminaries

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Consider the equation $1+1=2$ **(Eq. 1)**

We know that $1=\ln e$ and $1=\sin^2 x+\cos^2 x$

further $2=\sum_{n=0}^{\infty}\frac{1}{2^n}$

Therefore, Eq. (1) can be expressed as $\ln e+\left(\sin^2 x+\cos^2 x\right)=\sum_{n=0}^{\infty}\frac{1}{2^n}$ **(Eq. 2)**

This can be further simplified using the relations $1=\cosh y\sqrt{1-\tanh^2 y}$ and $e=\lim_{n\to 0}(1+n)^{1/n}$

Eq. (2) may therefore be rewritten: $\ln\left[\lim_{n\to 0}(1+n)^{1/n}\right]+\left(\sin^2 x+\cos^2 x\right)=\sum_{n=0}^{\infty}\frac{\cosh y\sqrt{1-\tanh^2 y}}{2^n}$ **(Eq. 3)**

It should be obvious that Eq. (3) is much clearer and more easily understood than Eq. (1).

Chapter 2 - 3

Mathematical preliminaries

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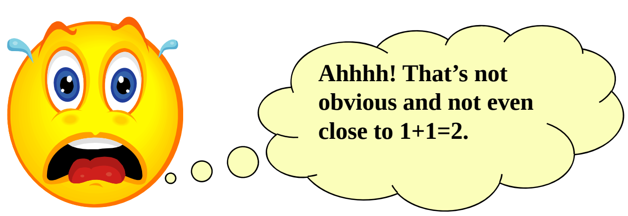
The Galerkin formulation of the transient heat conduction equation

$$\int_\Omega w_j \rho c\frac{\partial T}{\partial t}d\Omega=-\int_\Omega \nabla^T w_j k\nabla T d\Omega+\int_\Omega w_j Q d\Omega+\int_\Gamma w_j q d\Gamma$$



Chapter 2 - 4

Mathematical preliminaries

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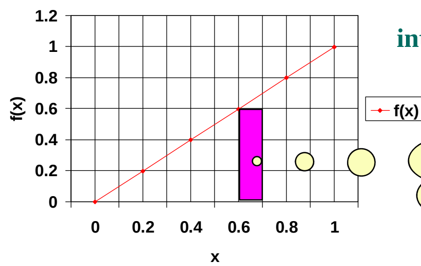
An integral is a number

$$\int_0^1 x\,dx = \frac{1}{2}x^2\Big|_0^1 = \frac{1}{2} - \frac{0}{2} = \frac{1}{2}$$



integral = area under curve = 1/2

Recall rectangle rule for numerical integration – we sum areas

Mathematical preliminaries

LSTC

The Galerkin formulation of the transient heat conduction equation

$$\int_\Omega w_j \rho c \frac{\partial T}{\partial t} d\Omega = -\int_\Omega \nabla^T w_j k \nabla T d\Omega + \int_\Omega w_j Q d\Omega + \int_\Gamma w_j q d\Gamma$$

From the previous vu-graph we know that each integral is a number representing, from left to right:

1. Change in internal energy
2. Conduction in and out of the volume
3. Energy generation inside the volume
4. Energy transfer (e.g., convection) from the surface

These integrals are difficult to evaluate and we will need numerical methods and a computer

Finite element method

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$$\int_{\Omega} w_j \rho c \frac{\partial T}{\partial t} d\Omega = -\int_{\Omega} \nabla^T w_j k \nabla T d\Omega + \int_{\Omega} w_j Q d\Omega + \int_{\Gamma} w_j q d\Gamma$$

Numerical methods needed to solve this equation:

1. **The FEM provides a technique to discretize the body which allows us to sum volumes for the numerical integration.**
2. ***CONTROL_THERMAL_SOLVER defines parameters to perform the numerical integration.**
3. ***CONTROL_THERMAL_TIMSTEP defines parameters for the time integration method used for the 1st term.**
4. ***CONTROL_THERMAL_NONLINEAR defines parameters to handle material and boundary condition nonlinearities.**

Final system of equations

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$$\left[\begin{array}{cccc}
\frac{C_{n+\alpha}}{\Delta t} & +\frac{\alpha}{\Delta t}\frac{dC_{n+\alpha}}{dT_{n+\alpha}}T_{n+1} & -\frac{\alpha}{\Delta t}\frac{dC_{n+\alpha}}{dT_{n+\alpha}}T_{n} & \\
+\alpha K_{n+\alpha} & +\alpha^2\frac{dK_{n+\alpha}}{dT_{n+\alpha}}T_{n+1} & -\alpha^2\frac{dK_{n+\alpha}}{dT_{n+\alpha}}T_{n} & +\alpha\frac{dK_{n+\alpha}}{dT_{n+\alpha}}T_{n} \\
+\alpha H_{n+\alpha} & +\alpha^2\frac{dH_{n+\alpha}}{dT_{n+\alpha}}T_{n+1} & -\alpha^2\frac{dH_{n+\alpha}}{dT_{n+\alpha}}T_{n} & +\alpha\frac{dH_{n+\alpha}}{dT_{n+\alpha}}T_{n} \\
-\alpha\frac{dg_{n+\alpha}}{dT_{n+\alpha}} & +\alpha\frac{df_{n+\alpha}}{dT_{n+\alpha}} & -\alpha\frac{dh_{n+\alpha}}{dT_{n+\alpha}} &
\end{array}\right]\Delta\theta = \begin{array}{l} g_{n+\alpha} - f_{n+\alpha} + h_{n+\alpha} \\ + \left(\frac{C_{n+\alpha}}{\Delta t} + \alpha K_{n+\alpha} + \alpha H_{n+\alpha}\right) T_n \\ \left(-K_{n+\alpha} - H_{n+\alpha}\right) T_n \\ - \left(\frac{C_{n+\alpha}}{\Delta t} + \alpha K_{n+\alpha} + \alpha H_{n+\alpha}\right) T_{n+1} \end{array}$$

Tangent Stiffness

For each element, each term is an 8x8 matrix

Nonlinear iterate update

$$T_{n+1}^{i+1} = T_{n+1}^{i} + \Delta\theta$$

Solving a non-linear problem is a lot of work.
Note that Cp and k are weak functions of T

Chapter 3 – Solvers

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We are spending much of this chapter on CG because that is what you should use – but not always!

*CONTROL_THERMAL_SOLVER

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1-processor

- 1 = symmetric Gauss profile solver (reset to 11)
- 2 = non-symmetric Gauss profile solver
- 3 = diagonal preconditioned conjugate gradient
- 4 = incomplete Choleski preconditioned CG
- 5 = diagonal preconditioned conjugate gradient (for non-symmetric)

MPP

- 11 = mpp direct (sparse Gauss type)
- 12 = mpp diagonal CG
- 13 = mpp symmetric Gauss-Seidel
- 14 = mpp SOR
- 15 = mpp modified Choleski CG

default

Stiffness matrix

Solve [A]{x}={b} where [A] is a (numnp x numnp) symmetric matrix

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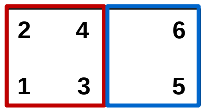
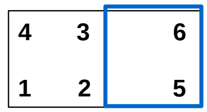
Consider the following element numbering schemes. Node numbering affects the size of the stiffness matrix.





	1	2	3	4	5	6
1	X	X	X	X		
2		X	X	X		
3			X	X	X	X
4				X	X	X
5					X	X
6						X

	1	2	3	4	5	6
1	X	X	X	X		
2		X	X	X	X	X
3			X	X	X	X
4				X	0	0
5					X	X
6						X

Accounting for embedded zeros is not desirable. It increases memory requirements.

Stiffness matrix

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	1	2	3	4	5	6	7	8	9	10
1	x	x							x	x
2		x	x		x	x			x	0
3			x	x	x	x			0	0
4				x	0	0	x		0	0
5					x	0	x		x	0
6						x	0	x	x	0
7							x	x	0	0
8								x	0	0
9									x	x
10										x

Obsolete solvers

- **symmetric Gauss solver needs upper triangle = 55 entries**
- **solver 1 (skyline Gauss) needs values + embedded 0's = 42 entries**

Recommended solvers to use:

- **solver 11 (sparse Gauss) needs only the non-zeros → 27 entries**
- **solver 3 & 12 (conjugate gradient) needs only the non-zeros → 27 entries**

Recommended solvers to use

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Conjugate gradient → faster

Faster but can have convergence problems

Gauss → slower

Slower but NO convergence issues

Check out each method for production runs and pick the best for that particular problem

CG non-convergence

Always check your results for convergence when using an iterative solver

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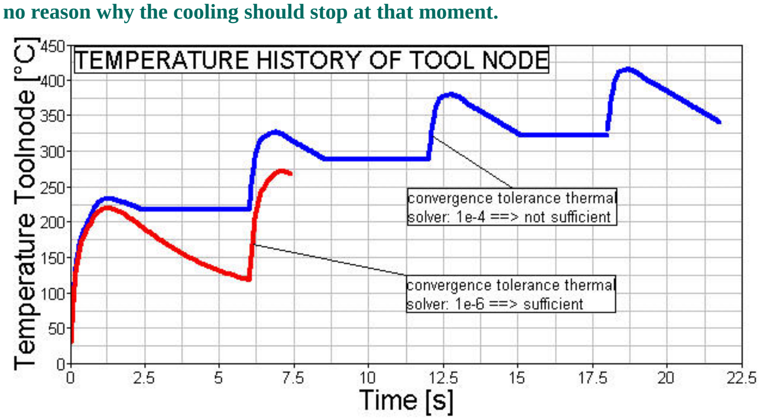
I have a coupled thermomechanical simulation of a b-pillar. I'm simulating several sheets after each other and looking at the heating up of the tools. The forming process lasts for 0.6 seconds and afterwards I keep the tools closed for continuous cooling. The tools heat up, then start to cool down until the cooling stops (blue curve) because of some unknown reason. I can absolutely find no reason why the cooling should stop at that moment.



CG non-convergence

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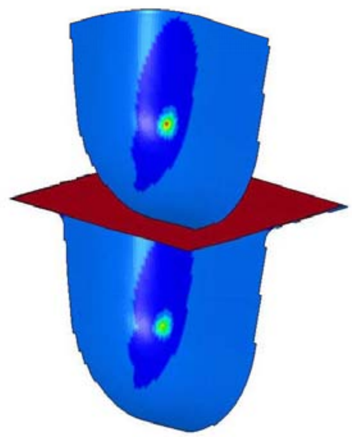
Note the hot spot. The temperature should be 303.1 There seems to be a bug in the code. My guess is the contact algorithm.

Tolerance	Temperature
E-04	304.8
E-06	303.0
E-08	303.1



Chapter 3 - 7

CG non-convergence

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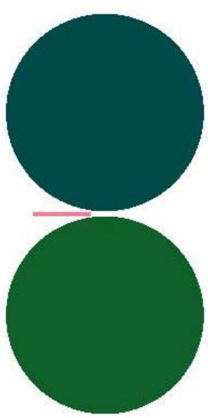
There are plastic strains but no change in temperature when using SOLVER=3. When switching to SOLVER=1 it works, but for a large model this is VERY slow.



tol = 1.e-04

tol = 1.e-08

Chapter 3 - 8

Pre-conditioned conjugate gradient

e.g., diagonal, Choleski, etc.

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solve $Ax = b$ by PCG $B^{-1}Ax = B^{-1}b$

Pre-conditioned Conjugate Gradient is a numerical method to solve Ax=b. We do not want to take the inverse of A because that procedure is very cpu expensive. We multiply both sides of Ax=b by another matrix B^{-1}. B^{-1} is called the pre-conditioning matrix. If $B^{-1} = A^{-1}$, then we get the solution in 1 iterative step but this means that we have actually calculated A inverse which is what we don't want to do. So, the trick is to get B^{-1} to be "close" to A^{-1} but much easier to calculate. A measure on how "close" is the condition number. Diagonal preconditioning is extremely easy. We set B^{-1} to 1/diag(A).

The condition number is $cond(B^{-1}A) = \|B^{-1}A\| \|B^{-1}A\|$

For thermal problems, cond is between 10 - 1000. Pre-conditioned conjugate gradient works well. However, the condition number for mechanical problems is >10**6. This is why CG does not work well for the mechanical problem.

Finding a good pre-conditioner is an active research area. The problem is that the pre-conditioners are problem dependent. It may work well on one problem and completely fail on another. LSTC has 10 problems we give to developers of pre-conditioners for testing their methods.

Pre-conditioned conjugate gradient

General iterative method algorithm

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solve $Ax = b$

Pre-conditioning $B^{-1}Ax = B^{-1}b$

iteration process $x^{i+1} = x^i + B^{-1}\underbrace{\left(b - Ax^i\right)}_{\text{residual – if 0, then we have converged}}$

```
*CONTROL_THERMAL_SOLVER
atype       ptype       solver    tol
message     max_iter    abstol    reltol
```

Optional input line when solver ≥ 12

Solver diagnostic information printed to screen or messg file.
Default=0, no info

Convergence criteria

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Solvers 3, 4, 5
Convergence reached if

$$\|residual\|_2 < tol * \|residual_{initial}\|_2$$

Solvers 12 - 15
Convergence reached if

$$\|residual\|_2 < abstol + reltol * \|residual_{initial}\|_2$$

The use of abstol in solvers 12-15

a) **If initial T=0 and no BCs, then initial residual=0. The use of abstol will account for this and satisfy convergence.**
b) **The initial residual can be small when using a very small time step. Then tol*residual is very very small. The use of abstol will account for this and satisfy convergence.**

Which method should I use

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DSCG requires about 35% less storage than ICCG.

ICCG takes fewer iterations than DSCG, but overall may be more time consuming due to the pre-conditioning cost.

ICCG seems best for steady state problems.

DSCG seems best for transient problems.

Gauss - if CG fails, then there is something wrong with the model. You can usually obtain a solution using Gauss to trouble shoot the input file.

Check out each method for production runs and pick the best for that particular problem

Chapter 4 – Time step

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CONTROL_THERMAL_TIMESTEP

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CONTROL_THERMAL_TIMESTEP

TS	eq.0 eq.1	fixed time step variable time step
TIP	eq.0 eq.1	default set to 0.5 for Crank-Nicolson full implicit
ITS		initial thermal time step (default = 0.01 thermal response time)
TMIN		minimum thermal time step (default = 0.01 thermal response time)
TMAX		maximum thermal time step (default = 100 * thermal response time)
DTEMP		maximum T change in a time step above which the time step will be reduced (default = 1.)
TSCP		the time step is decreased by this factor if DTEMP is exceeded (default = 0.5)
LCIDTS		Designates a load curve number which defines data pairs of (time breakpoint, new Δt). The time step will be adjusted to hit the time breakpoint exactly.

Response time

$$\Delta t = {l^2}/{\alpha}$$

$$\alpha = {k}/{\rho c}$$

Fourier heat equation

Background theory

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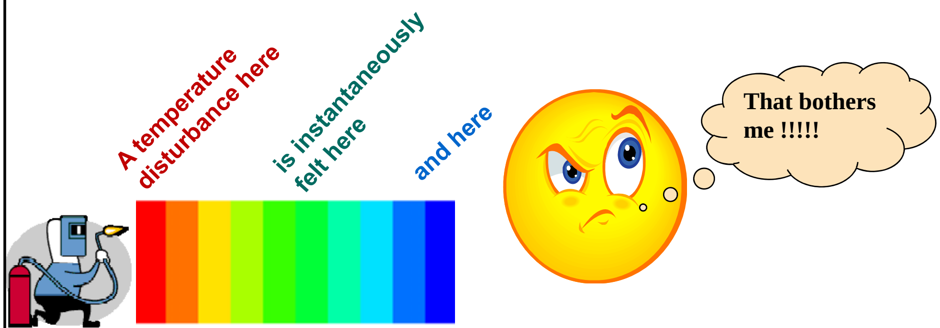
The solution of $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$ is such that the temperature change at every location is instantaneous. Note that there isn't a thermal acceleration term.



Hyperbolic heat equation

The thermal wave speed can be ignored for most applications.

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The **hyperbolic** form models the heat source disturbance as traveling with a finite speed of propagation through the medium.

metal: α = 1*10^{-5} m^2/sec

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T - \tau \frac{\partial^2 T}{\partial t^2}$$

	metals
τ [sec] at 298K	1*10^{-12}
τ [sec] at 50K	1*10^{-6}

Needed when modeling:

- cryogenic temperatures
- pico-second time scales
- nano-meter length scales

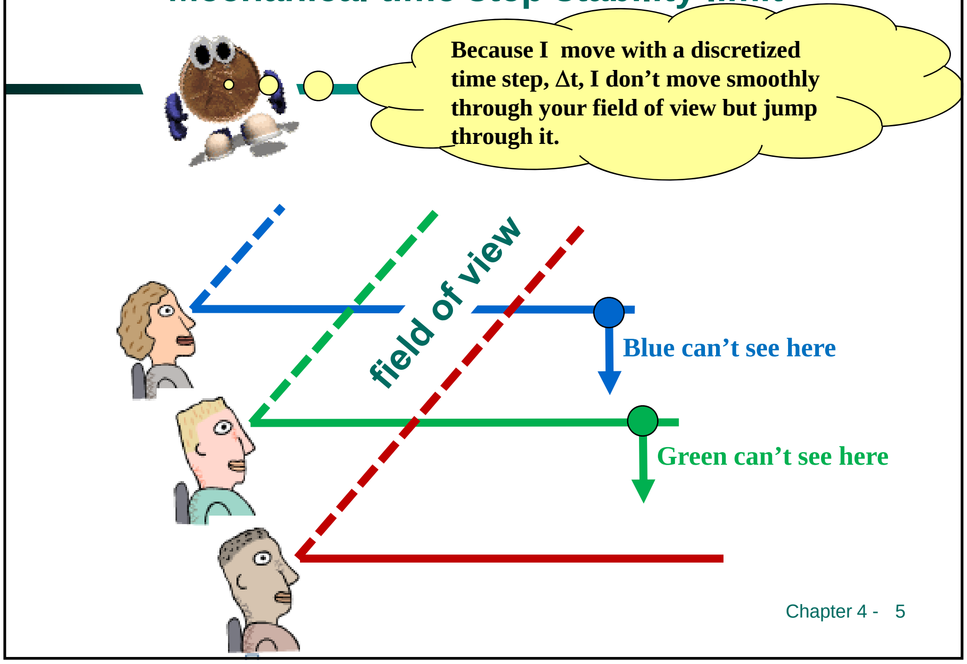
Time step criteria:

$$\Delta t \le \frac{l}{\sqrt{\alpha/\tau}} = \frac{l}{c_{thermal}} \approx \frac{l}{3200\ m/\sec}$$

speed of sound = 5400 m/sec

A. Vedavarz, "Significance on Non-Fourier Heat Waves in Microscale Conduction DSC-Vol. 32, Micromechanical Sensors, Actuators and Systems, ASME 1991.

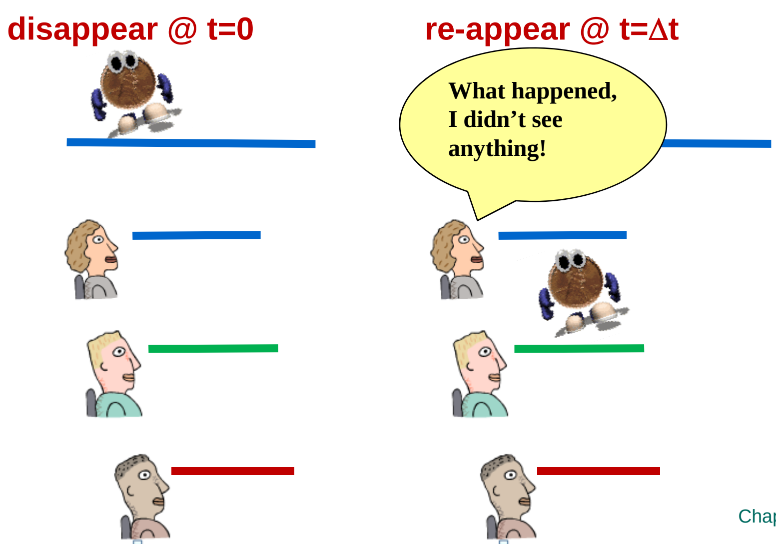
Mechanical time step stability limit



Chapter 4 - 5

Mechanical time step stability limit

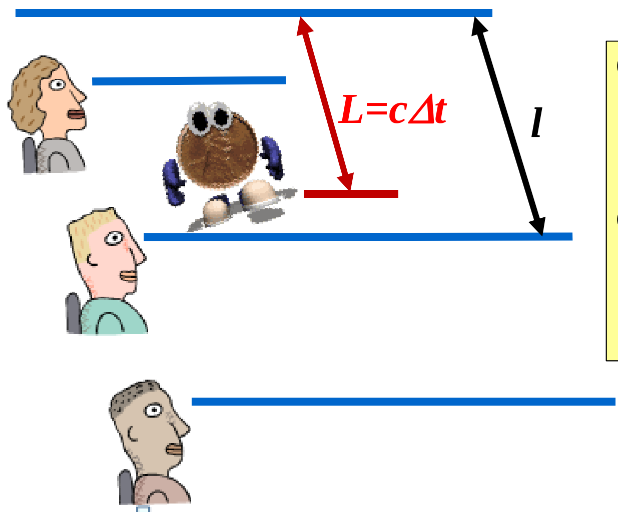
LSTC



Chapter 4 - 6

Mechanical time step stability limit

LSTC



Can see me, if

$$l > c\Delta t$$

Or

$$\Delta t < \frac{l}{c}$$

Mechanical time step stability limit

Explicit mechanical time step

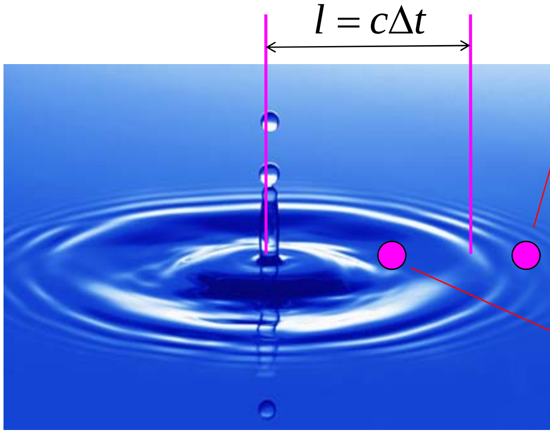
LSTC

The physics governing the transport operation (i.e., momentum, energy) in the model must respond like real life. The problem is that our model is discretized. Time and position are not continuous. We have discrete sampling locations (i.e., the nodes) at discrete time intervals (i.e., time step). We have to have the time resolved location of the disturbance peaks less than a nodal distance. Otherwise, the node doesn't see the disturbance

Mechanical time step stability limit

A snapshot at t=Δt shows the location of the disturbance peaks

LSTC



Nodes at $l > c\Delta t$
Have several disturbance peaks to respond to. Time step criteria:

$$\Delta t \le l/c$$

Nodes at $l < c\Delta t$
do not feel the disturbance. The peak and ripples have jumped over the sampling point (i.e., node). This leads to an unstable calculation.

Thermal time step vs. Mechanical time step

for aluminum

LSTC

explicit mechanical time step (shell element)

$$\Delta t \le \frac{l}{c} = \frac{l}{\sqrt{\frac{E}{\rho(1-\upsilon^2)}}} = \frac{0.005}{\sqrt{\frac{70*10^9}{2700(1-0.3^2)}}} \approx 1.*10^{-6}$$

thermal time step

$$\Delta t \le \frac{l^2}{\alpha} = \frac{l^2}{\frac{k}{\rho c}} = \frac{0.005^2}{\frac{(220)}{(2700)(900)}} \approx 3.*10^{-1}$$

Suggestions

1. Take 10 to 100 explicit mechanical time steps per thermal time step
2. Make sure the thermal time step is small enough to capture the mechanical motion

The numerical model must respond as fast as real life (e.g., heating due to rapid deformation can be faster than heat conduction

Thermal time step vs. Mechanical time step

LSTC

Δt mechanical = 0.01
Δt thermal = 0.01

The heat flux from the welding torch smoothly moves across the surface.

Δt mechanical = 0.01
Δt thermal = 1.

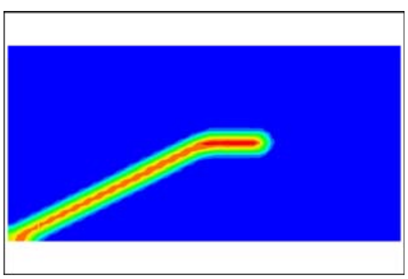
The heat flux is jumping elements.

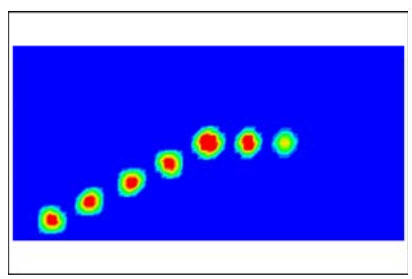
Chapter 4 - 11

Thermal time step vs. Mechanical time step

The thermal time step must be small enough to capture the mechanical motion

LSTC





Δt mechanical = 0.01
Δt thermal = 0.01

The heat flux from the welding torch smoothly moves across the surface.

Δt mechanical = 0.01
Δt thermal = 1.

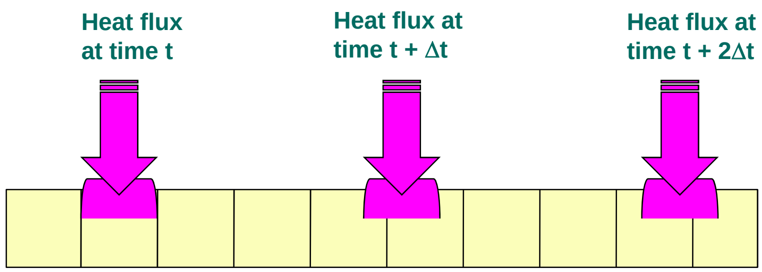
The heat flux is jumping elements.

Chapter 4 - 12

Thermal time step vs. Mechanical time step

The rate of mechanical motion must be considered in selecting an appropriate time step.

LSTC



In the numerical model the heat flux is jumping elements. This produces a discontinuous (i.e., ratcheting) temperature response. In reality, the flux from the welding torch smoothly moves across the surface.

Solution – decrease the time step so the flux has a resident time on each element.

Explicit vs. implicit time integration

thermal code

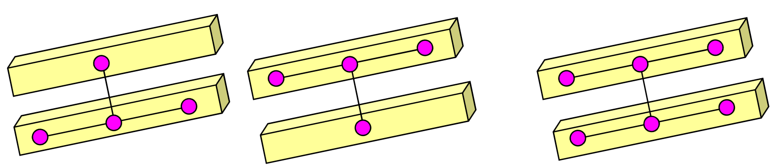
LSTC

$$\int_\Omega w_j \rho c \frac{\partial T}{\partial t} d\Omega = -\int_\Omega \nabla^T w_j k \nabla T d\Omega + \int_\Omega w_j Q d\Omega + \int_\Gamma w_j q d\Gamma$$

Node point time integration stencil

time t+Δt

time t



explicit
- easy formulation
- time convergence O(Δt)
- time step stability limit

implicit
- system of simultaneous equations -matrix solution
- time convergence O(Δt)
- unconditionally stable

Crank-Nicolson
- matrix solution
- time convergence O(Δt^2)
- unconditionally stable, but may oscillate

What is O(Δt), O(Δt²), O(Δx²)

LSTC

O(Δt), O(Δt^2), and O(Δx^2) refer to the **convergence rate** of the numerical solution.

- **O(Δt)** means that as the time step is halved, the error decreases by **1/2**
- **O(Δt^2)** means that as the time step is halved, the error decreases by **1/4**

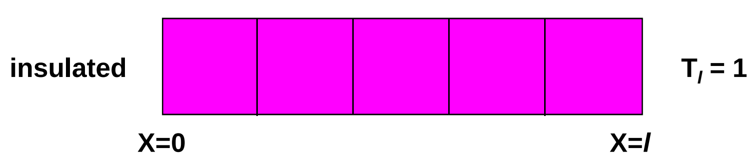
A **mathematical formulation** of convergence rate is presented in section 8.2.3 of T.J.R. Hughes, The Finite Element Method, Prentice Hall, 1987.

What is O(Δt), O(Δt²), O(Δx²)

A numerical experiment (converge_time.k)

LSTC

The region $0<x<l$ with zero initial temperature, with the surface x=0 insulated and the surface x = l kept at constant temperature T_l for t > 0.



Solve for T @ time = 2 on the insulated surface

What is O(Δt), O(Δt²), O(Δx²)

A numerical experiment (converge_time.k)

LSTC

Crank-Nicolson	
Δt	T
0.100	?
0.050	?
0.025	?
0.001	0.99054

? ? ?

As the time step is halved, the error decreases by? ________

implicit	
Δt	T
0.100	?
0.050	?
0.025	?
0.001	0.99049

? ? ?

As the time step is halved, the error decreases by? ________

What is O(Δt), O(Δt²), O(Δx²)

A numerical experiment (converge_time.k)

LSTC

implicit	
Δt	T
0.100	0.984155
0.050	0.987525
0.025	0.989080
0.001	0.990487

141 296 634

linear convergence

As Δt is halved, the error decreases by 1/2

$*10^{-5}$

Crank-Nicolson	
Δt	T
0.100	0.990769
0.050	0.990601
0.025	0.990558
0.001	0.990544

1.4 5.7 22.5

quadratic convergence

As Δt is halved, the error decreases by 1/4

Crank-Nicolson oscillations

Workshop problem crank_oscillations.k

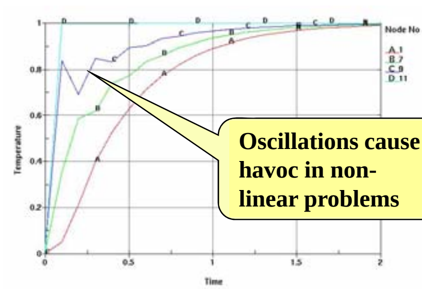
LSTC

From previous slide using a time step of $\Delta t = 0.1$

- Implicit error = 634.e-05
- Crank Nicolson error = 22.5e-05

Crank Nicolson is almost 30 times more accurate for this problem.

Why don't we use Crank Nicolson all the time → **OSCILLATIONS**



Oscillations cause havoc in non-linear problems

We eliminate the oscillations by using a smaller time step

Variable time step algorithm

LSTC

```
*CONTROL_THERMAL_TIMESTEP
$     TS     TIP     ITS     TMIN     TMAX    DTEMP     TSCP
                                   default    1.        0.5
```

Response time

$$\Delta t = l^2 / \alpha$$

$$\alpha = k / \rho c$$

TMIN	minimum thermal time step (default = 0.01 thermal response time)
TMAX	maximum thermal time step (default = 100 * thermal response time)
DTEMP	maximum T change in a time step above which the time step will be reduced (default = 1.)
TSCP	the time step is decreased by this factor if DTEMP is exceeded (default = 0.5)

Variable time step algorithm

LSTC

```
*CONTROL_THERMAL_TIMESTEP
$     TS     TIP     ITS    TMIN     TMAX   DTEMP     TSCP
                                 default    1.        0.5
```

Think of dtemp as a set point on a thermostat

Calculate ΔT_{big} → the largest temperature change of any node in the mesh

- If ΔT_{big} >> dtemp, decrease time step and retake $\Delta t_{new} = tscp * \Delta t_{old}$
- If ΔT_{big} ~ dtemp, then slowly increase or decrease time step
- If ΔT_{big} << dtemp, increase time step and continue $\Delta t_{new} = \Delta t_{old} / tscp$
- The rate of decrease/increase is controlled by tscp

Subject to $t_{\min} \leq \Delta t \leq t_{\max}$

Chapter 4 - 21

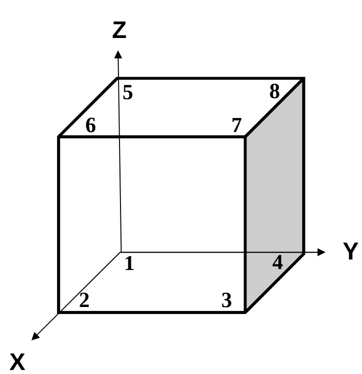
bcflux.k – investigate thermal time step parameters

Cube with flux boundary condition

LSTC

Problem description

A cube initially at 0C, is heated by a heat flux on all 6 faces.
What is the rate of temperature increase?



cube	edge = 1.0 m
density	ρ = 1 kg/m^3
heat capacity	c = 1 J/kg C
conductivity	k = 1 W/m K
flux	q = 1 W/m^2
initial temperature	T_0 = 0 C

Chapter 4 - 22

Time scaling

TSF → set "**thermal speedup factor**" on CONTROL_THERMAL_SOLVER keyword

LSTC

Punch velocities are artificially increased.

Deformation
1000 times faster

Baseline

time	displacement
0.	0.
1.6	-0.00792

$$\frac{t}{\Delta t} = \frac{1.6}{1.e-07} = 1.6e+07 cycles$$

Time scaled

time	displacement
0.	0.
0.0016	-0.00792

$$\frac{t}{\Delta t} = \frac{0.0016}{1.e-07} = 1.6e+04 cycles$$

"Thermal velocity" terms (i.e., those with units of W = J/s) must be scaled by the same ratio as the punch velocity. Thermal velocity terms include

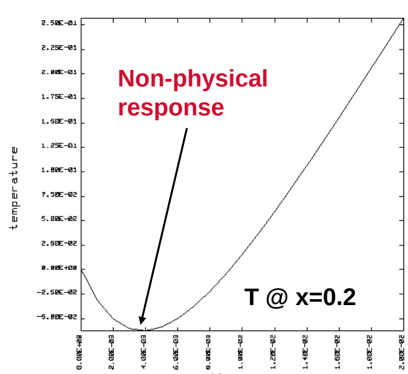
- Thermal conductivity
- Convection heat transfer coefficients
- Contact heat transfer coefficients
- Surface heat flux

Consistent vs. lumped mass matrix

Problem con_lump.k

LSTC

The numerical recipe given by the Galerkin finite element formulation for the mass matrix is termed "consistent mass". However, small time steps (instead of improving accuracy) may lead to physically unrealistic results. An alternative is constructing a "lumped" mass matrix.



Problem: 1-D heat transfer in a slab
ρ = k = c = 1, Δt = 0.001
initial condition: T=0
Boundary condition @ x=0: flux=10
Boundary condition @ x=1: insulated

940 – option
950 – lumped only
960 – lumped only
970 – option (default lumped)
971 – option (default lumped)

*CONTROL_THERMAL_SOLVER	
atype = 1	lumped
atype = 2	consistent

Conclusion – time step selection

LSTC

Rule-of-Thumb

The mathematical model must respond as fast as real life.

$$\Delta t \leq \frac{(\Delta l)^2}{\alpha}$$

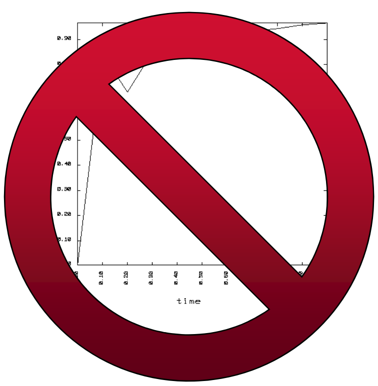
But don’t forget to consider the forcing frequency of boundary conditions

Chapter 4 - 25

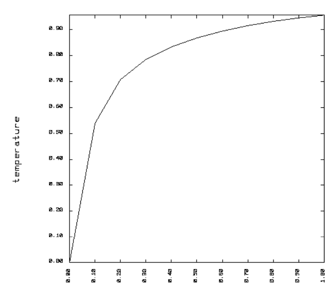
Conclusion – implicit vs. Crank-Nicolson

LSTC

Crank-Nicolson α = 0.5 has the wiggles



fully implicit α = 1

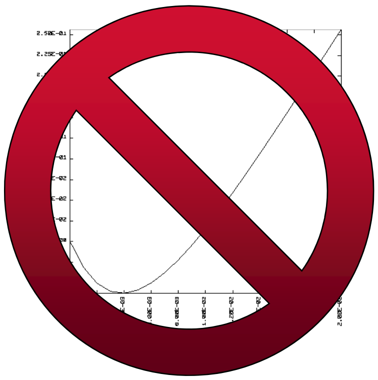


Chapter 4 - 26

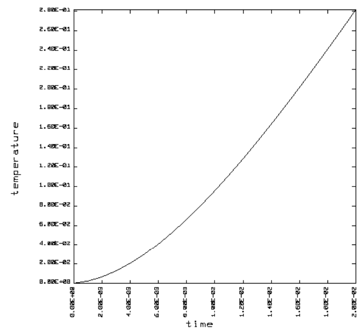
Conclusion – consistent vs. lumped

LSTC

consistent mass matrix



lumped mass matrix



LSTC

Chapter 5 – nonlinear problems

LSTC

*CONTROL_THERMAL_SOLVER

LSTC

```
*CONTROL_THERMAL_SOLVER
atype      pytpe      solver      cgtol
```

0 = linear

1 = nonlinear, properties evaluated at Gauss point T

2 = nonlinear, properties evaluated at element average T

The problem is nonlinear when any heat transfer parameter is a function of temperature. Then, iterations are needed to obtain the correct solution.

Note: It is more accurate, but takes more CPU time to evaluate properties at the Gauss point temperature. In 3D, 8 computations are required versus only 1 for element average.

For most problems, c and k are weak functions of temperature and element average is OK

*CONTROL_THERMAL_NONLINEAR

This KEYWORD is optional. Default values will be used if omitted when ptype>0.

LSTC

```
*CONTROL_THERMAL_NONLINEAR
Refmax   tol   dcp   lumpbc    thlstl   print   phcp
```

refmax = maximum number of iterations per time step (default 10)

tol = temperature convergence tolerance (default 1.e-04)

dcp = divergence control parameter (default 0.5)

lumpbc = lump boundary condition stiffness (not recommended)

thlstl = line search convergence tolerance (default 0, no line search)

print = print nonlinear convergence parameters

phcp = phase change penalty parameter (default 100.) [note: used only if HLAT & TLAT are defined on thermal material card]

Divergence control parameter (DCP)

LSTC

In a transient problem when using a variable time step, dcp is used to reduce the time step if the solution is diverging. (default = 0.5)

$$\Delta t_{new} = dcp * \Delta t_{old}$$

In very severe cases you may want to specify dcp = 0.1

TSCP (time step control parameter) modifies the time step based on the change in temperature.

0.5 suggested. Halve the time step for dT/dt out of range.

$$\Delta t_{new} = tscp * \Delta t_{old}$$

Nonlinear solution method

LSTC

Temperature and time dependent FEM equation

$$[C_{n+\alpha}]\{\dot{T}_{n+\alpha}\}+[K_{n+\alpha}]\{T_{n+\alpha}\}=\{F_{n+\alpha}\}\qquad \begin{cases} C_{n+\alpha}=C(T_{n+\alpha}\ ,t_{n+\alpha}) \\ K_{n+\alpha}=K(T_{n+\alpha}\ ,t_{n+\alpha}) \\ F_{n+\alpha}=F(T_{n+\alpha}\ ,t_{n+\alpha}) \end{cases}$$

***n* = time step counter**

Let

$$G(T)=[C(T)]\{\dot{T}\}+[K(T)]\{T\}-\{F(T)\}=0$$

Newton's method

$$G(T^{i+1})=G(T^{i})+\frac{dG}{dT}dT=0$$

***i* = nonlinear iteration counter**

Solve

$$\left[\frac{dG}{dT}\right]^{i}\{dT\}=-G(T^{i})$$

Tangent stiffness matrix

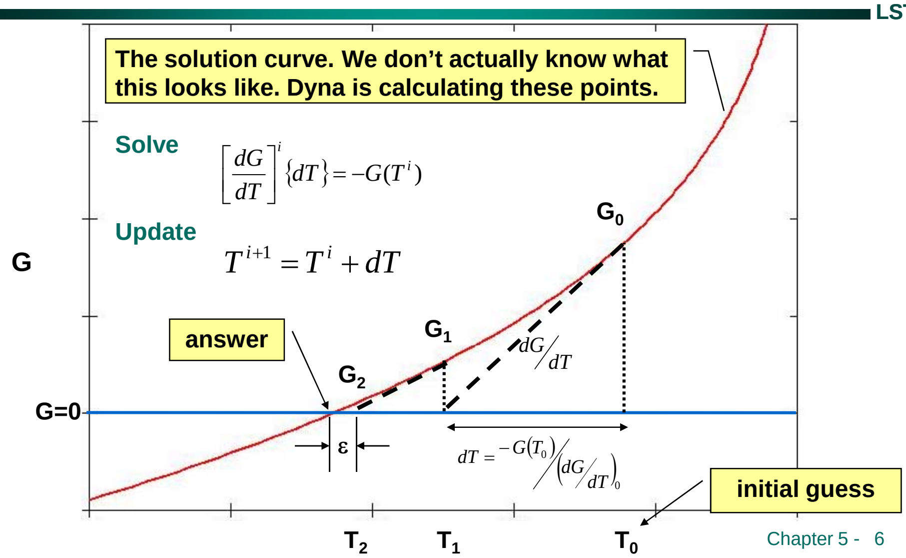
Update

$$T^{i+1}=T^{i}+dT$$

Nonlinear solution method

$$G(T)=[C(T)]\{\dot{T}\}+[K(T)]\{T\}-\{F(T)\}$$

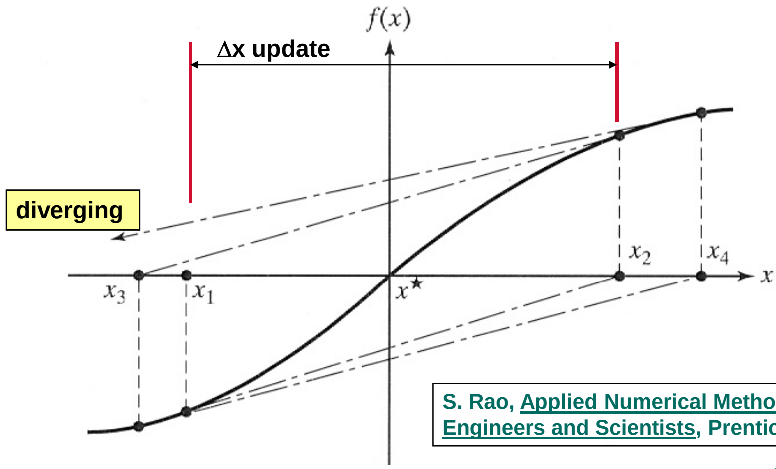
LSTC



Nonlinear solution method

Newton's method may not converge if the initial guess is far from the exact root.

LSTC

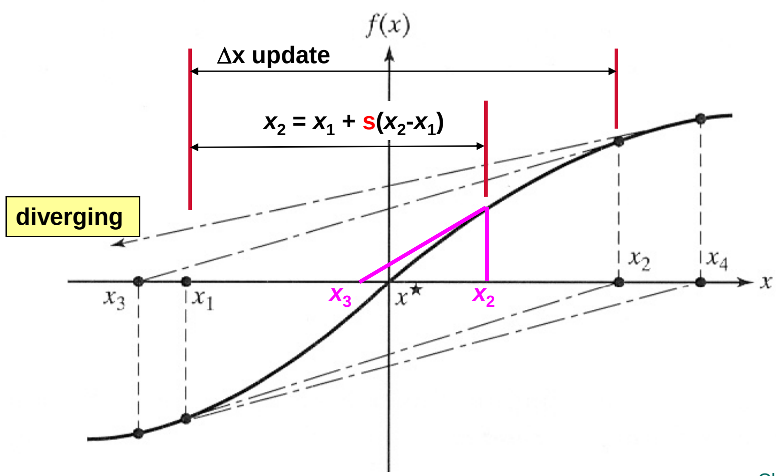


S. Rao, Applied Numerical Methods for Engineers and Scientists, Prentice Hall, 2002.

Nonlinear solution method

Upating by a fraction of the step size will help the problem converge. But, how do you chose a value for **s**.

LSTC



Nonlinear solution method

The optimum value for **s** is found by a "line search" optimization method

LSTC

Newton nonlinear iteration formula

$$\{dT\} = -\left[\frac{dG^i}{dT}\right]^{-1}\{G^i\}$$

Inverting the matrix [*dG/dT*] is expensive. Do this 1 time to get a search direction.

Update formula

$$T_2 = T_1 + sdT$$

residual

$$G(T) = [C(T)]\{\dot{T}\} + [K(T)]\{T\} - \{F(T)\}$$

Search (iterate) to find s which minimizes out of balance energy

thlstl = line search minimization tolerance
eq. 0, no line search (default)
eq. 0.9, otherwise

T. Belytschko, Nonlinear Finite Elements for Continua and Structures, Wiley, p333.
M.A. Crisfield, Nonlinear FEA of Solids and Structures, Wiley, p254.

Material nonlinearity

LSTC

```
*MAT_THERMAL_ISOTROPIC_TD_LC
      tmid       rho
      hclc      tclc

tmid → thermal material ID
rho  → material density
hclc → load curve for Cp vs. T
tclc → load curve for k vs. T
```

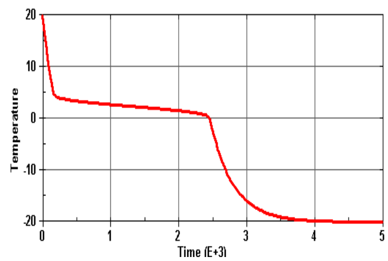
Material nonlinearity

There are 2 phase change algorithms

LSTC

1

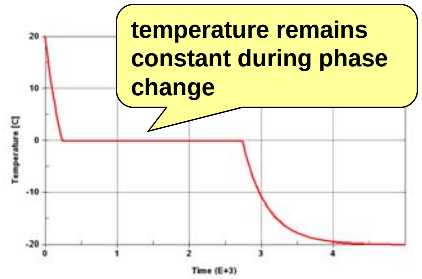
Heat capacity method



• use for alloys with a phase change temperature range

2

Energy balance method



• use for pure materials with a distinct phase change temperature

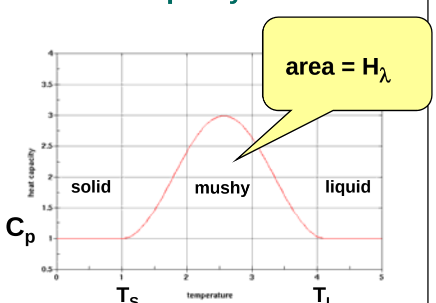
Material nonlinearity

1. Heat capacity phase change method

LSTC

1

Heat capacity method



Algorithm

1. Modify the Cp vs. T curve such that the area under the spike equals H_λ.
2. LS-DYNA fits a cosine curve to the data T_S, T_L, and H_λ defined using the keyword *MAT_THERMAL_ISOTROPIC_PHASE_CHANGE
3. Run just like any other nonlinear problem. Define line search convergence parameter thlstl.

```
*CONTROL_THERMAL_NONLINEAR
refmax  tol  dcp  lumpbc  thlstl  print  phcp
```

Material nonlinearity

2. Energy balance phase change method

LSTC

2

Energy balance method

use

*MAT_THERMAL_ISOTROPIC

solidification $T_{ele}^{t+\Delta t} < T_\lambda \le T_{ele}^{t}$

melting $T_{ele}^{t} \le T_\lambda < T_{ele}^{t+\Delta t}$

Algorithm

1. Check if the element average temperature has passed through phase change
2. If it has, then hold the temperature to T_λ , and add or subtract latent heat
3. Don't let the element change temperature until all H_λ is accounted for. Penalty factor phcp controls this.

```
*CONTROL_THERMAL_NONLINEAR
refmax  tol  dcp  lumpbc  thlstl  print  phcp
```


Material nonlinearity

material input for water

LSTC

1. Heat capacity method

```
*MAT_THERMAL_ISOTROPIC_PHASE_CHANGE
$     TMID       TRO
         1    1000.0
$       T1        T2        T3        T4 . . . .
    -100.0     100.0
$      HC1       HC2       HC3       HC4 . . . .
    2000.0    2000.0
$      TC1       TC2       TC3       TC4 . . . .
       1.0       1.0
$     SOLT      LIQT      HLAT
       0.0       2.0  3.00E+05
```

2. Energy method

```
*MAT_THERMAL_ISOTROPIC
$     TMID       TRO     TGRLC    TGMULT      TLAT      HLAT
         1    1000.0         0        0.        0.  3.00E+05
$       HC        TC
     2000.        1.
```

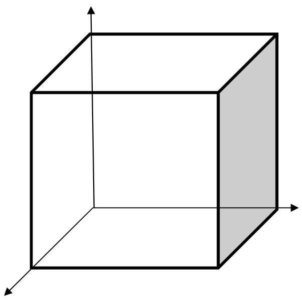
phase1.k & phase2.k – nonlinear thermal material

Workshop problem

LSTC

Problem description

A cube of water initially at 20C, is cooled by convection on all 6 faces. What is the length of time required to turn the water into ice?



Ice cube	edge = 0.1 m
density	ρ = 1000 kg/m^3
heat capacity	c = 2000 J/kg C
conductivity	k = 1 W/m K
latent heat	H_λ = 300,000 J/kg
phase temp.	T_λ = 0 C
Convection coef.	h = 100 W/m^2
environment T	T = -20 C
initial temperature	T_0 = 20 C

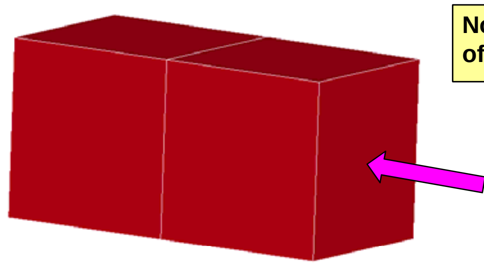
radiation.k – nonlinear radiation boundary condition

Workshop problem

LSTC

Calculate the steady state temperature distribution in the block.

T = 300K



Nonlinear because of 4th power

$$\dot{q}'' = \sigma\varepsilon\left(T_s^4 - T_\infty^4\right)$$

T_∞ = 600K

σ = 5.67e-08 W/m^{2}K

Chapter 6 – Thermal Boundary Conditions

LSTC

Thermal boundary conditions

3D heat diffusion equation

LSTC

$$\rho c \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \dot{q}'''$$

Must have 1 initial condition

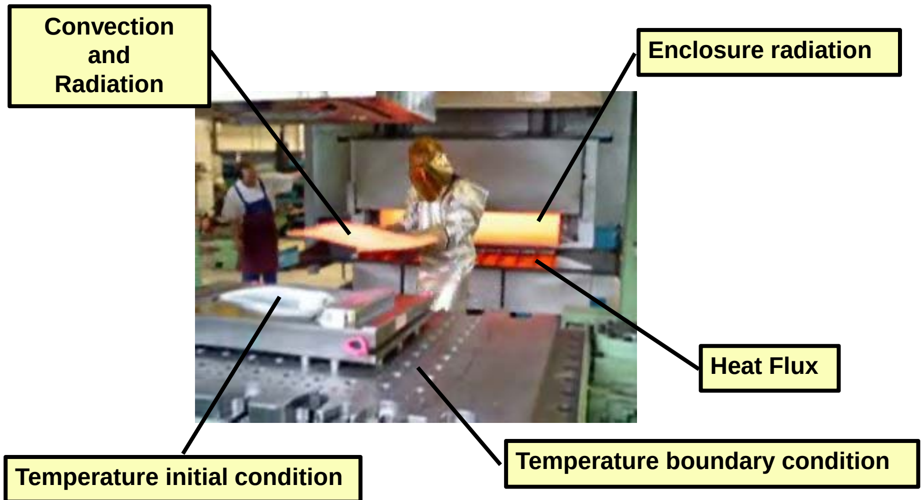
- **T = 0 (default)**

Must have 6 boundary conditions

- **insulated (default)**
- **temperature**
- **flux**
- **convection**
- **radiation**

Thermal boundary conditions

LSTC



Thermal boundary conditions

LSTC

1. *INITIAL_TEMPERATURE
2. *BOUNDARY_TEMPERATURE

applied to nodes

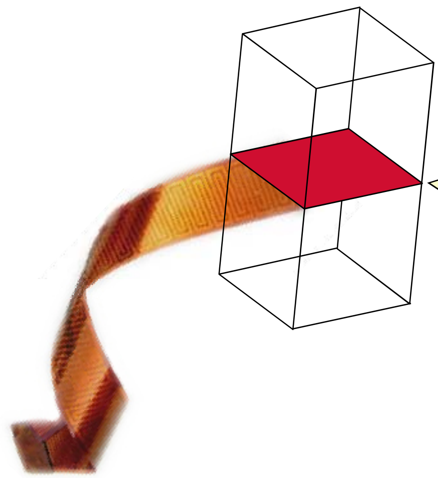
3. *BOUNDARY_CONVECTION
4. *BOUNDARY_FLUX
5. *BOUNDARY_RADIATION

applied to segments (function of surface area)

Thermal boundary conditions

Can be defined on interior segments

LSTC



A flux bc can be defined on this 4-node interior segment to model a thin film heater laminated between 2 plates. Modeling the assembly this way avoids very thin elements and small time steps in the mechanical computations.

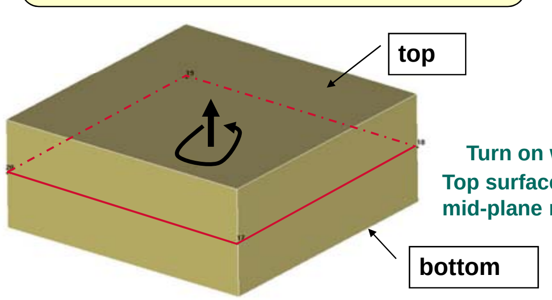
BOUNDARY THERMAL keyword

LSTC

LCID	MULT	LOC
load curve id	curve multiplier	shell surface

`gt.0:` function versus time
`eq.0:` use constant multiplier (MULT)
`lt.0:` function versus temperature

`eq.+1:` top
`eq. 0:` midplane
`eq.-1:` bottom



Thermal 12 node shell

Turn on with TSHELL=1 on *CONTROL_SHELL

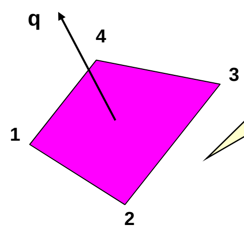
Top surface in direction of right hand rule using mid-plane nodes.

BOUNDARY THERMAL keyword

surface segment numbering is by right hand rule

LSTC

Sign convention
right hand rule



Surface heat flux is a vector having both magnitude and direction. Heat flux in the direction of the surface outward normal vector is positive. Therefore, energy into the body is negative.

INITIAL_TEMPERATURE_(option)

Option is NODE or SET

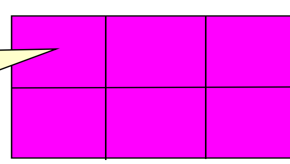
LSTC

```
*INITIAL_TEMPERATURE_option
    NSID/NID    TEMP    LOC
```

NSID=0: all nodes are included

Temperature at node or node set

T = 0 for all nodes unless defined differently by this keyword



Boundary condition temperatures are applied at the first time increment.

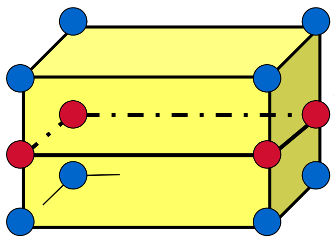
Suppose I have a block at room temperature. If the TBC is applied at t=0., then a temperature gradient exists across the surface elements and the block is no longer at room temperature at t=0. Instantaneous creation of thermal strains!

INITIAL_TEMPERATURE_(option)

Typical user error when using a thick thermal shell

LSTC

A thick thermal shell is used to model the blank so that a through thickness temperature gradient can be calculated



User specifies initial temperature on these nodes (e.g., T=800C)

User forgets to specify temperatures on the top (loc=+1) and bottom (loc=-1) surfaces. The default is T=0 at these nodes and the blank cools down very rapidly.

BOUNDARY_TEMPERATURE_(option)

Option is NODE or SET

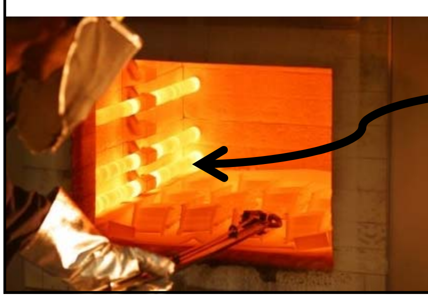
LSTC

```
*BOUNDARY_TEMPERATURE_(option)
   NSID/NID   LCID   CMULT   LOC
```

Node or node set

Load curve ID for temperature versus time. If LCID = 0, then T = CMULT.

Curve multiplier for temperature

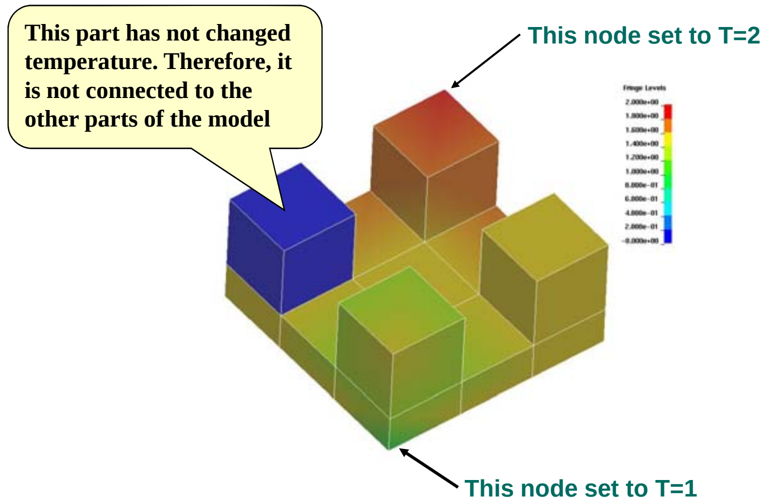


The heaters are modeled by specifying a temperature boundary condition.

BOUNDARY_TEMPERATURE_(option)

A steady state analysis with TBCs can be used to check part (mesh) connectivity.

LSTC



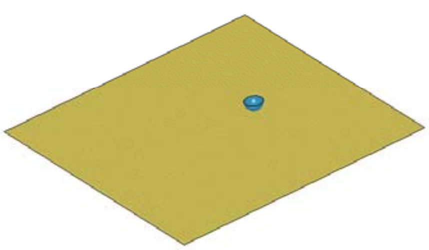
Chapter 6 - 11

BOUNDARY_FLUX_(option)

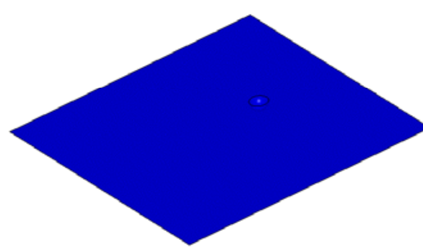
Application

LSTC

Laser assisted incremental sheet forming → the laser heats the material and softens it for forming. The energy from the laser is modeled using a flux boundary condition as a function of position and time.



deformation



temperature

Chapter 6 - 12

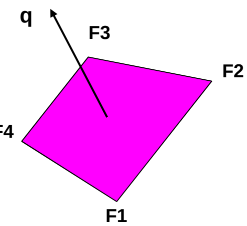
BOUNDARY_FLUX_(option)

Option is SEGMENT or SET

LSTC

Segment set ID

```
*BOUNDARY_FLUX_SET
SSID
LCID   F1   F2   F3   F4   LOC
```



Load curve ID
>0 function versus time
=0 use constant multiplier
<0 function versus temperature

Flux values at the node points which multiply the load curve value.

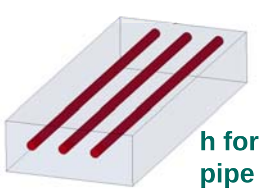
BOUNDARY_CONVECTION_(option)

The parameter with the greatest uncertainty is the convection coefficient, h

LSTC

h for vertical plates and cylinders

h for horizontal plates



h for turbulent pipe flow

h for water spray cooling

BOUNDARY_CONVECTION_(option)

Option is SEGMENT or SET

LSTC

Segment set ID

```
*BOUNDARY_CONVECTION_SET
SSID
HLCID    HMULT    TLCID    TMULT
```

Load curve ID
>0 function versus time
=0 use constant multiplier
<0 function versus temperature

h evaluated at

$$T_{film} = \frac{T_{surf} + T_\infty}{2}$$

h load curve ID and h multiplier

T_∞ load curve ID and T_∞ multiplier

$$\dot{q} = hA(T - T_\infty)$$

Convection heat transfer coefficient, h

Rules of thumb

LSTC

Situation	h [W/m^2 C]
free convection in air	5
forced convection in air	200
free convection in water	600
forced convection in water	1000
boiling in water	6000

Free convection

Forced convection

Convection heat transfer coefficient, h

How do you calculate h → use experimental curve fits

LSTC

Free Convection $N = C(G * P)^a$

Forced Convection $N = C(R * P)^a$

Convection heat transfer coefficient, h

We want to calculate h

$N = C(G*P)^a$ for **free** convection

$N = C(R*P)^a$ for **forced** convection

LSTC

Nusselt Number

$$N = \frac{hL}{k}$$

Prandtl Number

$$P = \frac{c_p \mu}{k}$$

Ratio of material properties

Reynolds Number

$$R = \frac{VL\rho}{\mu}$$

Grashof Number

$$G = \frac{\rho^2 g \beta (T - T_\infty) L^3}{\mu^2}$$

Velocity for forced convection

Buoyancy for free convection

Convection heat transfer coefficient, h

free convection over external surfaces (circa 1932)

LSTC

$$\frac{hL}{k} = C(GP)^a = C\left(\frac{\rho^2 g\beta\Delta TL^3}{\mu^2}\frac{c_p\mu}{k}\right)^a$$

geometry	laminar	turbulent	L
Vertical plate & pipe	GP < 10^9 C=0.59 a=1/4	GP > 10^9 C=0.13 a=1/3	height
Horizontal plate upper surface heated	GP < 10^7 C=0.54 a=1/4	GP>10^7 C=0.14 a=1/3	$\frac{2(length * width)}{length + width}$
Horizontal plate lower surface heated	GP < 10^{10} C=0.27 a=1/4		$\frac{2(length * width)}{length + width}$

h for vertical plates & cylinders

Calculate h for a woman of height 1.7m

LSTC

Properties for air at $T_{film} = \frac{T_{surf} + T_\infty}{2} = \frac{37+20}{2} = 29C$

ρ = 1.177 [kg/m^3]   k = 0.0258 [W/m C]

Cp = 1005 [J/kg C]   P = 0.72

μ = 1.846e-05 [kg/m sec] $g\beta\rho^2 / \mu^2$ = 1.42e+08 [1/C m^3]

$$G = \frac{g\beta\rho^2(T-T_\infty)L^3}{\mu^2} = (1.42e+08)(37-20)(1.7)^3 = 1.19e10$$

$$N = 0.13(GP)^{1/3} = 0.13[(1.19e10)(0.72)]^{1/3} = 266.$$

vertical cylinder

compare with rule of thumb h=5.

$$h = \frac{Nk}{l} = \frac{(266.)(0.0258)}{(1.7)} = 4.\ \text{W/m}^2\ \text{C}$$

h for vertical plates & cylinders

What about radiation?

LSTC

Let it be winter with a wall temperature of 20C (68F)

$$h_{rad} = \frac{\sigma \varepsilon F\left(T_1^4 - T_2^4\right)}{\left(T_1 - T_2\right)} = \frac{\left(5.67e-08\right)(1)(1)\left(310^4 - 293^4\right)}{(310-293)} = 6.22 \quad \text{W/m}^2\ \text{C}$$

This is a technique that gives a quick calculation to determine the importance of radiation

Note that the radiation transport coefficient is larger than the convection coefficient. More energy is transported by radiation to the surroundings than by convection to the air in the room. This is why you feel colder in the winter and warmer in the summer in a room with the same air temperature.

Surfaces do not have to be hot for radiation to be important. The critical parameter is the surface heat transfer coefficient.

Chapter 6 - 21

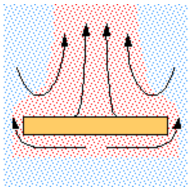
h for horizontal plates

Calculate h for a hot horizontal steel plate

LSTC

Convection currents are free to rise from top surface, h is larger

$$h_{top} = \left[0.14\left(Gr * \Pr\right)^{0.33}\right]\frac{k}{l}$$



Convection currents stagnate on bottom surface, h is smaller

$$h_{bot} = \left[0.27\left(Gr * \Pr\right)^{0.25}\right]\frac{k}{l}$$

Chapter 6 - 22

h for horizontal plates

Calculate h for a hot horizontal steel plate

LSTC

Slab temperature = 800C = 1073K
Room temperature = 20C = 293K

Film temperature = ½ (slab + room) = 410C = 683K

Properties for air at 410C

ρ = 0.5167 [kg/m^3]	k = 0.0515 [W/m C]
Cp = 1071 [J/kg C]	P = 0.68
μ = 3.283e-05 [kg/m sec]	$g\beta\rho^2 / \mu^2$ = 3.55e+06 [1/C m^3]

L=1m, W=0.3m

$$L = \frac{2(length * width)}{length + width} = \frac{2(1*0.3)}{1+0.3} = 0.46$$

h for horizontal plates

Calculate h for a hot horizontal steel plate

LSTC

$$G = \frac{g\beta\rho^2(T - T_\infty)L^3}{\mu^2} = (3.55*10^6)(800-20)(0.46)^3 = 2.7*10^8$$

$$G*P = (2.7*10^8)*(.68) = 1.8*10^8$$

turbulent on top surface
GP > 10^7

$$\frac{hL}{k} = 0.14(GP)^{1/3} = 0.14(1.8*10^8)^{1/3} = 79.$$

$$h = \frac{(79.)(0.0515)}{(0.46)} = 8.84 \text{ W/m}^2\text{ C}$$

Laminar on bottom surface
GP < 10^{10}

$$\frac{hL}{k} = 0.27(GP)^{1/4} = 0.27(1.8*10^8)^{1/4} = 31.$$

$$h = \frac{(31.)(0.0515)}{(0.46)} = 3.5 \text{ W/m}^2\text{ C}$$

Rule of thumb

h = 5 W/m² C

h for horizontal plates

What about radiation?

LSTC

$$\dot{q} = \sigma\varepsilon A\left(T_s^4 - T_\infty^4\right) = \sigma\varepsilon A \frac{\left(T_s^4 - T_\infty^4\right)}{\left(T_s - T_\infty\right)}\left(T_s - T_\infty\right) = h_r A\left(T_s - T_\infty\right)$$

$$h_{rad} = \frac{\sigma\varepsilon\left(T_1^4 - T_2^4\right)}{\left(T_1 - T_2\right)} = \frac{(5.67e-08)(.8)\left(1073^4 - 293^4\right)}{(1073-293)} = 76.7$$ W/m² C

This is a technique that gives a quick calculation to determine the importance of radiation

Radiation is almost 10x more important

h_{con} = 8.84

h_{rad} = 76.7

h for horizontal plates

h_{eff} → the effective heat transfer coefficient

LSTC

$$h_{eff} = h_{conv} + h_{rad} = 8.88 + 76.7 = 85.6$$

Method 1
- **define as linear**
- **use BOUNDARY_CONVECTION with h_{eff}**

fast
less accurate

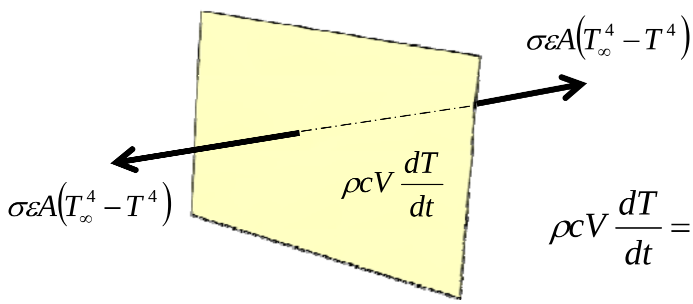
Method 2
- **define as nonlinear**
- **use BOUNDARY_CONVECTIONN with h_{conv}**
- **use BOUNDARY_RADIATION, code calculates h_{rad}**

slow
more accurate

Calculate cooling time for a hot slab

radiation lumped parameter model

LSTC



$$\rho c V \frac{dT}{dt} = 2\sigma\varepsilon A\left(T_\infty^4 - T^4\right)$$

The solution to this differential equation between the limits ($T=T_i$ @ t=0) and ($T=T_f$ at t), is

$$t = \frac{\rho c V}{2A\sigma\varepsilon}\left[\frac{1}{4T_\infty^3}\ln\frac{\left(T_f + T_\infty\right)/\left(T_f - T_\infty\right)}{\left(T_i + T_\infty\right)/\left(T_i - T_\infty\right)} + \frac{1}{2T_\infty^3}\left(\tan^{-1}\frac{T_f}{T_\infty} - \tan^{-1}\frac{T_i}{T_\infty}\right)\right]$$

Chapter 6 - 27

Calculate cooling time for a hot slab

Numisheet benchmark

LSTC

Numisheet 2008 Benchmark BM03 Process Steps

1. **Heating of the blank (1.95mm thick) to 940C**
2. **Transport from the oven into the tool (6.5 sec)**
3. **Temperature of the blank at the beginning of the die movement 810C**

$$T_\infty = 20C = 293K$$
$$T_i = 940C = 1213K$$
$$T_f = 810C = 1083K$$
$$\rho = 7870 \ \ kg/m^3$$
$$c = 650 \ \ W/kgK$$
$$\sigma = 5.67e-08 \ \ W/m^2K^4$$
$$\varepsilon = 1$$
$$A = lw = (1)(1) = 1$$
$$V = lwh = (1)(1)(0.00195) = 0.00195m$$

Using previous equation

Time = 6.68 sec

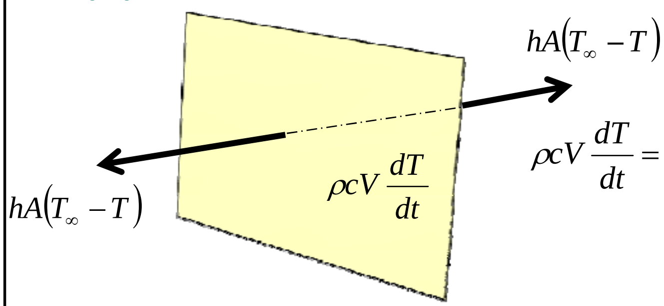
Chapter 6 - 28

Calculate cooling time for a hot slab

convection lumped parameter model

LSTC

Consider an object being heated from some uniform initial temperature, T_i. If the object is of high thermal conductivity, then its internal resistance can be ignored, and we can regard the heat transfer process as being controlled solely by surface convection.



$$\rho c V \frac{dT}{dt} = 2hA(T_\infty - T)$$

The solution is $$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\left(\frac{2hA}{\rho c V}\right)t}$$

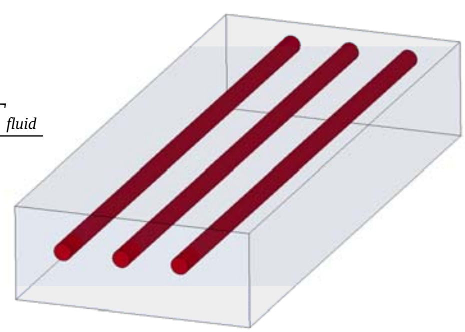
h for turbulent pipe flow

Turbulent forced convection in circular tubes

LSTC

For water $$\frac{hd}{k} = 0.023\left(\frac{V\rho d}{\mu}\right)^{0.8}\left(\frac{c_p \mu}{k}\right)^{0.4}$$

- **properties evaluated at** $T_{bulk} = \frac{T_{wall} + T_{fluid}}{2}$
- **Given mass flow rate** $V\rho = \frac{\dot{m}}{A_{pipe}}$



Knudson & Katz, Fluid Dynamics and Heat Transfer, McGraw Hill, Chapter 14,

h for turbulent pipe flow

Water properties

LSTC

T [C]	ρ [kg/m^3]	C_p [J/kg C]	μ [kg/m s]	k [W/m C]
20	998.	4182.	1.002e-03	0.603
40	992.	4179.	0.651e-03	0.632
60	983.	4185.	0.462e-03	0.653
80	972.	4197.	0.350e-03	0.670
100	958.	4216.	0.278e-03	0.681

h for turbulent pipe flow

Problem definition

LSTC

Pipe diameter = D = 15mm = 0.015 m

Pipe cross section area = A = $\pi D^2/4 = \pi(0.015)^2/4$ = 1.77e-04 m^2

Volumetric flow rate = G = 20 l/min = 0.02 m^3/min = 3.33e-04 m^3/sec

Flow velocity = G/A = 1.89 m/sec

Pipe wall temperature = T_{wall} =100C

Water temperature = T_{fluid} = 20C

h for turbulent pipe flow

Some preliminaries

LSTC

Fully developed – the effect of entrance conditions (e.g., pipe from a header) on h are negligible.

$$\frac{L}{D} > 40$$

Fluid properties are evaluated at the film temperature, T_{film}

$$T_{film} = \frac{T_{wall} + T_{fluid}}{2} = \frac{100 + 20}{2} = 60$$

Reynolds number

$$\mathrm{Re} = \frac{V\rho D}{\mu} = \frac{(1.89)(983)(0.015)}{0.462 * 10^{-3}} = 6.03 * 10^{4}$$

Prandtl number

$$\mathrm{Pr} = \frac{c_p \mu}{k} = \frac{(4185.)(0.462 * 10^{-3})}{0.653} = 2.96$$

Chapter 6 - 33

h for turbulent pipe flow

Classical empirical correlations

LSTC

Dittus-Boelter equation

$$h = 0.023 \frac{k}{D} \mathrm{Re}^{0.8} \mathrm{Pr}^{n}$$

n=0.3 for cooling of the fluid
n=0.4 for heating of the fluid

$$= 0.023 \frac{0.653}{0.015} \left(6.03 * 10^{4}\right)^{0.8} (2.96)^{0.4} = 10{,}300 \ \frac{W}{m^2 C}$$

What you do if the pipe is not perfectly smooth

Sieder-Tate equation

$$h = 0.023 \frac{k}{D} \mathrm{Re}^{0.8} \mathrm{Pr}^{n} \left(\frac{\mu_{bulk}}{\mu_{wall}}\right)^{0.14}$$

μ(T) correction factor

Chapter 6 - 34

h for turbulent pipe flow

Gnielinski correlation

LSTC

$$h = \left(\frac{k}{D}\right)\left[\frac{(f/8)(\mathrm{Re}-1000)\mathrm{Pr}}{1+12.7(f/8)^{0.5}\left(\mathrm{Pr}^{2/3}-1\right)}\right] = 11{,}400\ W/m^2C$$

***f* = Darcy–Weisbach friction factor (see next vu-graph for value)**

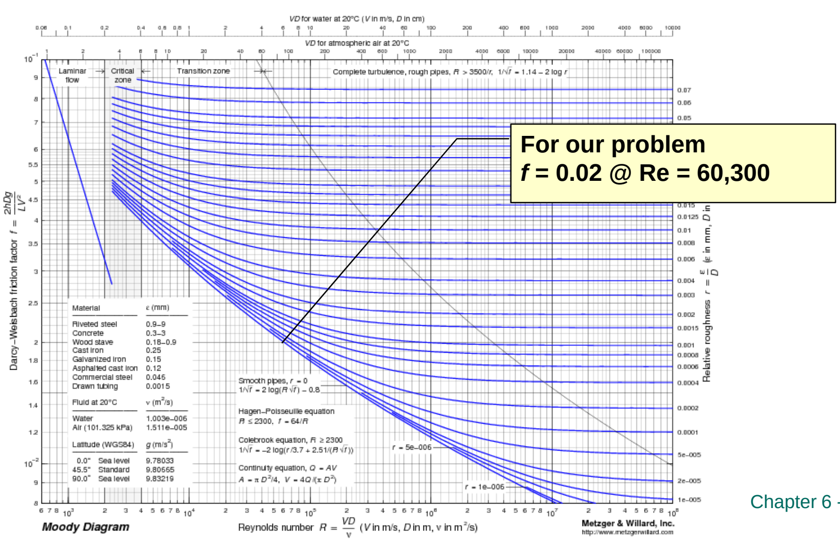
There are 2 definitions for *f*. The Darcy–Weisbach friction factor is 4 times larger than the Fanning friction factor, so attention must be paid to note which one of these is meant in any "friction factor" chart or equation being used. The Darcy–Weisbach factor is more commonly used by civil and mechanical engineers, and the Fanning factor by chemical engineers, but care should be taken to identify the correct factor regardless of the source of the chart or formula.

Chapter 6 - 35

h for turbulent pipe flow

Friction factor from: http://www.mathworks.com/matlabcentral/fx_files/7747/1/moody.png

LSTC



Chapter 6 - 36

h for water spray cooling

LSTC

$$h = 190 + \tanh\left(\frac{V_s}{8}\right) * \left\{140 V_s \left(1 - \frac{V_s \Delta T}{72000}\right) + 3.26 \Delta T^2 \left[1 - \tanh\left(\frac{\Delta T}{128}\right)\right]\right\}$$

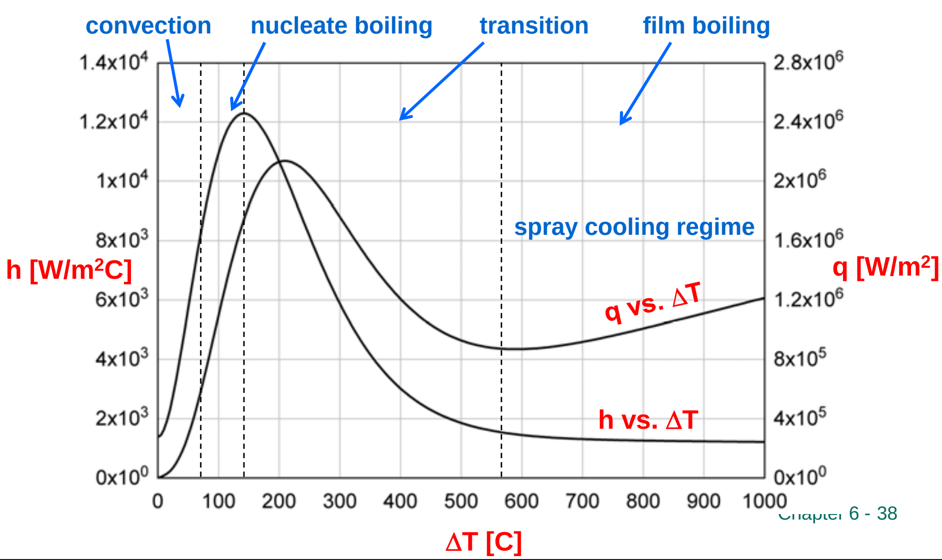
V_s = spray density [kg/m²s]

$\Delta T = T_{surface} - T_{liquid}$

J. Wendelstorf, "Spray Cooling heat transfer and Calculation of Water Impact Density for Cooling of Steel Sheet materials by Inverse Process Modeling", Steel Research International, V80, September, 2009, pp. 639-644.

h for water spray cooling

LSTC



DEFINE_FUNCTION keyword

See workshop problem rod_gr_pr_nu.k

LSTC

How do you enter $h = \left[0.14 \left(Gr * \Pr \right)^{0.33} \right] \frac{k}{l}$

1 Hand calculate h and enter it as a load curve using the **BOUNDARY CONVECTION** keyword

2 Use the **DEFINE FUNCTION** keyword and enter the equation

Chapter 6 - 39

DEFINE_FUNCTION keyword

inline FORTRAN syntax

LSTC

```
*BOUNDARY_CONVECTION_SEGMENT
$     LCIDH       HM      LCIDT       TM        LOC
          4        0.          0      20.          0

*DEFINE_FUNCTION
          4
h(x,y,z,vx,vy,vz,temp,tinf,time)= k/y *
 (0.14*((grav*beta*rho**2*abs(temp-tinf)*length**3/mu**2)*pr)**.33)
```

Use *PARAMETER to define constants, such as grav, beta, rho, etc.

Chapter 6 - 40

DEFINE_FUNCTION keyword

C program

LSTC

```
*BOUNDARY_CONVECTION_SEGMENT
$    LCIDH        HM      LCIDT        TM         LOC
         4          0.        0        20.          0

*DEFINE_FUNCTION
       4
float h(float x,float y,float z,float vx,float vy,float vz,
 float temp, float tinf,float time)
{
 float gr, nu, h ;
 gr=grav*beta*rho**2*abs(temp - tinf)*length**3 / mu**2 ;
 nu=0.14 * (gr * pr)**.33 ;
 h = nu*k/y ;
 return (h) ;
}
```


DEFINE_FUNCTION keyword

h defined using a function with a load curve argument

LSTC

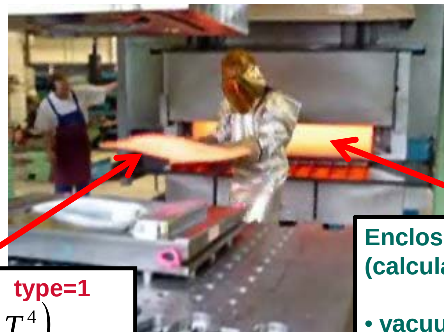
```
*BOUNDARY_CONVECTION_SEGMENT
      1213      1214      1216      1215
$    LCIDH        HM     LCIDT        TM         LOC
         7         0.         8        0.          0
*DEFINE_FUNCTION_TABULATED
         6 load curve function
tinfy
0.,20.
1.,30.
*DEFINE_FUNCTION
         7
h7(time,temp)= k/y *
 (0.14 * ( (grav*beta*rho**2*abs(temp - tinfy(time))*leng**3 / mu**2)
 * pr)**.33 )
*DEFINE_FUNCTION
         8
t8(time)=tinfy(time)
```

BOUNDARY_RADIATION_(option1)_(option2)

Option1 is SEGMENT or SET

LSTC

```
*BOUNDARY_RADIATION_(option)
SSID    TYPE
```



Surface radiation type=1

$$q = hA\left(T^4 - T_\infty^4\right)$$

$$h = \sigma\varepsilon F = f(time, T)$$

$$T_\infty = f(time)$$

Enclosure radiation type=2 (calculate VF or EF)

- vacuum heat transfer
- high T heating furnace
- annealing oven

*BOUNDARY_RADIATION_(option1)_(option2)

Option 2 is VF or EF read/calculate

LSTC

*BOUNDARY_RADIATION_ { SEGMENT | SET } { VF_READ | VF_CALCULATE | EF_READ | EF_CALCULATE }

For type 1 & 2: SEGMENT, SET

For type 2: VF_READ, VF_CALCULATE, EF_READ, EF_CALCULATE

- diffuse
  - VF_READ
  - VF_CALCULATE
- specular
  - EF_READ
  - EF_CALCULATE

Remember to define σ on *CONTROL_THERMAL_SOLVER

Surface radiation

TYPE 1: surface radiation to the environment

LSTC

```
*BOUNDARY_RADIATION_SET
SSID   TYPE
HLCID  HMULT  TLCID  TMULT  LOC
```

Load curve ID for h=εσF
>0 versus time
=0 use constant multiplier
<0 versus temperature

Load curve ID for T_∞
>0 versus time
=0 use constant multiplier

$$\dot{q} = \varepsilon\sigma F A\left(T^4 - T_\infty^4\right)$$

Usually, F = 1.

Enclosure radiation

TYPE 2: radiation in an enclosure

LSTC

***BOUNDARY_RADIATION_SET_VF_(calculate or read)**

The ***view factor*** (or shape factor, configuration factor, geometric factor) is the fraction of energy leaving a black surface that arrives at a second black surface. The view factor is based on surface geometry. LS-DYNA does the diffuse gray body calculations.

***BOUNDARY_RADIATION_SET_EF_(calculate or read)**

The ***exchange factor*** is the fraction of energy leaving a surface that arrives at a second surface both directly and by all possible intermediate diffuse and specular reflections. Values are calculated using Monte Carlo numerical methods. The exchange factor is based on surface geometry and surface radiation properties.

For a description of exchange factors and how to calculate them, see (Hottel & Sarofim, Radiative Transfer, McGraw Hill, 1967.)

Enclosure radiation

View Factors

LSTC

```
*BOUNDARY RADIATION SET VF  { READ
                             { CALCULATE
  SSID   TYPE   RAD_GRP   FILE_NO   BLOCK
  SELCID  EMULT
```

Set BLOCK=1 if this segment blocks the view between other segments

Surface ε(T) load curve ID and multiplier

Number of separate radiation enclosures

FILE_NO is added to the root name **viewfl_** to form the name of the file containing the view factors. If 0, then file name is just viewfl.

1 encl. 10 surf => 10^2 =100 vf

2 encl. 5 surf => $2*5^2$ = 50 vf

Enclosure radiation

View factors

LSTC

Load curve ID for ε
`>0` versus time
`=0` use constant multiplier
`<0` versus temperature

Curve multiplier for surface emissivity

```
SELCID          SEMULT
```

The surface emissivity is used to calculate diffuse gray body radiation exchange within the enclosure.

Warning

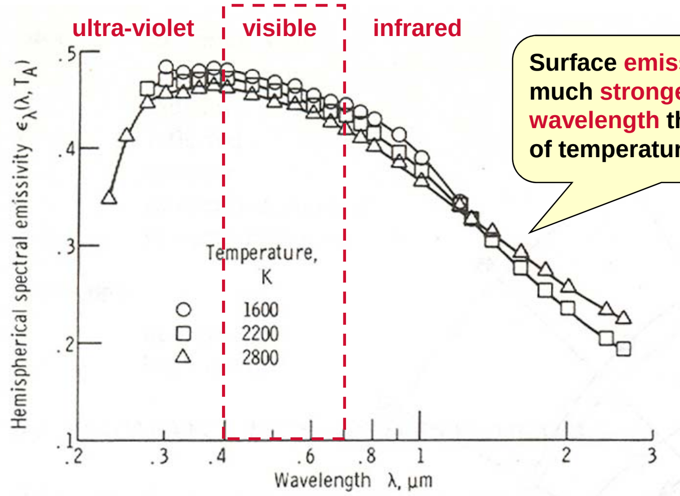
If ε(T,t), then a matrix inversion is needed for each nonlinear iteration. This is cpu expensive

Emissivities are tabulated in Appendix D of R. Siegel & J.R. Howell, Thermal Radiation Heat Transfer.

Enclosure radiation

Surface emissivity

LSTC



Surface **emissivity** is a much **stronger function of wavelength** than a function of temperature

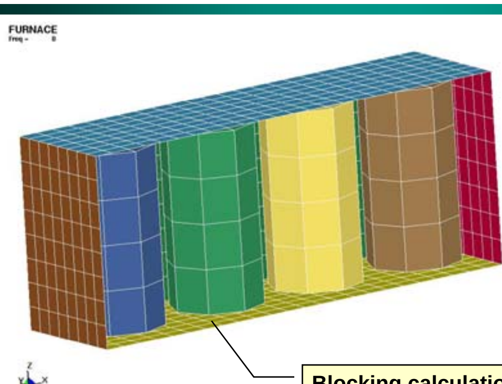
Hemispherical spectral emissivity of tungsten

Chapter 6 - 49

Enclosure radiation

Blocking surfaces

LSTC



This problem has 5 enclosures
4 → inside of cans
1 → inside of furnace

Blocking calculations take a lot of time. All surfaces can be flagged as blocking. However, to speed things up, specify only those surfaces that are actually blocking surfaces (e.g., only the cans inside the furnace are blocking).

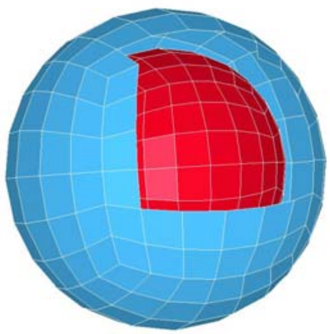
Chapter 6 - 50

Enclosure radiation

view factors versus exchange factors

LSTC

Two concentric spheres with 432 surface segments



View Factors

- 0.49 hours to calculate VF
- can read existing VIEWFL for a different surface ε
- 1 sec to calculate radiation transport

Exchange Factors

- 3.9 hours to calculate EF
- must recalculate EF for a different surface ε
- only method for specular surfaces

example
Dewar vessel

Chapter 6 - 51

Enclosure radiation

VF and EF matrix values (see frustum.k)

LSTC

$$\begin{bmatrix} F_{11}=0.0000 & F_{12}=0.8556 & F_{13}=0.1444 \\ F_{21}=0.3735 & F_{22}=0.4955 & F_{23}=0.1310 \\ F_{31}=0.3249 & F_{32}=0.6751 & F_{33}=0.0000 \end{bmatrix} \begin{matrix} *(A_1=176.7) \\ *(A_2=404.8) \\ *(A_3=78.54) \end{matrix} = \begin{bmatrix} 0.0000 & 151.2 & 25.52 \\ 151.2 & 200.6 & 53.03 \\ 25.52 & 53.03 & 0.000 \end{bmatrix}$$

view factors
row sum = 1.

exchange factors
row sum = ε

The file VIEWFL contains the viewfactors.

The file EXCHFL contains the exchange factors.

The matrix [F]*{A} is symmetric

$$\begin{bmatrix} f_{11}=0.0263 & f_{12}=0.4692 & f_{13}=0.1045 \\ f_{21}=0.2048 & F_{22}=0.4557 & f_{23}=0.1395 \\ f_{31}=0.2352 & f_{32}=0.7186 & f_{33}=0.0462 \end{bmatrix} \begin{matrix} *(A_1=176.7) \\ *(A_2=404.8) \\ *(A_3=78.54) \end{matrix} = \begin{bmatrix} 4.645 & 82.91 & 18.47 \\ 82.91 & 184.5 & 56.45 \\ 18.47 & 56.45 & 3.627 \end{bmatrix}$$

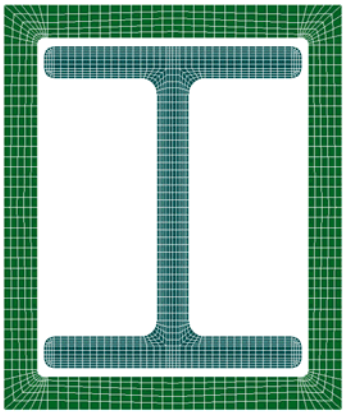
Chapter 6 - 52

Enclosure radiation

VF error diagnostic at top of TPRINT file

LSTC

```
view factor calculation error diagnostic
 row sum total should be       =  1.2120E+03
 row sum total is              =  1.3321E+02
 row sum total after smoothing =  1.2120E+03
```



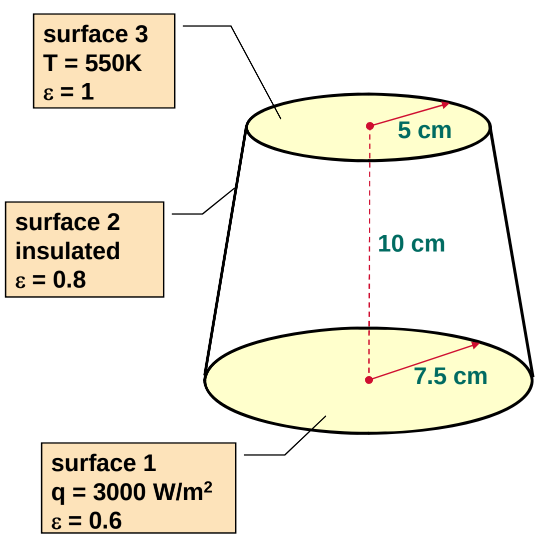
view factor smoothing activated

by entering $-\sigma$

Chapter 6 - 53

Workshop problem frustrum.k

LSTC



A frustum of a cone has its base heated as shown. The top is held at 550K while the side is perfectly insulated. Surfaces 1 and 2 are gray and diffuse, while surface 3 is black. What is the temperature of surfaces 1 and 2.

Siegel & Howell, Thermal Radiation Heat Transfer, 2nd ed., p. 245.

Chapter 6 - 54

Workshop problem cask_ss.k & cask_tr.k

Reference: L.C. Sanchez, “Performance Testing of Thermal Analysis Codes for Nuclear Fuel Casks”, Sandia National Laboratories, rpt. SAND84-1854, January 1987.

LSTC

Chapter 6 - 55

LOAD_HEAT_GENERATION_(option)

Where option is SOLID or SET

LSTC

```
*LOAD_HEAT_GENERATION_option
   SID   LCID   CMULT
```

Solid → element ID
Set → element set ID

Load curve ID for Q [W/m^3]
>0 function versus time
=0 use constant multiplier
<0 function versus temperature

Curve multiplier for Q [W/m3]

Heat generation can also be defined by material using the *MAT_THERMAL_option keyword

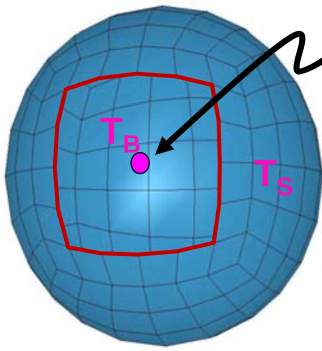
Specifying heat generation by element and material is additive

Chapter 6 - 56

BULKNODE – modeling a gas or fluid in a container

*BOUNDARY_THERMAL_BULKNODE

LSTC



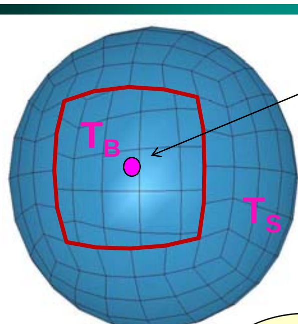
BULKNODE -This is a lumped parameter approach to model a fluid inside a rigid container. A node is defined with a specified volume, density, and heat capacity. The node coordinates are arbitrary, but it makes sense to place the node in the correct geometric position for visualization. The surface segments of the container are also defined so the bulk node can exchange heat by convection and radiation to the container.

Note that we are not modeling conduction in the fluid. The entire fluid volume is homogeneous at temperature T_B. The fluid temperature changes due to convection and radiation heat exchange with the container segments at T_S.

Chapter 10 - 57

BULKNODE – modeling a gas or fluid in a container

LSTC



The heat flow between the bulk node, B, and the surrounding surface, S, is given by

$$\dot{q}'' = h\left(T_S^a - T_B^a\right)^b$$

The value of h has the greatest uncertainty. The section on "How do you determine h" shows a hand calculation. Or, you may run a CFD code to numerically determine h.

Chapter 10 - 58

BULKNODE – modeling a gas or fluid in a container

BOUNDARY_THERMAL_BULKNODE keyword

LSTC

```
*BOUNDARY_THERMAL_BULKNODE
NID    PID    NBNSEG    VOL    LCID    H    A    B
```

NID	bulk node number
PID	this bulk node is assigned a PID which in turn assigns material properties
NBNSEG	number of surface segments surrounding the bulk node
VOL	volume of bulk node (i.e., cavity volume – calculated by LSPP during mesh generation)
LCID	load curve ID for heat transfer coefficient h
H	heat transfer coefficient h
A	exponent a
B	exponent b

LOAD_THERMAL_(option)

LSTC

WARNING

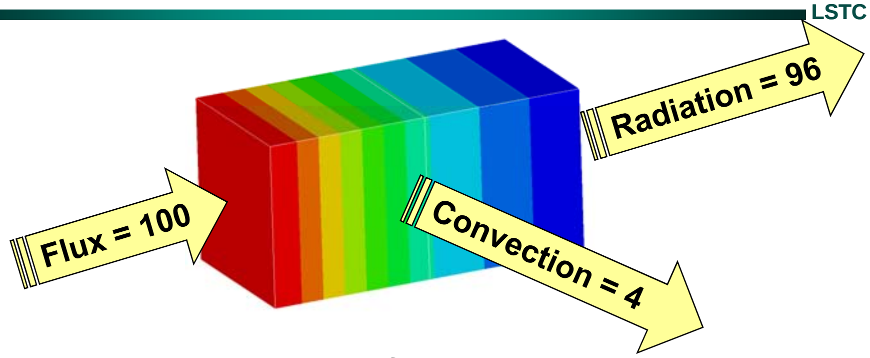
Use for mechanical only model

This keyword is used to define nodal temperatures that thermally load the structure for a mechanical analysis. Heat transfer is not performed.

Energy balance

boundary condition segment sets dumped to TPRINT file

LSTC



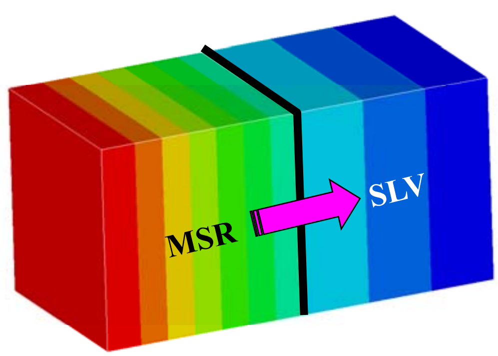

```
boundary condition segment set heat transfer rate [energy/time]
positive heat flow is in direction of the surface outward normal vector
 if temperture BCs are defined, then sum will not be 0
    bc type       segment set      [energy/time]
    flux                    1        -1.0000E+02
    convection              2         9.6060E+01
    radiation               3         3.9402E+00
                                    -------------
                energy balance sum = -4.2547E-08
```

Chapter 6 - 61

Energy balance

Energy transferred across contact surfaces dumped to TPRINT file

LSTC




```
contact surface heat transfer rate [energy/time]
positive heat flow is in direction of master-->slave
         order#  contact_id       [energy/time]
              1           1          1.4359E+01
```

Chapter 6 - 62

LSTC

Chapter 6 - 63

LSTC

Chapter 6 - 64

Chapter 7 – Thermal Contact

LSTC

http://www.pitt.edu/~dwilley/fire.html

$$\text{contact conductance} = \frac{1}{\text{contact resistance}}$$

Fire walking

The contact conductance is the controlling heat transfer mechanism

LSTC

Fire walking

Workshop problem: firewalking.k

LSTC

http://www.pitt.edu/~dwilley/fire.html

Bio – heat equation

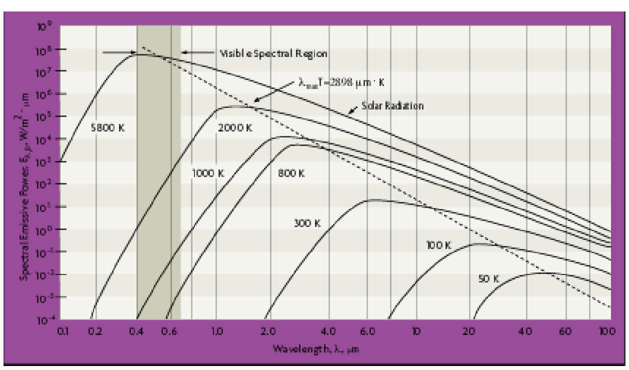
$$\rho C \frac{\partial T}{\partial t} = \nabla \cdot k \nabla T - \underbrace{W_b C_b \left(T - T_b\right)}_{\text{heat removal due to blood flow}}$$

The contact conductance is the controlling heat transfer mechanism.

Fire walking

Workshop problem: firewalking.k

LSTC



http://www.omega.com/literature/transactions/volume1/theoretical2.html

The visible wavelength is between 0.4μm (violet) to 0.7μm (red). The 1000K (727C) curve has an amount of radiant energy sufficient to be observed by the human eye between wavelengths of 0.4 to 0.7 microns. Since a larger percentage of the radiant energy is toward the longer wavelength of 0.7μm, an object at that temperature glows with a dull-red color.

The coals are 1000K (727C)

Fire walking

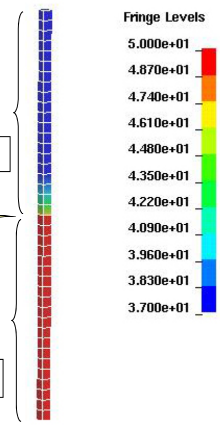
Workshop problem: firewalking.k

LSTC

material	density, ρ [kg / m3]	heat capacity, c [J / kg C]	conductivity, k [W / m C]
epidermis	1200.	3440.	0.34
muscle / fat	1060.	3350.	1.60
wood charcoal	240.	838	0.052

Interface resistance between the coals and the bottom of your foot is the controlling heat transfer mechanism

foot, epidermis

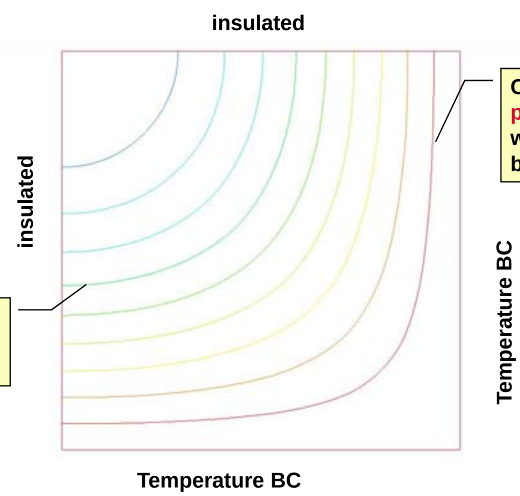


charcoal

LS-Dyna model showing temperature – the bottom of your foot reaches a temperature of 46.4C (115F). This is well below 155F that McDonalds now sells its coffee at to prevent skin burns after the Scalding Coffee lawsuit.

Interpreting temperature contours

LSTC



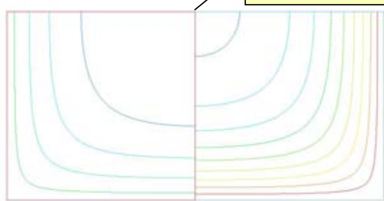
Contours are parallel to surfaces with a temperature boundary condition

Contours are perpendicular to insulated surfaces

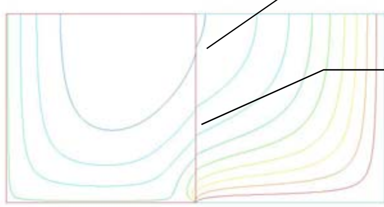
Interpreting temperature contours

LSTC

Contact surface



Note that the contours are perpendicular to the contact surface. This indicates that there is no heat transfer between the 2 parts.

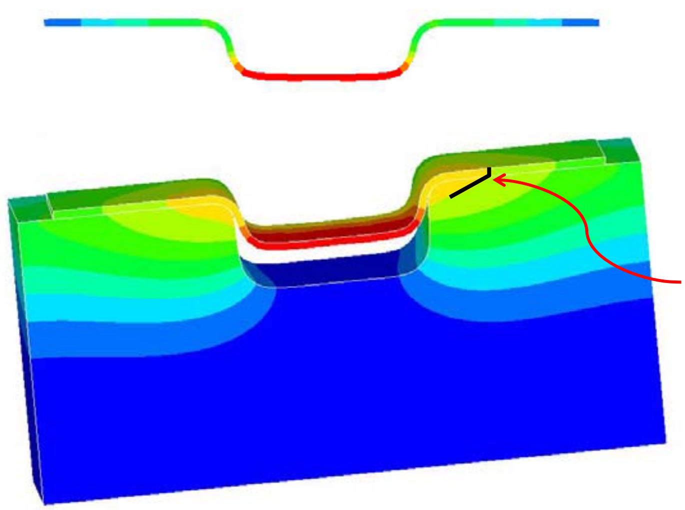


The discontinuity in the contour indicates a temperature drop across the interface

The change in slope indicates a thermal conductivity difference between the parts in contact

Interpreting temperature contours

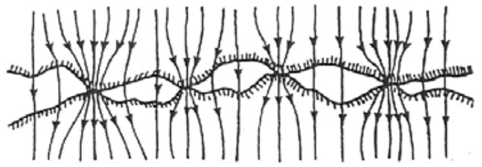
LSTC



Contours are perpendicular through the thin shell (no temperature gradient through thickness) but when considering the interface, we see that the T contour is not perpendicular on the tool side of the interface.

Contact conductance

LSTC



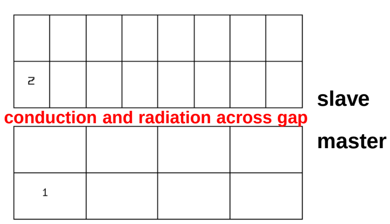
Parameters influencing contact conductance

- contact pressure (elastic/plastic deformation)
- contact temperature
- material in gap (vacuum, gas, lubricant)
- surface flatness
- surface roughness
- surface finish (e.g., oxide layer)

Chapter 7 - 9

Contact conductance

LSTC



In metal forming, by convention, the blank is defined as the slave surface.

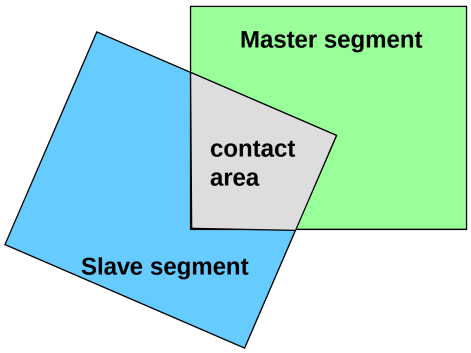
Three modes of heat transfer

1. Conduction across fluid filling gap with variable gap thickness
2. Radiation across gap
3. Metal-to-metal contact with voids
   - h = constant
   - h (t), h(T), h(P), general function

Chapter 7 - 10

Only use segment based contact

LSTC



$$\dot{q} = h_{contact} A \left(T_{master} - T_{slave} \right)$$

Contact area between master and slave surfaces → only use segment based contact

*CONTACT_SURFACE_TO_SURFACE_THERMAL

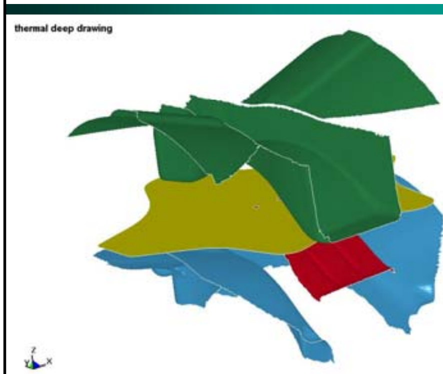
*CONTACT_AUTOMATIC_SURFACE_TO_SURFACE_THERMAL

🚫 *CONTACT_AUTOMATIC_NODES_TO_SURFACE_THERMAL

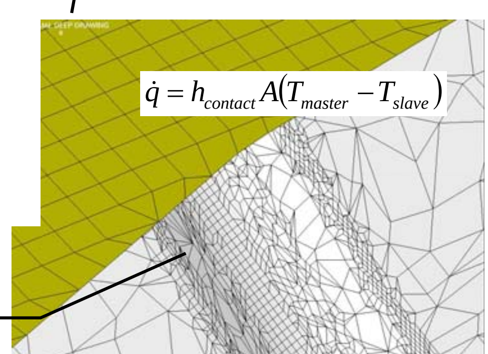
Only use segment based contact

Stamping model courtesy of ThyssenKrupp

LSTC



The BLANK mesh is made to accurately calculate deformation.

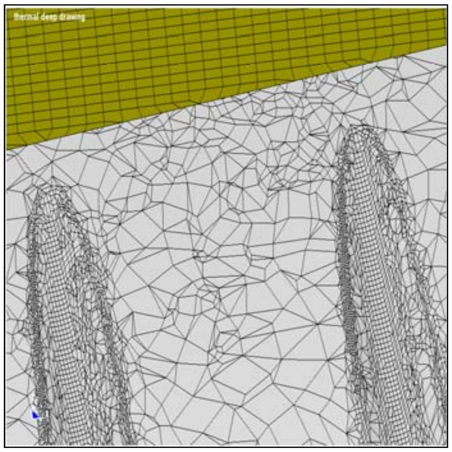


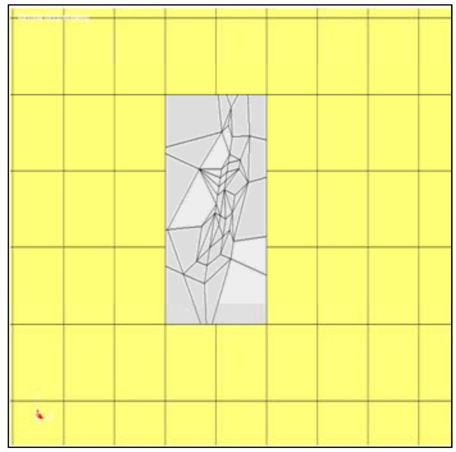
The TOOL mesh is created to accurately represent the surface shape.

Only use segment based contact

Note mesh mismatch between tool and blank

LSTC

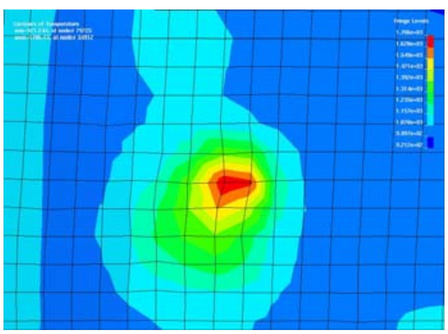


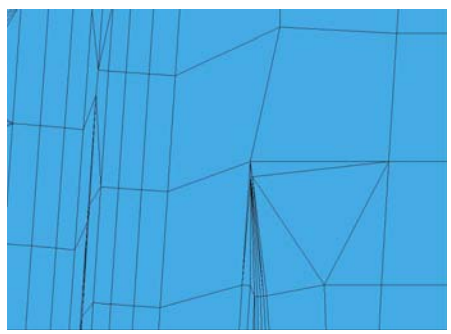


Only use segment based contact

Anomalous blank temperatures can occur with segment based contact when the die mesh has badly defined elements.

LSTC





Blank mesh – original mesh had square elements

Die mesh – created to capture surface geometry

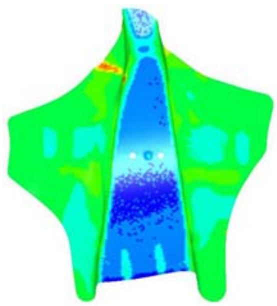
A good mesh is a prerequisite for good temperatures

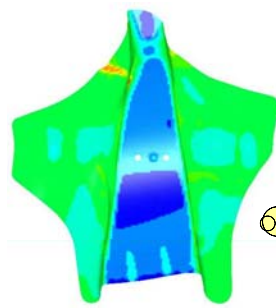
Thermal contact keyword

Use of 1-way flag

LSTC

```
*CONTACT_(OPTION1)_THERMAL
k  hrad  hcont  Lmin  Lmax  chlm  bc_flg  1_way
```





Assumption: Rigid tools don't deform or change temperature

2 way thermal contact
- **note blotchy region**
- **both blank & tool change temperature**

1 way thermal contact
- **smooth T fringes**
- **only blank changes T**

Chapter 7 - 15

Thermal contact keyword

Use of bc_flag

LSTC

```
*CONTACT_(OPTION1)_THERMAL
k  hrad  hcont  Lmin  Lmax  chlm  bc_flag  1_way
```

During the transfer process, heat is lost by convection and radiation from both surfaces to the environment.

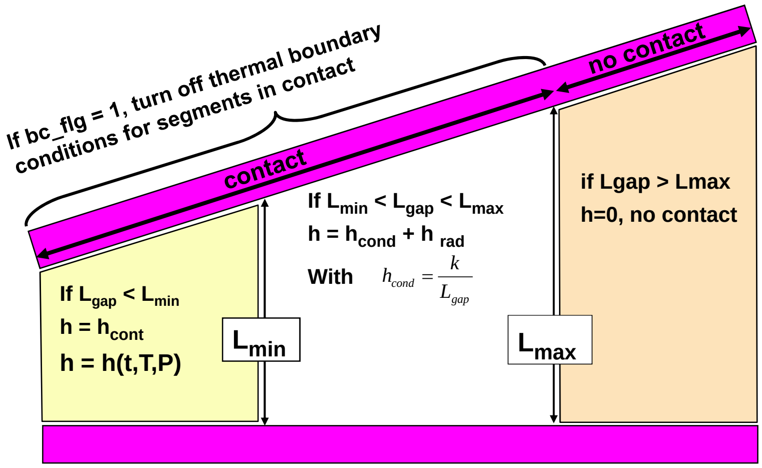
If bc_flag=1, then convection & radiation boundary conditions are turned off when contact occurs.

Chapter 7 - 16

Thermal contact keyword

LSTC

```
*CONTACT_(OPTION1)_THERMAL
k   hrad   hcont   Lmin   Lmax   chlm   bc_flg   1_way
```



Thermal contact keyword

Thermal-mechanical contact for stamping

LSTC

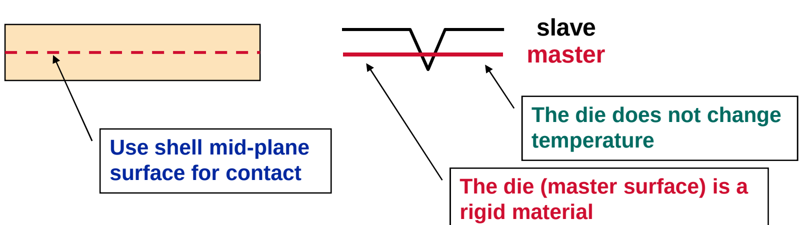
*CONTACT_FORMING_ONE_WAY_SURFACE_TO_SURFACE_THERMAL_FRICTION

FORMING: Do not account for master surface (die) shell thickness in calculating contact gap.

ONE_WAY: Only consider slave surface (blank) penetration into master surface (die).

THERMAL: Thermal *ONE WAY* - master surface (die) does not change temperature due to contact.

FRICTION: Define $\mu_s(T)$ $\mu_d(T)$ h(P,T)



See, "LS-DYNA Contact User's Manual"

Thermal contact keyword

(1) Friction function of T (2) heat transfer function of P

LSTC

```
*CONTACT_(option)_THERMAL_FRICTION
lcfst  lcfdt  formula  a  b  c  d  lch
```

Mechanical friction coefficients vs. temperature

Static → $\mu_s = \mu_s$ * lcfst(T)

Dynamic→ $\mu_d = \mu_d$ * lcfdt(T)

1	h(P) is defined by load curve "a"	such as GE data
2	h(P) = a + bP + cP2 + dP3	polynomial curve fit
3	$h(P)=\frac{\pi k_{gas}}{4\lambda}\left[1.+85\left(\frac{P}{\sigma}\right)^{0.8}\right]=\frac{a}{b}\left[1.+85\left(\frac{P}{c}\right)^{0.8}\right]$	I.T. Shvets, "Contact Heat Transfer between Plane Metal Surfaces", Int. Chem. Eng., Vol4, No. 4, p621, 1964.
4	$h(P)=a\left[1-\exp\left(-b\frac{P}{c}\right)\right]^{d}$	Li & Sellers, Proc. Of 2nd Int. Conf. Modeling of Metals Rolling Processes, The Institute of Materials, London, 1996.

Thermal contact keyword

LCH defined by in-line FORTRAN function

LSTC

LCH	= 0	not defined
	< 0	h(temperature)
	> 0	h(time)
	> nlcur	function(time, Tavg, Tslv, Tmsr, pres, gap)

```
*CONTACT_SURFACE_TO_SURFACE_THERMAL_FRICTION
lcfst  lcfdt  formula  a  b  c  d  101
*DEFINE_FUNCTION
101
h101(pres)=25.+25.e-07*pres+25.e-14*pres**2+25.e-21*pres**3
```

Thermal contact keyword

LCH defined by in-line FORTRAN function with load curve

LSTC

```
*DEFINE FUNCTION TABULATED
$#      fid     definition
        100     acoef(tavg)
$# title
acoef
$#               tavg                acoef
                   0.                  25.
                1000.                  25.
*DEFINE_FUNCTION
101
h101(pres,tavg)=acoef(tavg)+25.e-07*pres+25.e-14*pres**2
+25.e-21*pres**3
```

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Thermal contact keyword

LCH defined by Function specified by C program

LSTC

```
*DEFINE_FUNCTION
$#      fid     defintion
        101     h a function of pressure
float contact(float tslv, float tmsr, float pres)
{
 float tmean, acoef, h ;
 tmean=(tslv+tmsr)/2. ;
 acoef=.125*tmean ;
 h=acoef+25.e-07*pres+25.e-14*pres**2+25.e-21*pres**3 ;
 printf ("tmean= %f   acoef= %f   h= %f \n",tmean,acoef,h);
 return (h) ;
}
```

Chapter 7 - 22

Thermal contact keyword

Dumping contact pressure information

LSTC

Master side pressure is dumped

Slave side pressure is dumped

```
*CONTACT_option_ID
  CID
 SSID  MSID  SSTYP  MSTYP  SBOXID  MBOXID  SPR  MPR
*DATABASE_NCFORC
 DT
*DATABASE_BINARY_INTFOR
 DT
```

Text file
- Contact surface is identified by CID
- Nodes in contact are printed
- Pressure at node is printed

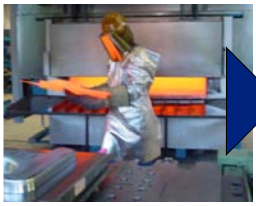
Binary file s=file_name
- contact force
- contact pressure
- contact gap

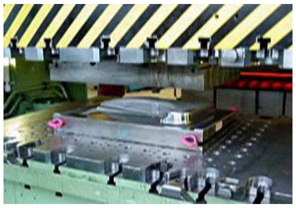
Chapter 7 - 23

Hot stamping → h(pressure)

Courtesy of Mercedes Car Group, Sindelfingen , Germany

LSTC





Transfer Positioning Hot forming & Quenching

Chapter 7 - 24

Hot stamping → h(pressure)

LSTC

When the blank touches the tool, we have to model both contact heat transfer and heat loss to the environment.

The top surface loses heat to the environment by convection and radiation. (h_{conv} ~ 10, h_{rad} ~ 100)

The bottom surface loses heat to the tool. The contact heat transfer to the tool is 10x greater than conv. + rad. loss.

What is h contact ?

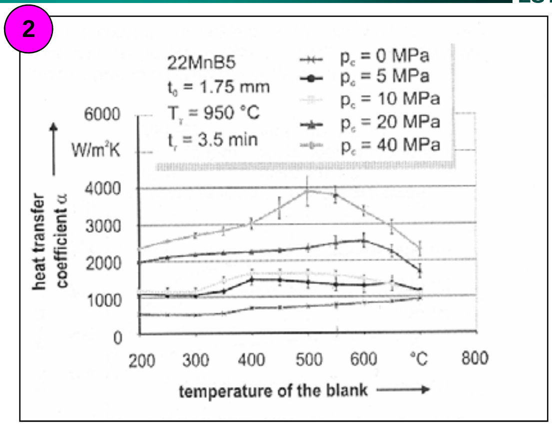
Hot stamping → h(pressure)

LSTC

1 Numisheet BM03 data

P [MPa]	h [W/m²K]
0	1300
20	4000
35	4500

2



M. Merklein and J. Lechler, SAE Technical Paper 2008-01-0853, April, 2008.

3 Shvets' formula

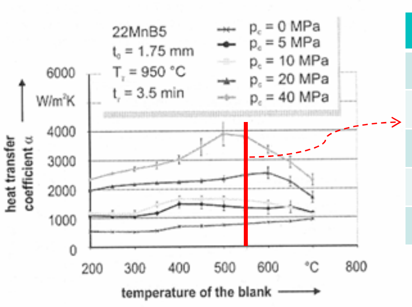
$$h = \frac{k\pi}{4\lambda}\left[1+85\left(\frac{P}{\sigma_r}\right)^{0.8}\right]$$

I.T. Shvets, "Contact Heat Transfer Between Plane Metal Surfaces", Int. Chem. Eng, Vol 4, No 4, p621, 1964.

Hot stamping → h(pressure))

How do you calculate h(P) at the interface

LSTC



P	h @ 550C (curve)	h calculated
0	750	750
5	1330	1330
10	1750	1770
20	2500	2520
40	3830	3830

$$h = \frac{k\pi}{4\lambda}\left[1 + 85\left(\frac{P}{\sigma_r}\right)^{0.8}\right]$$

h = contact conductance [W/m²C]
k = air thermal conductivity
0.059 W/mC at 550 C
λ = surface roughness [m]
P = interface pressure [MPa]
σ_r = rupture stress [MPa]

M. Merklein and J. Lechler, "Determination of Material and process Characteristics for Hot Stamping Processes of Quenchable Ultra High Strength Steels with Respect to a FE_based Process design", SAE Technical Paper 2008-01-0853, April, 2008.

I.T. Shvets, "Contact Heat Transfer Between Plane Metal Surfaces", Int. Chem. Eng, Vol 4, No 4, p621, 1964.

Hot stamping → h(pressure)

How do you calculate h(P) at the interface

LSTC

1. Using curve data, solve the equation for λ at (P, h) = (0, 750).

$$750 = \frac{(0.059)\pi}{4\lambda}\left[1 + 85\left(\frac{0}{\sigma_r}\right)^{0.8}\right]$$

λ = 61.8e-05

2. Using curve data and the above value for λ, solve the equation for σ_r at (P, h) = (40, 3830).

$$3830 = \frac{(0.059)\pi}{4(6.18*10^{-5})}\left[1 + 85\left(\frac{40}{\sigma_r}\right)^{0.8}\right]$$

σ_r = 1765

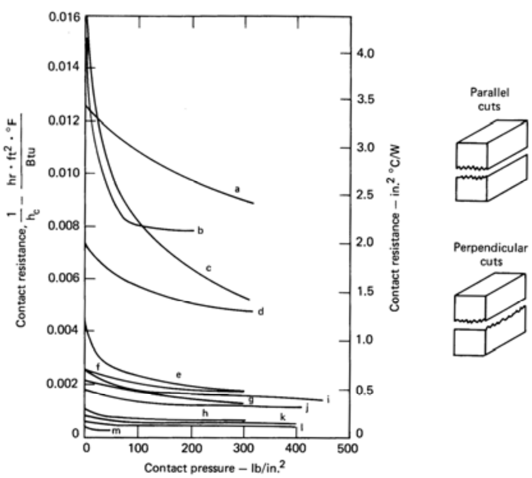
3. Now use the equation to calculate h(P)

$$h = \frac{(0.059)\pi}{4(6.18*10^{-5})}\left[1 + 85\left(\frac{p}{1765}\right)^{0.8}\right]$$

h from GE heat transfer data book

contact resistance vs. pressure

LSTC



Ref: N.D. Fitzroy, Ed., <u>Heat Transfer Data Book</u>, General Electric Company, 1970.

h from GE heat transfer data book

contact resistance vs. pressure

LSTC

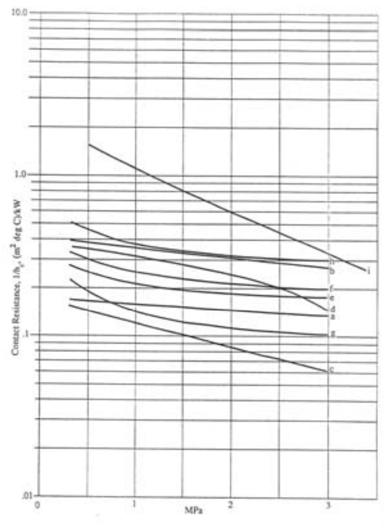
Curve	Material	Roughness Rms (μ in)	Temp. (F)	condition
a	steel	1000	200	parallel, rusted
b	steel	1000	200	parallel, clean
c	steel	1000	200	perpendicular, clean
d	steel	125	200	parallel, rusted
e	steel	125	200	parallel, clean
f	steel	63	200	perpendicular, clean
g	steel	63	200	parallel, clean
h	steel	4	200	clean
i	416 ss	100	200	clean
j	416 ss	100	400	clean
k	416 ss	30	200	clean
l	416 ss	30	400	clean

Ref: N.D. Fitzroy, Ed., <u>Heat Transfer Data Book</u>, General Electric Company, 1970.

h from GE heat transfer data book

contact resistance vs. pressure

LSTC



Curve	Material[4]	Roughness Rms (μm) Block 1	2	Grease or Oil in Gap	Temp. (°C)	Lubricant Material	Ref. No.[5]
a	Stainless Steel 304	0.38	- 0.51	Lubricant	39	Silicone Spray	22
b	Stainless Steel 304	0.38	- 0.51	Lubricant	94	Silicone Spray	22
c	Stainless Steel 304	0.38	- 0.51	Grease	35	Lithium Grease	22
d	Stainless Steel 304	0.38	- 0.51	Grease	86.5	Lithium Grease	22
e	Stainless Steel 304	0.38	- 0.51	Grease	38.5	Molykote g Grease	22
f	Stainless Steel 304	0.38	- 0.51	Grease	90.5	Molykote g Grease	22
g	Stainless Steel 304	0.38	- 0.51	Grease	37	Graphite Grease	22
h	Stainless Steel 304	0.38	- 0.51	Grease	86.5	Graphite Grease	22
i	Stainless 17-4PH	0.15	- 0.20	Grease	-3	DC FS-1280 Grease	19

Ref: N.D. Fitzroy, Ed., Heat Transfer Data Book, General Electric Company, 1970.

h from GE heat transfer data book

Picking h_{cont} [Btu / hr ft^2 F]

LSTC

	50 psi	100 psi	200 psi
Aluminum	790	1430	2290
Carbon steel	405	493	545

h is greater for softer material due to increased deformation of contact surface with P

h increases with applied P

perfect contact ~ 10,000 Btu/hr ft^2 F
50,000 W/m^{2}C

WARNING – h has units and must be scaled to fit the units of the problem

Calculating h

1. S. Song and N.M. Yovanovich, "Thermal Gap Conductance: Effect of Gas Pressure and Mechanical Load", AIAA-89-0429, 1989.
2. S. Song and N.M. Yovanovich, "Correlation of Thermal Accommodation Coefficient for Engineering Surfaces", ASME National Heat Transfer Conference, August, 1987

http://www.mhtl.uwaterloo.ca/paperlib/papers/contact_gap_cond.html

LSTC

1

$$\mu = \frac{MW_{gas}}{MW_{solid}} = \frac{MW_{air}}{MW_{iron}} = \frac{29}{55.8} = 0.52$$

2

$$\alpha = \frac{2\mu}{(1+\mu)^2} = 0.45$$

mean free path

specific heat ratio

3

$$M = 2\Lambda\left(\frac{2\gamma}{N_{\Pr}(\gamma+1)}\right)\left(\frac{2-\alpha}{\alpha}\right) = 2(2.08e-07)\left(\frac{2*1.4}{0.707(1.4+1)}\right)\left(\frac{2-0.45}{0.45}\right) = 2.4e-06 \;\; ft$$

Prandtl number

Compare to GE curve e

4

$$h = \frac{k_{gas}}{M+\delta} = \frac{k_{gas}}{M+0.61(\sigma_1+\sigma_2)} = \frac{0.0154}{2.4e-06+0.61(10e-06+10e-06)} = 1060 \;\; Btu/hr\cdot ft^2\cdot F$$

Gap parameter – ref. 1 gives several formulas including δ(P)

RMS surface roughness 120 μin = 10.0e-06 ft

Chapter 7 - 33

Calculating h with fluid (or gas) in gap

LSTC

gap thickness is calculated by mechanics

$$\text{contact conductance} = \frac{\text{thermal conductivity of fluid in gap}}{\text{gap thickness}}$$

$$h = \frac{k_{air}}{2\sigma_{rms}} = \frac{0.0154}{2(10.0e-06)} = 770 \;\; Btu/hr\cdot ft^2\cdot F$$

This is only valid for small gaps. Convection flow begins as the gap increases.

Chapter 7 - 34

Calculating h

I.T. Shvets and E.P. Dyban, "Contact Heat Transfer Between Plane Metal Surfaces", Int. Chem. Eng., Vol. 4, No. 4, p621, 1964.

LSTC

$$h = \frac{\pi k_{gas}}{4\lambda}\left[1 + 85\left(\frac{P}{\sigma_{rupture}}\right)^{0.8}\right]$$

k air	= 0.0154 Btu/hr ft F	
λ roughness	= 120 μin	h = 2280 Btu/hr ft^2 F
P	= 200 lb/in^2	
$\sigma_{rupture}$ steel	= 60,000 lb/in^2	

σ_u for 1020 cold rolled steel

Calculating h

D. Gilmore, Satellite Thermal Control Handbook, The Aerospace Corporation, 1994. (vol. 2 is cryogenics)

LSTC

1020 cold rolled steel

E = 30e+06 lb/in^2	υ = 0.3
σ = 120 μin = 10.0e-06 ft	k = 30 Btu / hr ft F
tan θ = 0.125 (roughness slope)	P = 100 lb/in^2

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{(10.e-06)^2 + (10.e-06)^2} = 1.4e-05 \ \ ft$$

$$\overline{E} = \frac{E_1 E_2}{E_1(1-\upsilon_2^2) + E_2(1-\upsilon_1^2)} = 1.65e+07 \ \ lb/in^2$$

Vacuum in gap

$$h = 1.55\frac{k\tan\theta}{\sigma}\left(\frac{P/\sqrt{2}}{\overline{E}\tan\theta}\right)^{0.94} = 26 \ \ Btu/hr \cdot ft^2 \cdot F$$

Calculating h

Vertical enclosed air space

LSTC

Grashof number $G = \frac{\rho^2 g \beta}{\mu^2} l^3 (T_{s2} - T_{s1})$

G < 2000. $h = \frac{k}{l}$ — conduction dominates

2,000 < G < 20,000 $h = \frac{k}{l} \frac{0.2}{\left(L/l\right)^{1/9}} [G * P]^{1/4}$

G > 20,000 *h* is independent of l — convection dominates

Double pane window design

For air at room temperature with 25C between inside and outside temperatures, *l* < 8 mm to eliminate convection flow

ref: W. McAdams, Heat Transmission, p181, McGraw-Hill, 1954.

Chapter 7 - 37

Calculating h

Horizontal enclosed air space (heat flow upward)

LSTC

Grashof number $G = \frac{\rho^2 g \beta}{\mu^2} l^3 (T_{s2} - T_{s1})$

G < 1000. $h = \frac{k}{l}$ — conduction dominates

1,000 < G < 3.0e+05 $h = \frac{k}{l} 0.21 [G * P]^{1/4}$

G > 3.0e+05 *h* is independent of l — convection dominates

ref: W. McAdams, Heat Transmission, p182, McGraw-Hill, 1954.

Chapter 7 - 38

Calculating gap radiation, h_{rad}

LSTC

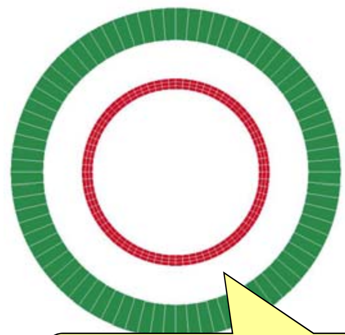
assumption – The gap is small enough such that the segment-to-segment view factor is 1.

Gray body radiation between 2 flat plates

$$f_{rad} = \frac{\sigma}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

Warning – this is not radiation transfer between a surface to the environment for which $f_{rad} = \sigma \varepsilon$

$$h_{rad} = \underbrace{f_{rad}}_{\text{user input}} \underbrace{\left(T + T_\infty\right)\left(T^2 + T_\infty^2\right)}_{\text{calculated by code}}$$



Enclosure radiation must be used for this geometry

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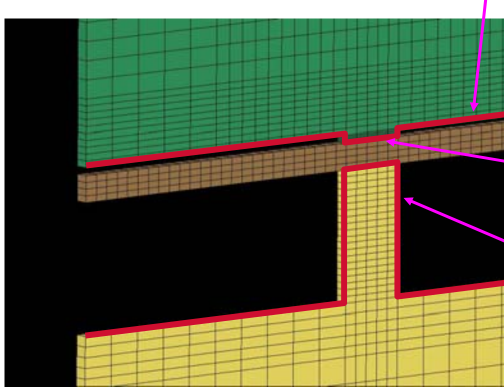
Calculating gap radiation, h_{rad}

LSTC



This is borderline between contact and an enclosure. A segment on 1 surface can see several segments on the other.

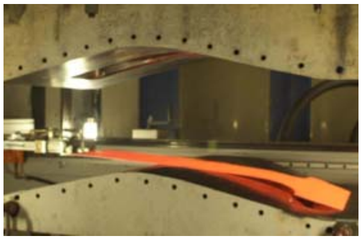
This is contact radiation.

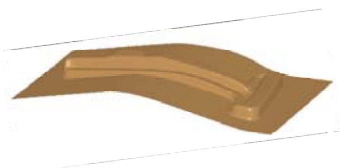
This is an enclosure and not contact radiation. A segment on 1 surface can see many segments on the other.

Chapter 7 - 40

Defining surfaces in contact

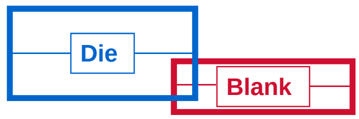
There are 2 choices for contact between parts defined with shells



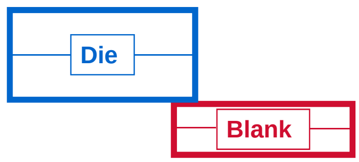


FE model

1. die mid-plane (CAD defined surface) contacts blank surface.



2. die surface contact blank surface



Defining surfaces in contact

Contact keyword parameters

***CONTACT_(option)_FORMING_(option)**

do not account for master surface (die) shell thickness in calculating contact.

***CONTROL_CONTACT attribute SHLTHK (shell thickness)**

0 → shell thickness is not considered

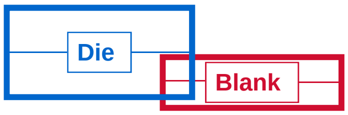
1 → shell thickness is considered but rigid bodies are excluded

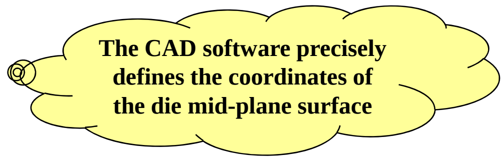
2 → shell thickness is considered including rigid bodies

Defining surfaces in contact

1. die mid-plane contacts blank surface

LSTC





Activated with

***CONTACT_FORMING_SURFACE_TO_SURFACE**

Modeling technique

1. **The die is a rigid material and does not deform.**
2. **The thermal analog is that the die does not change temperature.**
3. **The die is given a rigid body motion and a temperature boundary condition.**
4. **The mid-plane of the die is the reference surface for mechanical and thermal contact.**

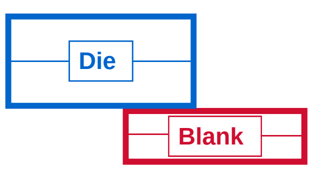
Workshop problem: bouncing_shell_5.k

Chapter 7 - 43

Defining surfaces in contact

2. Die surface contacts blank surface

LSTC



Activated with

***CONTACT_SURFACE_TO_SURFACE with SHLTHK=2**

Modeling technique

1. **The die is a rigid material or an elastic material.**
2. **The die is given an initial temperature.**
3. **Due to contact, a temperature gradient is calculated through the die thickness.**

Workshop problem: bouncing_shell_6.k

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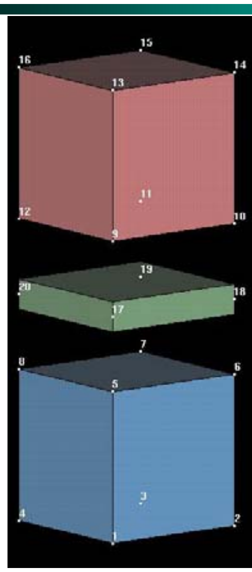
Workshop problem: bouncing_shell.k

LSTC



Note that shell has a top and bottom surface needing 2 contact definitions – get outward normal correct

Problem description

- Cold bottom block is stationary
- Top hot block is displaced downward
- Shell bounces between blocks and changes temperature when hit.

LS-POST commands to display shell thickness
1. click ‘Appear’
2. click ‘thick’
3. click ‘All Vis’

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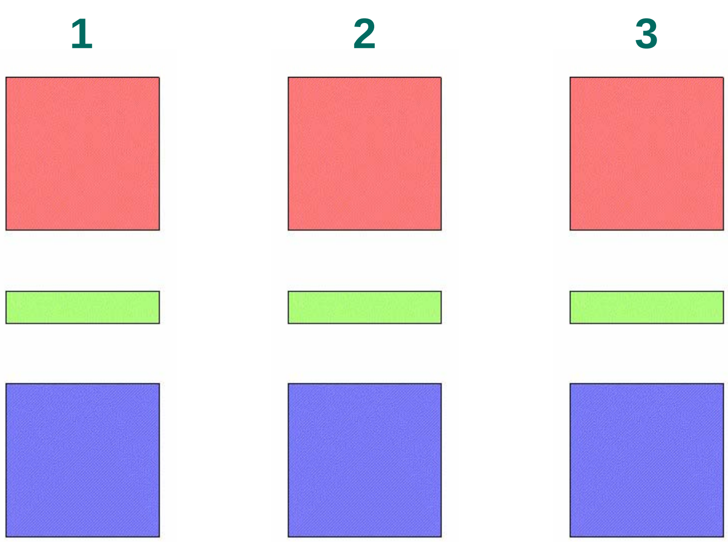
Workshop problem: bouncing_shell_# .k

6 input files where # = 1, 2, … , 6

LSTC



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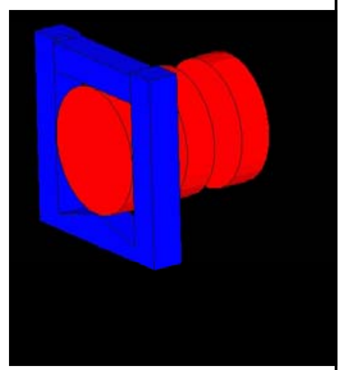
LSTC

LSTC

Chapter 8 – Thermal Stress

LSTC



Coupled Thermal - Stress

LSTC

Mechanical Calculations

Based on current temperature, calculate:
- plastic work
- part contact gap thickness
- temperature dependent material properties
- thermal expansion
- update geometry

Thermal Calculations

Based on current geometry, calculate:
- heat from plastic work
- contact conductance based on gap thickness
- heat from interface friction
- update temperature

Time Step → The thermal time step is independent of the mechanical time step. For most problems, the implicit thermal time step is chosen to be 10 – 100 times greater than the explicit mechanical time step. The rate of mechanical motion, mechanical deformation, and rate of heat transfer must all be considered in selecting an appropriate time step.

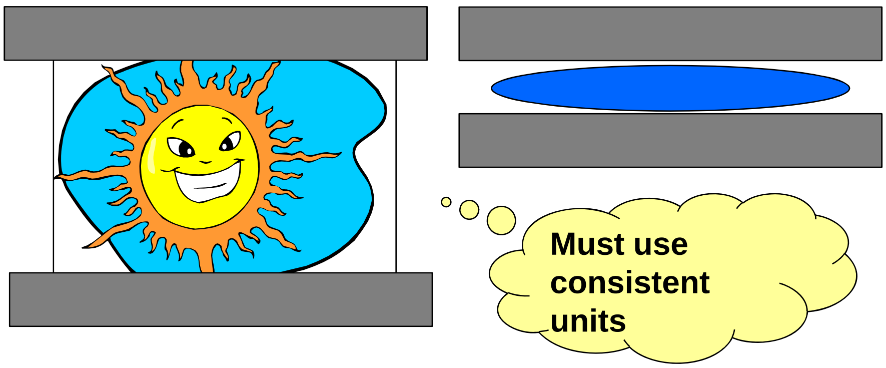
Conversion of mechanical work to heat

Error: plastic work to heat error caused by different mechanical and thermal unit sets

LSTC

I have small deformation, but I have reached the temperature of the sun.

I have turned the part into a pancake and there is no temperature change.



Conversion of mechanical work to heat

LSTC

The mechanical work per volume, *w*, expended in deformation is equal to the area under the stress-strain curve.

$$\left(\frac{N}{m^2}\right)\left(\frac{m}{m}\right)=\left(\frac{J}{m^3}\right)$$

$$w=\int_0^{\varepsilon}\sigma d\varepsilon=\rho c\Delta T$$

$$\left(\frac{kg}{m^3}\right)\left(\frac{J}{kgK}\right)(K)=\left(\frac{J}{m^3}\right)$$

The above equation is dimensionally consistent when using SI units. However, a units conversion factor must be included when using other units.

Conversion of mechanical work to heat

EQHEAT & FWORK

LSTC

$$(eqheat)(fwork)\ w = \rho c \Delta T$$

```
*CONTROL_THERMAL_SOLVER
   ATYPE    PTYPE    SOLVER    CGTOL    GPT    EQHEAT    FWORK
```

EQHEAT = mechanical equivalent of heat conversion factor

eqheat → 1 Nm = J

eqheat → 778 ft lb_f = BTU

FWORK = fraction of mechanical work converted into heat.

Conversion of mechanical work to heat

However, if you want to make things difficult, this is how to evaluate EQHEAT for the unit set ton, mm, msec, J

LSTC

quantity	unit
mass	ton
length	mm
time	msec
energy	Joule

$$work = Fd = (ma)d \left[(ton)\left(\frac{mm}{ms^2}\right)(mm) \right]$$

EQHEAT is the conversion factor (i.e., multiplier) to convert (*ton*)(*mm*²) / (*ms*²) to SI units of N m = J.

$$\frac{(ton)(mm^2)}{(ms^2)} * \underbrace{\left[\frac{10^3 kg}{ton} * \frac{m^2}{10^6 mm^2} * \frac{10^6 ms^2}{\sec^2} \right]}_{\text{EQHEAT}} = 10^3 (kg)\left(\frac{m}{\sec^2}\right)(m) = 10^3 (N)(m)$$

Conversion of mechanical work to heat

LSTC

Properties for 1020 cold rolled steel

Quantity	SI	ton, mm, ms, J
ρ	7.87e+03 kg/m^3	7.87e-09 ton/mm^3
E	2.05e+11 N/m^2	2.05e-01 TPa
C_p	4.86e+02 J/kg C	4.86e+05 J/ton C
k	5.19e+01 J/sec m C	5.19e-05 J/ms mm C
eqheat	1.	10^3

EQHEAT just converts the work term units, you must still convert length and time for other terms.

I don't like this because for k we have m and sec in the numerator [i.e., J=Nm=(kg m/s^2)(m)] and mm & ms in the denominator.

Chapter 8 - 7

Conversion of mechanical work to heat

Using a consistent set of units is the best approach

LSTC

Properties for 1020 carbon steel

mass	Length	Time	ρ	E	Cp	k	h
kg	m	sec	7.83e+03	2.10e+11	4.60e+02	7.1e+01	1
kg	mm	sec	7.83e-06	2.10e+08	4.60e+08	7.1e+04	1
kg	mm	msec	7.83e-06	2.10e+02	4.60e+02	7.1e-05	1.e-09
ton	mm	sec	7.83e-09	2.10e+05	4.60e+08	7.1e+01	1.e-03
g	mm	msec	7.83e-03	2.10e+05	4.60e+02	7.1e-02	1.e-06
g	mm	sec	7.83e-03	2.10e+11	4.60e+08	7.1e+07	1.e+03
g	cm	msec	7.83e+00	2.10e+00	4.60e-06	7.1e-12	1.e-15

Properties in SI

Density	ρ	[kg/m^3]
Elastic modulus	E	[Pa]
Heat capacity	Cp	[J/kg C]
Thermal conductivity	k	[W/m C]
Convection coefficient	h	[W/m^2 C]

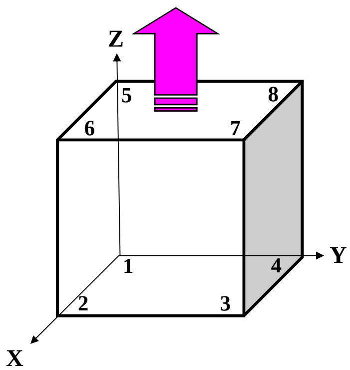
Chapter 8 - 8

Conversion of mechanical work to heat

Workshop problem work_to_heat.k

LSTC

Z-displacement ramps up from $\Delta z=0$ to $\Delta z=1$ in 1 second.



Hypothetical mechanical properties

density	2000 kg/m^3
modulus of elasticity	70.e+09 Pa
Poisson Ratio	0.3
coeff. of expansion	0.0 m/m K
yield stress	2.0e+06 Pa
Tangent modulus	0.0

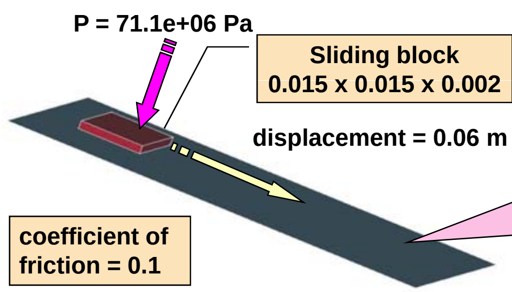
Hypothetical thermal properties

density	2000 kg/m^3
heat capacity	250 J/kg K
thermal conductivity	200 W/m K

Conversion of sliding friction to heat

friction_to_heat.k

LSTC



Sliding block 0.015 x 0.015 x 0.002

The frictional heat is divided equally between the surfaces.
½ to the sliding block
½ to the stationary block

coefficient of friction = 0.1

```
*CONTROL_CONTACT
  slsfac    rwpnal    islchk    shlthk
  usrstr    usrfrc    nsbcs .........
   sfric     dfric      edc .........
  ignore    frceng   skiprwg .........
```

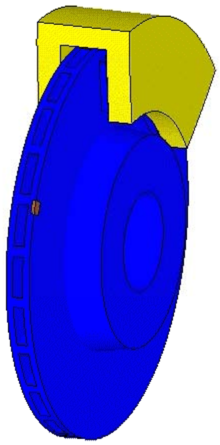
Set shlthk = 1 to account for shell thickness

Set frceng = 1 to calculate contact frictional energy

Conversion of sliding friction to heat

Warped brake rotor

LSTC



Coefficient of Thermal Expansion

Note: a hypothetical value for α is used to exaggerate differences in calculation methods

LSTC

Your job is to calculate the final length of a 1 meter long rod heated from 20C to 1020C. The coefficient of thermal expansion is α = 5.e-04 m/m C

You use the formula

$$\frac{L - L_0}{L_0} = \alpha(T - T_0)$$

$$L = L_0 + L_0\alpha(T - T_0)$$

And calculate

$$L = 1 + (1)(5.e-04)(1020-20) = 1 + 0.5 = 1.5 \ m$$

Coefficient of Thermal Expansion

Note: a hypothetical value for α is used to exaggerate differences in calculation methods

LSTC

In checking your work, a colleague uses the thermodynamic definition for the coefficient of thermal expansion

$$\alpha = \frac{1}{L}\left(\frac{\partial L}{\partial T}\right)_P \quad \text{or} \quad \frac{dL}{L} = \alpha dT$$

Integrating $\int_1^L \frac{dL}{L} = 5.e-04 \int_{20}^{1020} dT$

He gets $\ln(L) - \ln(1) = (5.e-04)(1020-100) = 0.5$

$$L = \exp[\ln(1) + 0.5] = 1.65 \ m$$

There is a large difference between the 2 answers. Which one is correct?

Chapter 8 - 13

Coefficient of Thermal Expansion

LSTC

Both are correct plus 1 other is also correct!

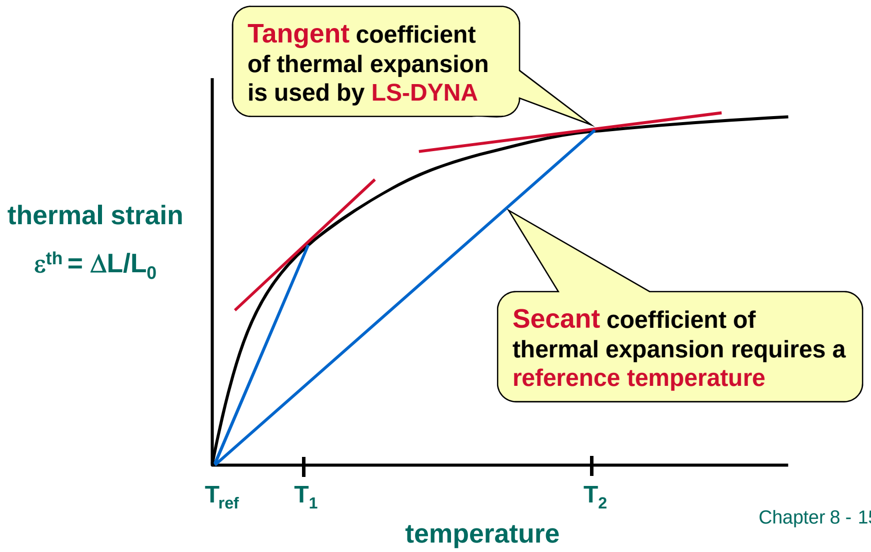
The difference is related to the definition of the CTE – there are 3. When using a CTE from a reference publication, you must determine how the CTE is defined. The 3 definitions are:

1. Tangent CTE using current length $\alpha_t = \frac{1}{L}\left(\frac{\partial L}{\partial T}\right)_P$
2. Tangent CTE using reference length $\alpha_{t,r} = \frac{1}{L_r}\left(\frac{\partial L}{\partial T}\right)_P$
3. Secant (or mean) CTE $\alpha_s = \frac{L - L_r}{L_r(T - T_r)}$

Chapter 8 - 14

Coefficient of Thermal Expansion

LSTC



Chapter 8 - 15

Coefficient of Thermal Expansion

tangent coefficient $\alpha_t = \frac{1}{L}\left(\frac{\partial L}{\partial T}\right)_P$

LSTC

The tangent coefficient of thermal expansion is a very convenient value to use in an explicit finite element code. An explicit analysis is an incremental method where calculations are based on the instantaneous properties of the material. A reference state (e.g., reference temperature, reference length) is not required. The same tangent CTE values are applicable for heating an object up from room temperature or, cooling it down from an elevated temperature (e.g., hot stamping, casting). This is not true for secant CTEs which have different values for heating and cooling because the secant CTE is a function of a strain free reference state. The secant CTE values depend on whether the strain free reference state is at room temperature or at the elevated temperature.

Chapter 8 - 16

Coefficient of Thermal Expansion

secant coefficient $\alpha_s = \dfrac{L - L_r}{L_r(T - T_r)}$

LSTC

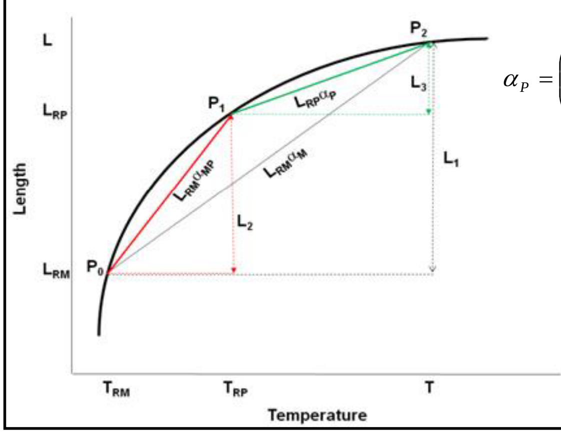
The secant coefficient of expansion, α_s is easily obtained in the laboratory and I'm sure many of you performed this experiment in a college physics lab. Take a rod at room temperature (i.e. reference temperature), T_r, and measure its length, L_r. Then uniformly (usually by an electric current) heat the rod and measure its new length, L, (or, change in length) and temperature, T. Note that a reference temperature, T_r, must be specified when using the secant value of thermal expansion. α_s is also referred to as the "mean" or "effective" coefficient of thermal expansion. The main disadvantage in using α_s is the requirement of a reference state. If the part initial temperature is different from the material reference state temperature, then the α_s values are no longer valid. They must be adjusted to account for the new strain free condition at the part initial temperature.

Coefficient of Thermal Expansion

How do you adjust a_s values at a new reference state?

LSTC

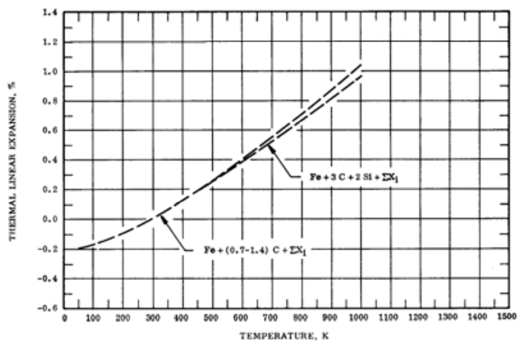
If the part initial temperature state (P_1), is different from the material reference temperature state (P_0), then the α_s values must be modified. The subscript RM means 'reference material', and the subscript RP means 'reference part'. The figure graphically depicts the computational method.



$$\alpha_P = \left(\frac{L_{RM}}{L_{RP}}\right)\frac{\alpha_M(T - T_{RM}) - \alpha_{MP}(T_{RP} - T_{RM})}{(T - T_{RP})}$$

Coefficient of Thermal Expansion

LSTC



PROVISIONAL VALUES

[Temperature, T, K; Linear Expansion, $\Delta L/L_0$, %; α, K^{-1}]

T	[Fe + (0.7 – 1.4) C + ΣX_i] $\Delta L/L_0$	[Fe + (0.7 – 1.4) C + ΣX_i] $\alpha \times 10^6$	(Fe + 3 C + 2 Si + ΣX_i) $\Delta L/L_0$	(Fe + 3 C + 2 Si + ΣX_i) $\alpha \times 10^6$
50	-0.199	5.6		
100	-0.169	6.7		
200	-0.091	8.9		
293	0.000	10.7	0.000	11.9
400	0.127	12.4	0.128	12.4
500	0.258	13.7	0.255	13.1
600	0.401	14.8	0.389	13.7
700	0.554	15.6	0.528	14.1
800	0.713	16.2	0.671	14.5
900	0.876	16.4	0.817	14.7
1000	1.041	16.5	0.965	14.9

Instantaneous coefficient of linear thermal expansion = tangent to curve at the given temperature

Y.S. Touloukian, <u>Thermophysical Properties of Matter, Thermal Expansion</u>, Vol. 12, Plenum Publishers.

Coefficient of Thermal Expansion

Consider the expansion of a steel cube with *l*=1 and α=15.e-06 between T=0 and T=1000. expansion_mat004.k

LSTC

$$\int_{l_0}^{l} \frac{dl}{l} = \int_{T_0}^{T} \alpha dT$$

$$\ln(l) - \ln(l_0) = \alpha(T - T_0)$$

$$l = \exp[\ln(l_0) + \alpha(T - T_0)]$$

$$l = \exp[\ln(1) + 15.e - 06(1000 - 0)]$$

$$l = 1.015113$$

$$\frac{\Delta l}{l} = \alpha \Delta T$$

$$l - l_0 \Big/ l_0 = \alpha(T - T_0)$$

$$l = l_0 + l_0 \alpha (T - T_0)$$

$$l = 1 + 1(15.e - 06)(1000 - 0)$$

$$l = 1.015000$$

Both methods give approximately the same answer for small strain.

MAT_106
+$\alpha \rightarrow$ tangent
- $\alpha \rightarrow$ secant

Coefficient of Thermal Expansion

Volumetric coefficient, β, versus linear coefficient, α

LSTC

Coefficient of volumetric thermal expansion

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

For an ideal gas β = 1/T

Coefficient of linear thermal expansion

$$\alpha = \frac{1}{L}\left(\frac{dL}{dT}\right)$$

$$\left.\begin{aligned} V &= l^3 \\ dV &= 3l^2 dl \\ \beta &= \frac{1}{V}\frac{dV}{dt} = \frac{1}{l^3}\frac{3l^2 dl}{dT} \\ \beta &= 3\frac{1}{l}\frac{dl}{dT} = 3\alpha \end{aligned}\right\} \quad \alpha = \frac{1}{3}\beta$$

Coefficient of Thermal Expansion

expansion_mat004_50percent.k

LSTC

How do you calculate α to produce a 50% volume change between T=30,000 and T=50,000.

$$\frac{dV}{V} = \beta dT$$

$$\ln\frac{V}{V_0} = 3\alpha\left(T - T_0\right)$$

$$\ln\left(\frac{1.5}{1}\right) = 3\alpha\left(50000 - 30000\right)$$

$$\alpha = 6.758e - 06$$

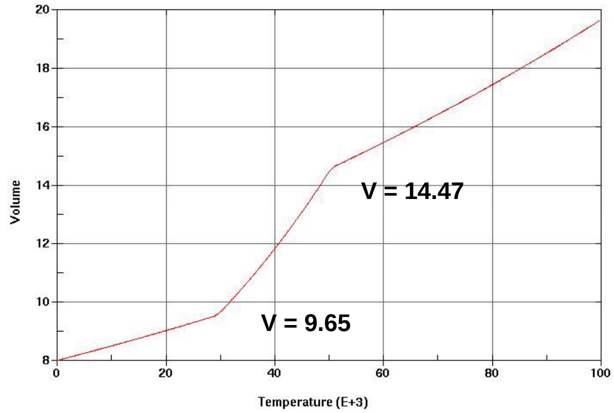
```
*MAT_ELASTIC_PLASTIC_THERMAL
         1      2700.
        0.     29999.     30000.     50000.     50001.     1.e+06 → T
   70.e+09    70.e+09    70.e+09    70.e+09    70.e+09    70.e+09 → E
        .3         .3         .3         .3         .3         .3 → ν
    2.e-06     2.e-06 6.758e-06 6.758e-06     2.e-06     2.e-06 → α
```

Coefficient of Thermal Expansion

expansion_mat004_50percent.k

LSTC

How do you calculate α to produce a 50% volume change between T=30,000 and T=50,000.



Thermal & mechanical material models

LSTC

Mechanical materials with coefficient of expansion

Type 4	*MAT_ELASTIC_PLASTIC_THERMAL
Type 21	*MAT_ORTHOTROPIC_THERMAL
Type 106	*MAT_ELASTIC_VISCOPLASTIC_THERMAL

To calculate thermal strains for all other materials, use the keyword (expansion_mat024_add.k)

*MAT_ADD_THERMAL_EXPANSION

PID LCID MULT

- MULT: curve multiplier
- LCID: load curve ID for thermal coefficient of expansion vs. temperature
- PID: part ID

Thermal & mechanical material models

MAT_106

LSTC

Material model 106 allows many of the mechanical properties to be a function of temperature. The temperature dependence is defined by load curves LC(option). See workshop problem upset_mat106.k as an example.

```
*MAT_106
MID    RO    E    PR    SIGY    ALPHA    LCSS
QR1    CR1   ........
C      P     LCE  LCPR  LCSIGY  LCR      LCX    LCALPH
```

σ vs. ε

α(T)

Load curve for scaling the stress given by the load curve LCSS as a function of temperature

E(T)

σ_Y(T)

ν(T)

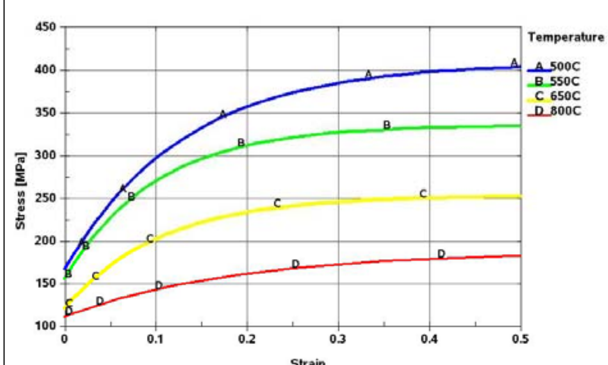
Thermal & mechanical material models

MAT_106: how to enter σ vs. ε vs. T

LSTC

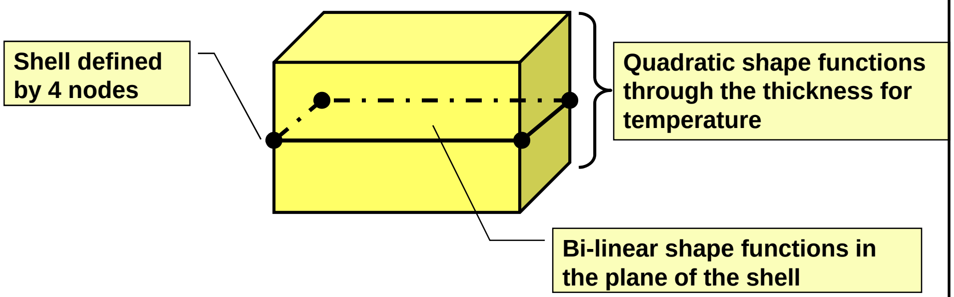
*DEFINE_TABLE
500
550
650
800
*DEFINE_CURVE
(stress,strain) at T=500
.
*DEFINE_CURVE
(stress,strain) at T=550
.
*DEFINE_CURVE
(stress,strain) at T=650
.
*DEFINE_CURVE
(stress,strain) at T=800
.

Material: 22MnB5 ($d\varepsilon/dt$=0.1 s^{-1})
Data from University of Erlangen



Thick thermal shell

LSTC



12 node quadratic shell (G. Bergman & M. Oldenburg, “A Finite Element Model for Thermo-mechanical Analysis for Sheet Metal Forming”, Dept. of Mechanical Engineering, Lulea University of Technology, Lulea Sweden)

Thick thermal shell

Cantilever beam: thermal_induced_shell_bending.k

LSTC

```
*CONTROL_SHELL
$   WRAPNG     ESORT     IRNXX    ISTUPD
                                       1
$  ROTASCL    INTGRD    LAMSHT    CSTYP6    TSHELL
                                                 1
```

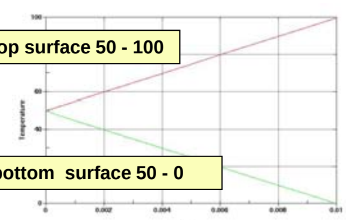
Calculate thickness change

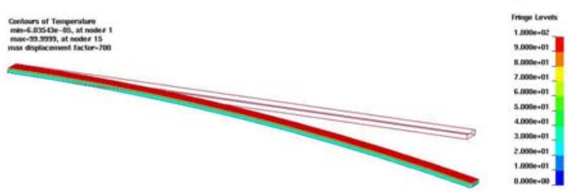
Turn on thick thermal shell

```
*INITIAL_TEMPERATURE_SET
         1        50.
         1        50.        -1
         1        50.        +1
*BOUNDARY_TEMPERATURE_SET
         1         1         1.        +1
         1         2         1.        -1
```

Ramp top surface 50 - 100

Ramp bottom surface 50 - 0

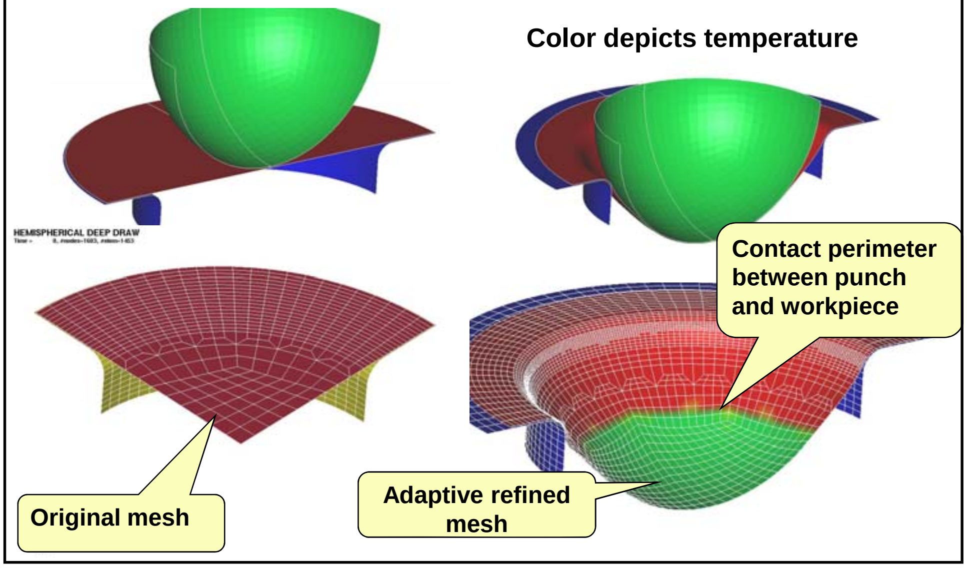




Adaptive meshing

hemi-draw-adapt-thermal-thin.k

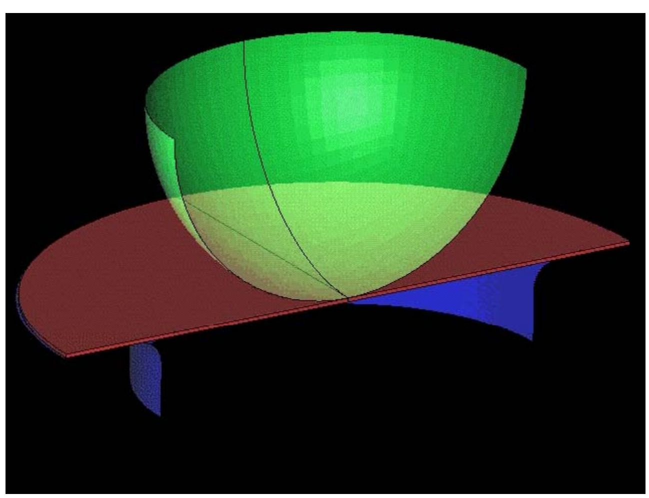
LSTC



Adaptive meshing

Thermal Mechanical Deep Draw Analysis

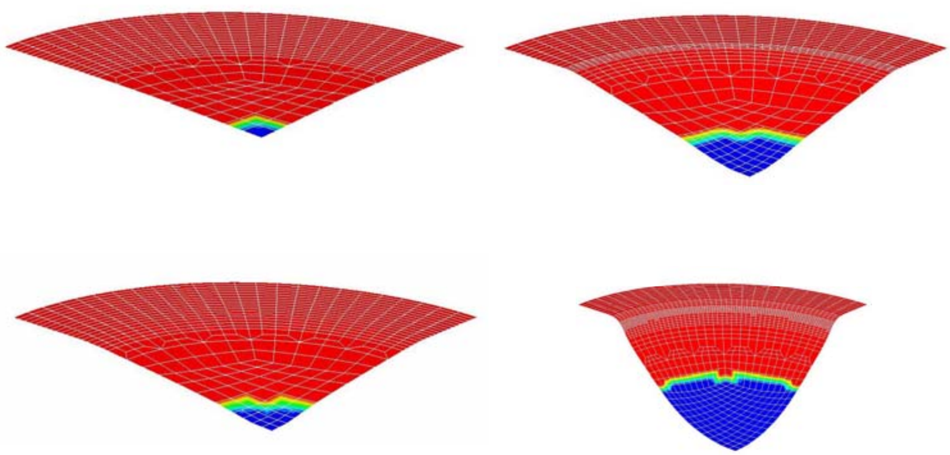
LSTC



Adaptive meshing

Thermal Mechanical Deep Draw Analysis

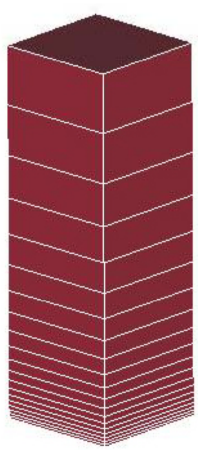
LSTC



Rod contraction problem

Solution method: implicit mechanics & heat transfer

LSTC



The rod is initially at 25C. The base is welded (i.e., x,y,z constrained) to a cold plate at T=0. The top of the rod transfers heat by convection to the environment at 25C. Slow process.

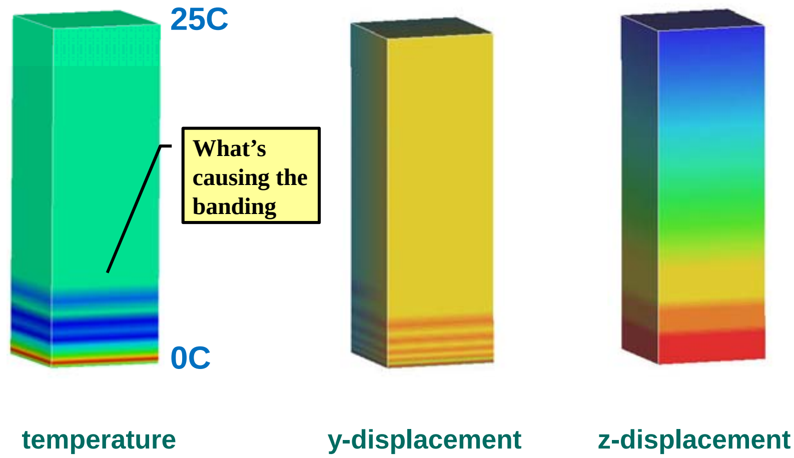
```
*SECTION_SOLID
$     secid    elform
          1         1
*HOURGLASS
$      hgid    hgtype
          1         5
```

Default 1-pt integration

Must use HOURGLASS for an under integrated element (e.g., ELFORM=1). Hourglass type 5 recommended for low velocity.

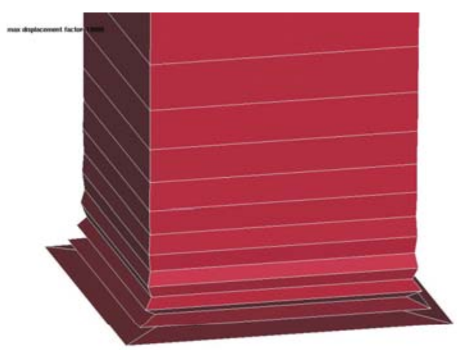
Rod contraction problem

LSTC

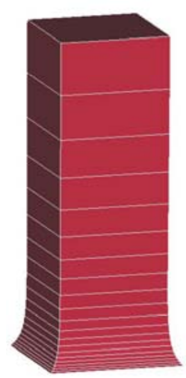


Rod contraction problem

LSTC



- displacement scale factor = 10,000
- results using **elform=1**
- note hourglass even with hourglass control turned on



Results using a fully integrated element, **elform=2**

Hourglass

D. Benson, "Zero Energy Modes in 1-Dimension: An Introduction to Hourglass Modes", FEA Information News, Feb, 2003.

LSTC

- Hourglass modes are caused by insufficient integration points (i.e., 1 point quadrature, elform=1)
- Fully integrated elements have no hourglass (elform=2)
- Hourglass often occurs in small displacement situations
- Hourglass modes are orthogonal to the real deformation
- General rule: hourglass energy < 10% of internal energy (see GLSTAT & MATSUM file)
- If hourglass occurs in an area where it does not influence the design area of concern, then it may be admissible.

Recommendation

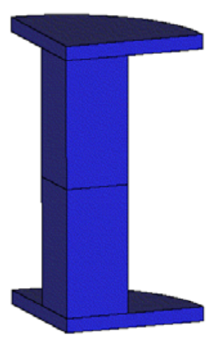
Use reduced integration (i.e., elform=1) until you experience hourglassing, it is much faster

Upset workshop problem

LSTC

The upsetting process is defined as the axial compression of an axisymmetric body between two perfectly rough, insulated plates.

- material is low carbon steel
- initial temperature: 20C
- no heat transfer to the environment – all plastic work goes into heating the part
- initial geometry: 9mm radius, 36 mm height
- imposed height reduction: $\Delta h/h = 0.44$
- loading time: 1.6 seconds

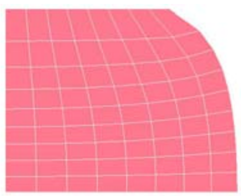


J. Van der Lugt, "Thermal Mechanically Coupled Finite Element Analysis in Metal Forming Processes", Computer Methods in Applied Mechanics and Engineering, 54 (1986) p. 145-160.

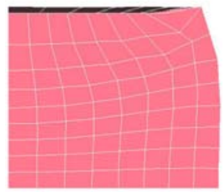
Upset workshop problem

Upset_mat003.k

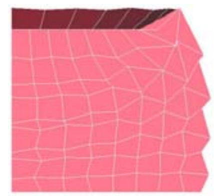
LSTC



baseline



Fully integrated solid element – a stiffer response



Default viscous hourglass

Upset workshop problem

Mass and Time scaling to reduce run time

LSTC

Exercise 6 → results

baseline: upest_mat003.k
mass scaled: upset_mat003_dt2ms.k
mass & time scaled: upset_mat003_dt2ms_time_scaled.k

	Baseline	Mass scaled	Mass & time scaled
run time	**7 h**	**15 s**	**2 s**
Δt mech	1.e-07	1.e-04	1.e-06
cycles mech		16,000	1600
Δt thermal	1.e-06	1.e-03	1.e-05
cycles thermal		1,600	160

If you are doing a code shootout to see which is faster, make sure you understand the scaling that is going on.

Upset workshop problem

Time scaling to reduce run time

LSTC

5. "Thermal velocity" terms (i.e., those with units of W = J/s) must be scaled by the same ratio as the punch velocity. Thermal velocity terms include
   - Thermal conductivity
   - Convection heat transfer coefficients
   - Contact heat transfer coefficients
   - Surface heat flux

$$k = (46)(1000) = 46000 \quad \frac{W}{m\ C}$$

```
*CONTROL_THERMAL_SOLVER (optional card 2)
  TSF = thermal speedup factor
```


LSTC

Chapter 9 - ALE coupled thermal-mechanics

LSTC

1. ALE and Euler element formulations
   1. ELFORM = 1 pure Lagrangian
   2. ELFORM = 5 ALE single material
   3. ELFORM = 6 Eulerian single material
   4. ELFORM = 7 Eulerian ambient element
   5. ELFORM = 11 ALE multi-material element
   6. ELFORM = 12 ALE single material and void
2. ALE post-processing using *LS-POST*
3. 3 ways to solve the upset problem
   1. Lagrangian
   2. ALE
   3. Eulerian

ALE and EULER element formulations

LSTC

There are many ways to model the same physical process. The different methods depend on different element formulations. The element formulation "ELFORM" is defined on the *SECTION_SOLID keyword.

ELFORM:

1 = Constant stress solid (pure Lagrangian formulation).

5 = 1-point ALE (single material in each cell).

6 = 1-point Eulerian (single material).

7 = 1-point Eulerian Ambient element.

11 = 1-point ALE multi-material element.

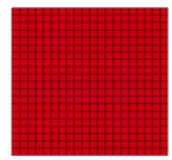
12 = 1-point ALE single-material-and-void.

ELFORM = 1: Pure Lagrangian Formulation

LSTC

```
*SECTION_SOLID
$     SECID     ELFORM          AET
          1          1
```

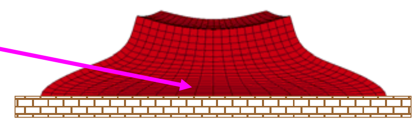
Element formulation 1 = Constant-stress solid



NOTE:
* The mesh deforms with the material.
* Pure Lagrangian method has no mesh smoothing.
* Only 1 material in each element.
* The mesh bunches up near the impact surface.

Advantage:
Free surface is followed automatically.

Disadvantage:
Not accurate at large deformation.



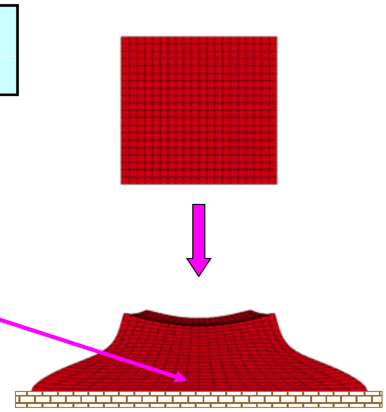
ELFORM = 5: Single Material ALE Formulation with Smoothing

LSTC

```
*SECTION_SOLID
$     SECID     ELFORM          AET
          1          5
```

Element formulation 5 = 1-point ALE solid

ALE mesh-smoothing activated ➔ *CONTROL_ALE



NOTE:
* The mesh deforms with the material.
* ALE method allows mesh smoothing.
* Only 1 material in each element.

Advantage:
Free surface is followed automatically.
Less element error for large deformation.

Disadvantage:
Loss in fidelity due to smoothing.

ELFORM = 6: Eulerian Formulation with Single Material

LSTC

```
*SECTION_SOLID
$     SECID      ELFORM          AET
          1           6
```

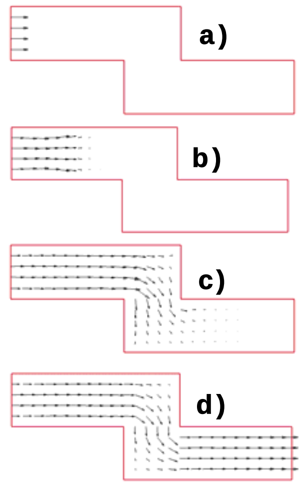
Element formulation 6 (or 7) = 1-point 3D Eulerian element

Advection activated ➔ *CONTROL_ALE

NOTE:
* The mesh is spatially fixed (no mesh smoothing).
* The material (fluid) flows through the mesh.
* Only 1 material / element.

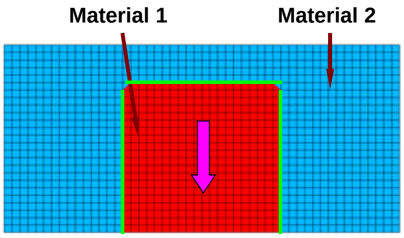
Advantage:
Efficient for very large element deformation (flow).
Can create new free surfaces automatically.

Disadvantage:
Can be expensive in computation.
Difficulty in tracking thin material interfaces.



ELFORM = 11: Multi-Material ALE Formulation

LSTC



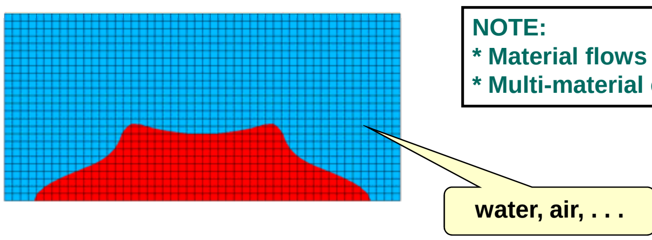

```
*SECTION_SOLID
$     SECID      ELFORM          AET
          1          11
```

Element formulation 11 = ALE multi-material

Advection activated ➔ *CONTROL_ALE

To track multi-material interfaces ➔ *ALE_MULTI-MATERIAL_GROUP

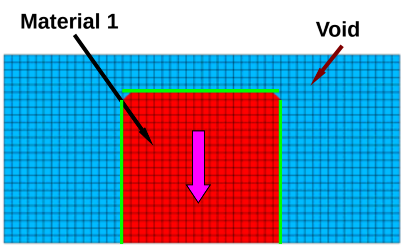
Merged nodes on mat1- mat2 mesh boundaries.



NOTE:
* Material flows through mesh.
* Multi-material element.

ELFORM = 12: ALE Formulation with Single Material + Void

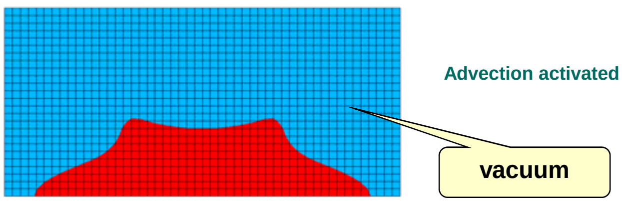
LSTC




```
*SECTION_SOLID
$     SECID     ELFORM        AET
          1         12
```

Element formulation 12 = 1-point integration 3D-element with 1 material and void

Merged nodes on material-mesh and void-mesh boundaries.

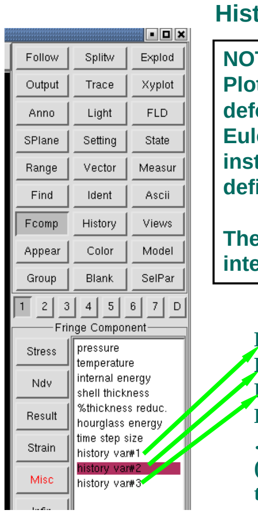


Advection activated ➔ *CONTROL_ALE

ALE Post-Processing using *LS-POST*

LSTC

History variable plotting: Density and volume fractions (vf)



NOTE:
Plotting of Lagrangian <u>parts</u> readily shows the material deformation because the mesh follows the material. Since Eulerian or ALE materials "flow" in their meshes, we need to plot, instead, their <u>volume fractions</u> which describe the interfaces defining the material boundaries.

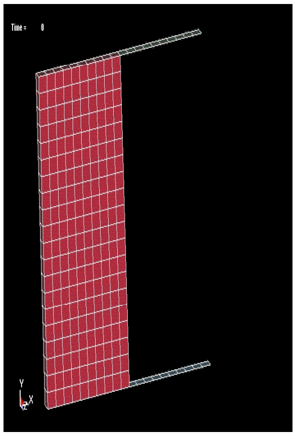
The resolution of the mesh defines the resolution of the interfaces.

History var # 1 = Density
History var # 2 = vf of the 1st ALE material
History var # 3 = vf of the 2nd ALE material
History var # 4 = vf of the 3rd ALE material
… etc.
(Additional history variables may depend on the material model used).

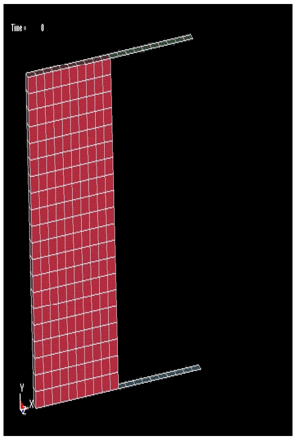
3 ways to solve the upset problem

1. Lagrangian upset_lagr.k
2. ALE upset_ale.k
3. Eulerian upset_euler.k

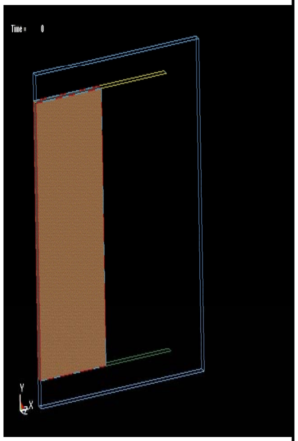
LSTC



Langrange



ALE



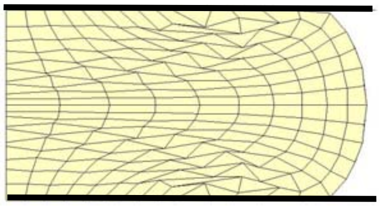
Euler

Chapter 9 - 9

upset_lagr.k versus upset_ale.k

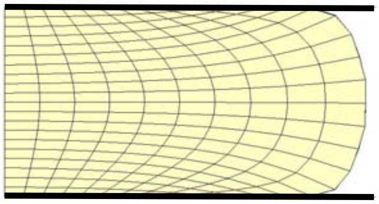
LSTC

upset_lagr.k



1. Langrangian - Mesh deforms with material and can become tangled.

upset_ale.k



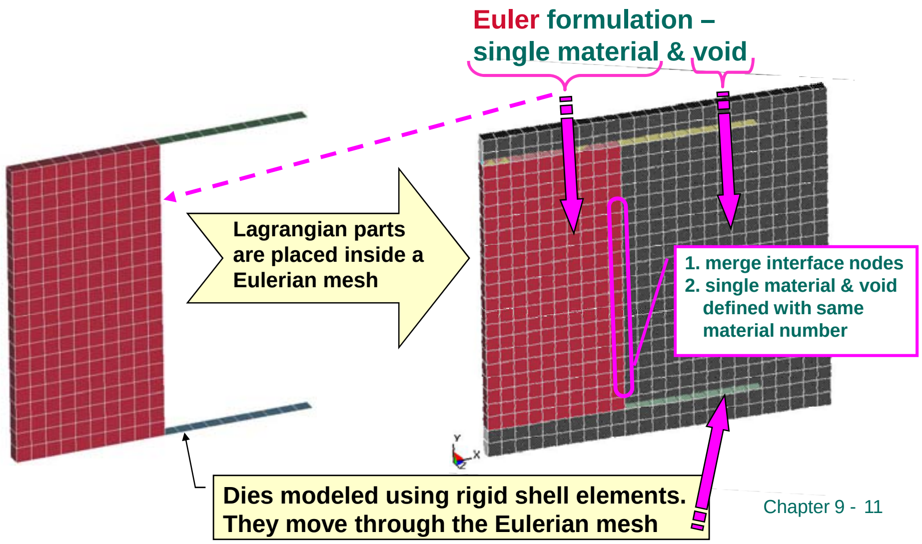
2. ALE - Mesh deforms with material and is smoothed.

Disadvantage:
Loss in fidelity due to smoothing.

Chapter 9 - 10

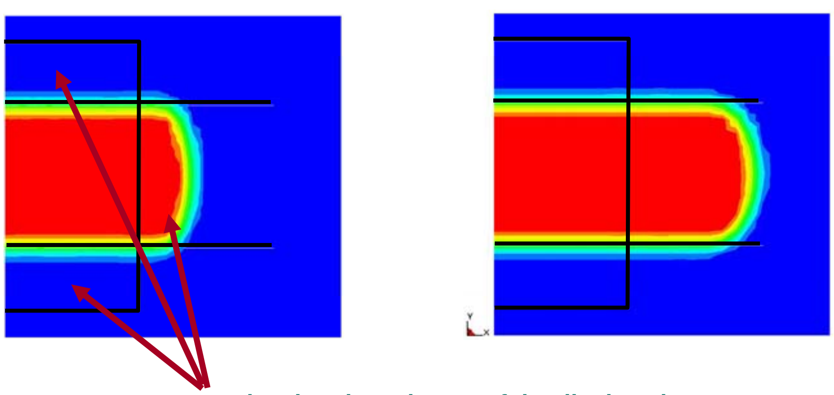
Eulerian upset_euler.k

LSTC



ALE (euler element) – watch out for Leakage

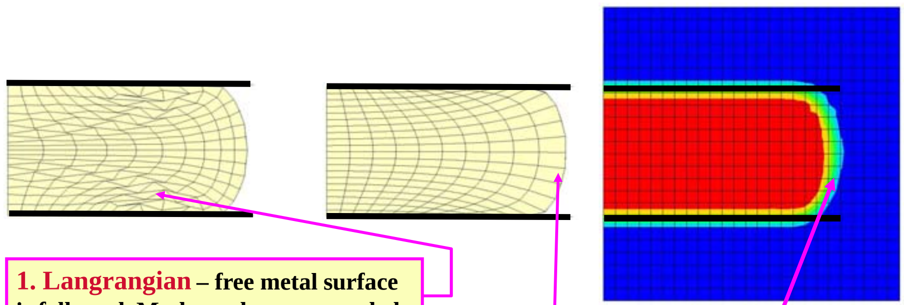
LSTC



Notice that the volumes of the displaced material are not equal. Nodes on the z-surfaces must be constrained to prevent material outflow in this problem.

Final deformed shape

LSTC



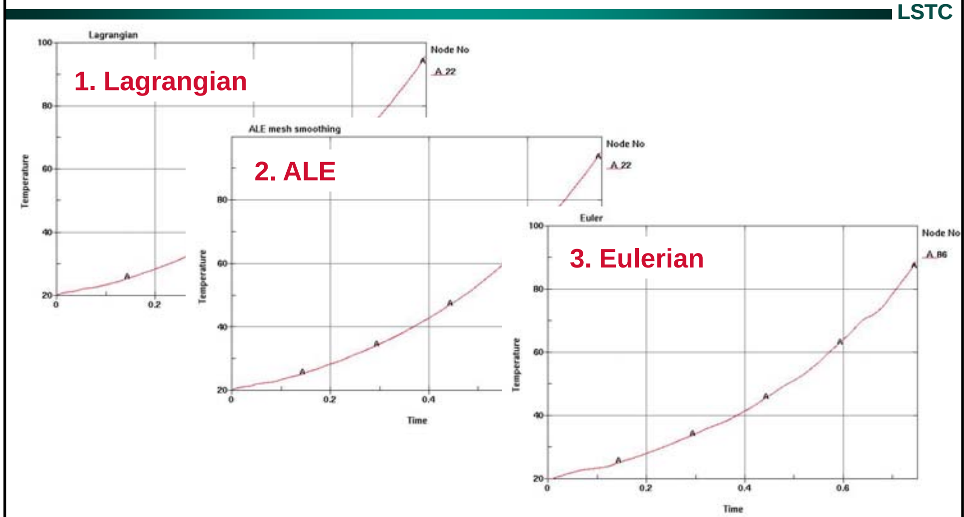
1. Langrangian – free metal surface is followed. Mesh can become tangled.

2. ALE – free metal surface is followed. Mesh deforms with material and is smoothed.

3. Euler - free metal surface is inferred from contours of "volume fraction". Material flows through mesh.

Thermal – Mechanical Analysis plastic work converted to heat

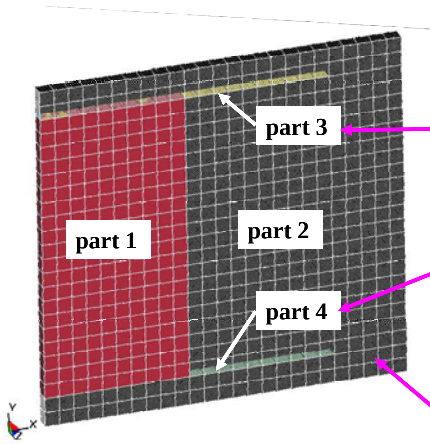
LSTC



*CONSTRAINED_LAGRANGE_IN_SOLID

defines the coupling interaction between a Lagrangian entity and a Eulerian entity (upset_euler.k)

LSTC




```
*CONTROL_ALE
0,1,1,-1.

*CONSTRAINED_LAGRANGE_IN_SOLID
3,1,1,0,2,4,2
0,0,100000.,1.

*CONSTRAINED_LAGRANGE_IN_SOLID
4,1,1,0,2,4,2
0,0,100000.,1.

*SET_PART_LIST
1
1,2

*INITIAL_VOID_PART
2
```

P1 & P2 are combined into a single entity for coupling

P1 & P2 are defined with the same material type. P2 is declared as initially void.

*CONTROL_ALE keyword

upset_ale.k

LSTC

```
*CONTROL_ALE
   DCT     NADV     METH     AFAC     BFAC     CFAC     DFAC
             25        1       1.
```

Arbitrary Lagrangian Eulerian (ALE) formulations may be thought of as automatic rezoning algorithms:

1. stopping the calculation when the mesh is distorted
   NADV = number of cycles between advection

Steps 2 & 3 are expensive

2. smoothing the mesh (minimize element distortion)
   AFAC = simple average smoothing
   BFAC = volume weighted smoothing
   CFAC = isoparametric smoothing
   DFAC = equipotential smoothing

Advection is dissipative - the solution variable fields are smeared out.

3. remapping the solution from the distorted mesh to the smooth mesh
   eq. 1: donor cell (1st order accurate)
   eq. 2: Van Leer (2nd order accurate)

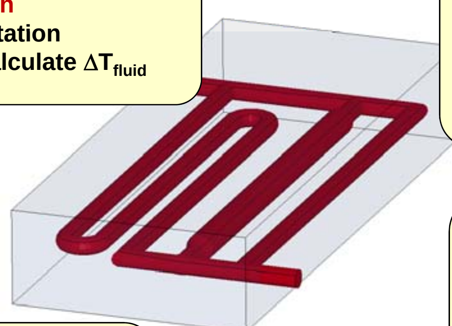
Chapter 10 - thermal-fluid coupling

LSTC

Modeling flow through a pipe with fluid structure interaction

LSTC

1. Boundary convection
   - must know h
   - fast computation
   - does not calculate ΔT_{fluid}



2. Bulk Flow
   - must know h & flow rate in each pipe
   - calculates ΔT_{fluid}
   - Slow computation, non-sym stiffness

3. ALE
   - must know h
   - laminar flow only
   - slow computation
   - fluid structure interaction (leakage)

4. Navier Stokes CFD
   - solution models boundary layer, calculates h
   - very slow computation
   - LS-980

How do you determine h

Problem definition

LSTC

Pipe diameter = D = 15mm = 0.015 m

Pipe cross section area = A = $\pi D^2/4 = \pi(0.015)^2/4$ = 1.77e-04 m^2

Volumetric flow rate = G = 20 l/min = 0.02 m^3/min = 3.33e-04 m^3/sec

Flow velocity = G/A = 1.89 m/sec

Pipe wall temperature = T_{wall} =100C

Water temperature = T_{fluid} = 20C

How do you determine h

Water properties

LSTC

T [C]	ρ [kg/m^3]	C_p [J/kg C]	μ [kg/m s]	k [W/m C]
20	998.	4182.	1.002e-03	0.603
40	992.	4179.	0.651e-03	0.632
60	983.	4185.	0.462e-03	0.653
80	972.	4197.	0.350e-03	0.670
100	958.	4216.	0.278e-03	0.681

How do you determine h

Some preliminaries

LSTC

Fully developed – the effect of entrance conditions (e.g., pipe from a header) on h are negligible.

$$\frac{L}{D} > 40$$

Fluid properties are evaluated at the film temperature, T_{film}

$$T_{film} = \frac{T_{wall} + T_{fluid}}{2} = \frac{100 + 20}{2} = 60$$

Reynolds number

$$\mathrm{Re} = \frac{V\rho D}{\mu} = \frac{(1.89)(983)(0.015)}{0.462*10^{-3}} = 6.03*10^{4}$$

Prandtl number

$$\mathrm{Pr} = \frac{c_p \mu}{k} = \frac{(4185.)(0.462*10^{-3})}{0.653} = 2.96$$

Chapter 10 - 5

How do you determine h

Classical empirical correlations

LSTC

Dittus-Boelter equation

$$h = 0.023 \frac{k}{D} \mathrm{Re}^{0.8} \mathrm{Pr}^{n}$$

n=0.3 for cooling of the fluid
n=0.4 for heating of the fluid

$$= 0.023 \frac{0.653}{0.015} \left(6.03*10^{4}\right)^{0.8} (2.96)^{0.4} = 10{,}300 \ \frac{W}{m^2 C}$$

Sieder-Tate equation

$$h = 0.023 \frac{k}{D} \mathrm{Re}^{0.8} \mathrm{Pr}^{n} \left(\frac{\mu_{bulk}}{\mu_{wall}}\right)^{0.14}$$

What do you do if the pipe is not perfectly smooth

μ(T) correction factor

Chapter 10 - 6

How do you determine h

Gnielinski correlation

LSTC

$$h = \left(\frac{k}{D}\right)\left[\frac{(f/8)(\mathrm{Re}-1000)\mathrm{Pr}}{1+12.7(f/8)^{0.5}\left(\mathrm{Pr}^{2/3}-1\right)}\right] = 11{,}400\ W/m^2C$$

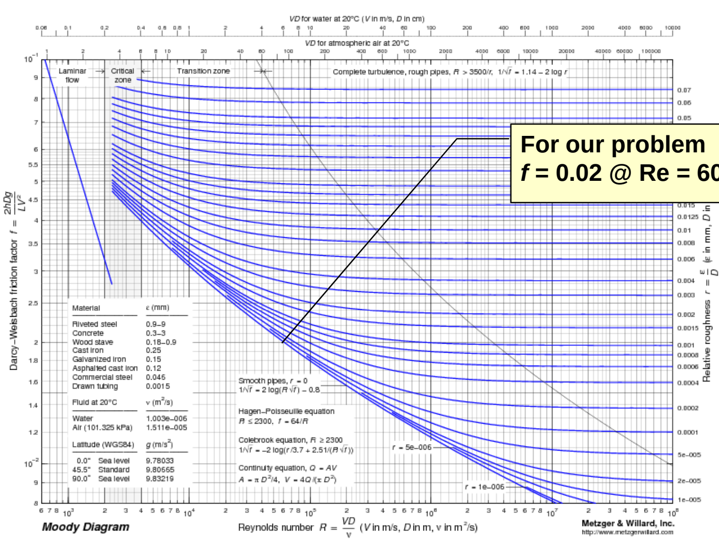
f **= Darcy–Weisbach friction factor (see next vu-graph for value)**

There are 2 definitions for *f*. The Darcy–Weisbach friction factor is 4 times larger than the Fanning friction factor, so attention must be paid to note which one of these is meant in any "friction factor" chart or equation being used. The Darcy–Weisbach factor is more commonly used by civil and mechanical engineers, and the Fanning factor by chemical engineers, but care should be taken to identify the correct factor regardless of the source of the chart or formula.

How do you determine h

Friction factor from: http://www.mathworks.com/matlabcentral/fx_files/7747/1/moody.png

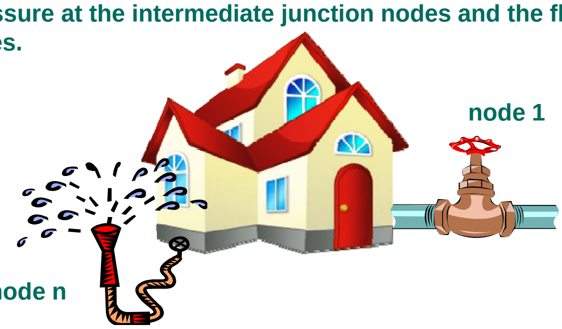
LSTC



Pipe network

LSTC

Think about pipes in your house. The starting point is the valve on the pipe entering your house. We will call this NODE 1. Node 1 is special and has a boundary condition specified. The BC is the pressure you would read on a pressure gauge at this location. The water enters your house and passes through several pipe junctions before it exits through your garden hose. Every junction is represented by a NODE. The last node also needs a BC specified. This BC is the mass flow rate. The pipe flow code will calculate the pressure at the intermediate junction nodes and the flow rate through the pipes.

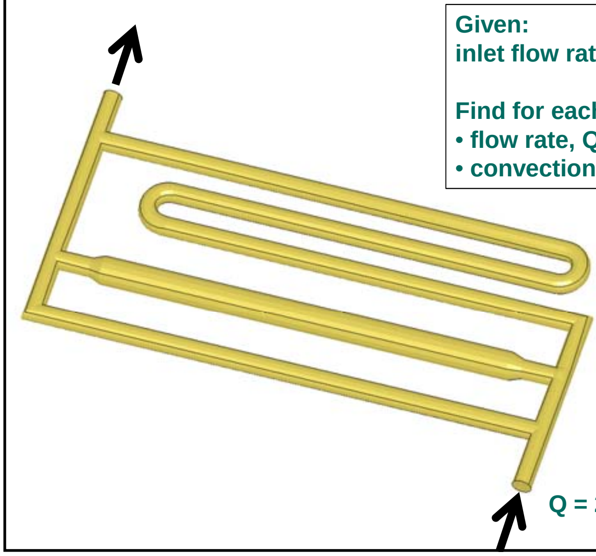


Pipe network

LSTC

Given:
inlet flow rate = 20 liter/min

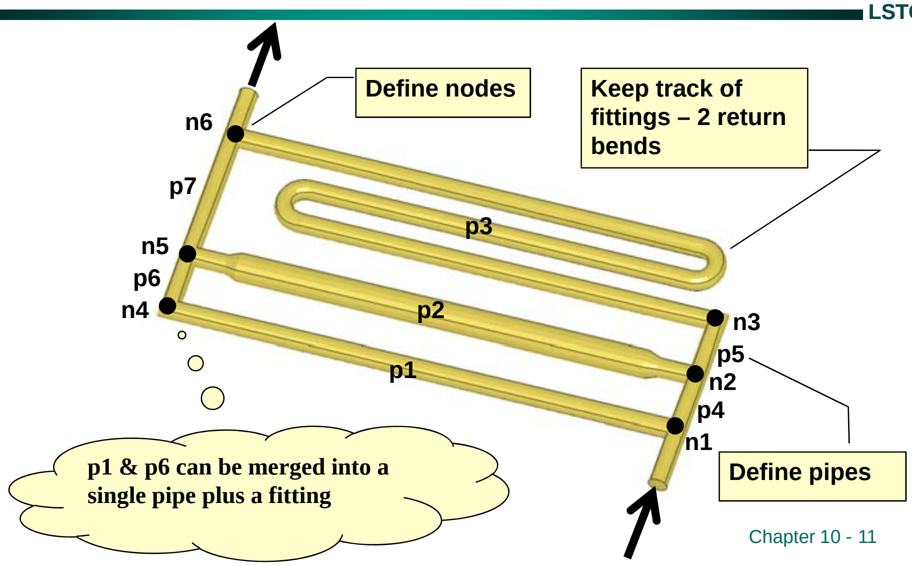
Find for each pipe:
- flow rate, Q
- convection heat transfer coefficient, h



Pipe network

Define nodes and pipes

LSTC



Pipe network

Solution algorithm

LSTC

Solve:

Bernoulli equation

$$\left(\frac{V_1^2-V_2^2}{2g}\right)+\left(\frac{P_1-P_2}{\rho g}\right)+\left(z_1-z_2\right)=H_f$$

Friction equation

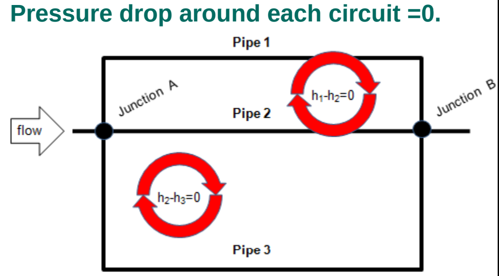
$$H_f=f\,\frac{L}{D}\frac{V^2}{2g}+H_{fitting}$$

Gnielinski equation

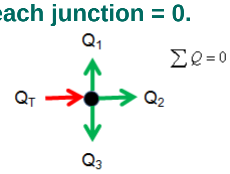
$$h=\left(\frac{k}{D}\right)\left[\frac{(f/8)(\mathrm{Re}-1000)\mathrm{Pr}}{1+12.7(f/8)^{0.5}\left(\mathrm{Pr}^{2/3}-1\right)}\right]$$

Subject to:

Pressure drop around each circuit =0.



Flow into each junction = 0.



Pipe network

LSTC

Pipe type	Roughness, e [mm]
Cast iron	0.25
Galvanized iron	0.15
Steel or wrought iron	0.046
Drawn tubing	0.0015

Fitting type	Equivalent length L_e/D
Globe valve	350
Gate valve	13
Check valve	30
90º std. elbow	30
90º long radius	20
90º street elbow	50
45º elbow	16
Tee flow through run	20
Tee flow through branch	60
Return bend	50

Pipe network

LSTC

input							output	
Pipe	N1	N2	Length [m]	Dia. [mm]	Rough [mm]	Ftg. [L_e/D]	Q [l/min]	h [W/m²C]
1	1	4	1	10	0.05		5.7	5600
2	2	5	1	20	0.05		9.7	2400
3	3	6	3	10	0.05	100	4.5	4600
4	1	2	0.2	10	0.05		14.2	11000
5	2	3	0.2	10	0.05		4.5	4600
6	4	5	0.2	10	0.05		5.7	5600
7	5	6	0.4	10	0.05		15.5	12000

3 methods to model pipe flow

LSTC

1
BOUNDARY_CONVECTION_SET
SID
HLCID HMULT

2
*BOUNDARY_THERMAL_BULKNODE
NID PID NBNSEG VOL HLCID HMULT

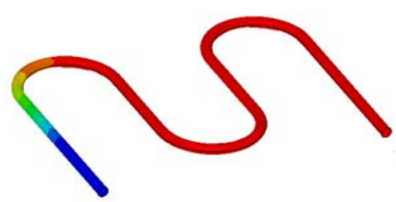
3
CONSTRAINED_LAGRANGE_IN_SOLID (i.e., ALE)
card 1
card 2
CQ = h

Chapter 10 - 15

Modeling flow through a pipe

LSTC

BULK FLOW is a lumped parameter approach to model fluid flow in a pipe. This keyword defines a contiguous set of beam elements that trace the centerline of the flow path. The beam node points are called **BULK NODES** and have special attributes in addition to their (x,y,z) location.

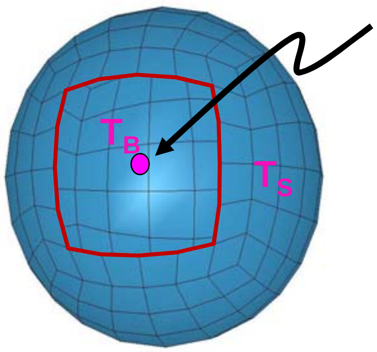


Chapter 10 - 16

BULKNODE – modeling a gas or fluid in a container

***BOUNDARY_THERMAL_BULKNODE**

LSTC

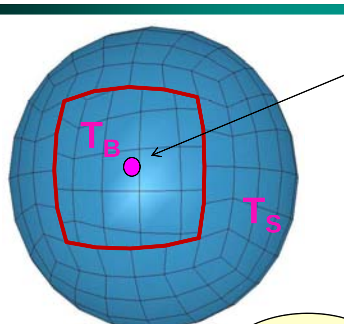


BULKNODE -This is a lumped parameter approach to model a fluid inside a rigid container. A node is defined with a **specified volume, density, and heat capacity.** The node coordinates are arbitrary, but it makes sense to place the node in the correct geometric position for visualization. The **surface segments** of the container are also defined so the bulk node can exchange heat by convection and radiation to the container.

> **Note that we are not modeling conduction in the fluid. The entire fluid volume is homogeneous at temperature T_B. The fluid temperature changes due to convection and radiation heat exchange with the container segments at T_S.**

BULKNODE – modeling a gas or fluid in a container

LSTC



The heat flow between the bulk node, B, and the surrounding surface, S, is given by

$$\dot{q}'' = h\left(T_S^a - T_B^a\right)^b$$

The value of h has the greatest uncertainty. The section on "How do you determine h" shows a hand calculation. Or, you may run a CFD code to numerically determine h.

BULKNODE – modeling a gas or fluid in a container

BOUNDARY_THERMAL_BULKNODE keyword

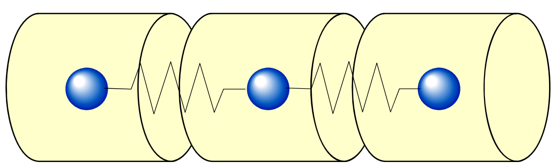
LSTC

```
*BOUNDARY_THERMAL_BULKNODE
NID    PID    NBNSEG    VOL    LCID    H    A    B
```

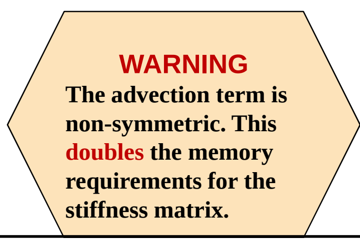
NID	bulk node number
PID	this bulk node is assigned a PID which in turn assigns material properties
NBNSEG	number of surface segments surrounding the bulk node
VOL	volume of bulk node (i.e., cavity volume – calculated by LSPP during mesh generation)
LCID	load curve ID for heat transfer coefficient h
H	heat transfer coefficient h
A	exponent a
B	exponent b

BULKFLOW – modeling flow through a pipe

LSTC



Each BULKNODE represents a homogeneous slug of fluid. The BULKNODES are connected with a contiguous set of beam elements defining the fluid flow path. Using the BULKFLOW keyword we define a mass flow rate (or velocity V) for the beams. We then solve the advection-diffusion equation.



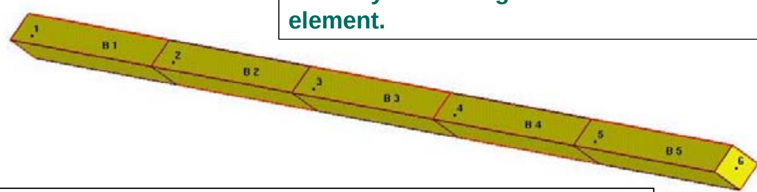
$$\rho c \frac{\partial T}{\partial t} + \rho c V \frac{\partial T}{\partial x} = K \frac{\partial^2 T}{\partial x^2}$$

BULKFLOW – modeling flow through a pipe

BOUNDARY_THERMAL_BULKFLOW keyword

LSTC

This keyword assigns a mass flow rate to a beam element.



*BOUNDARY_THERMAL_BULKFLOW_ { ELEMENT / SET

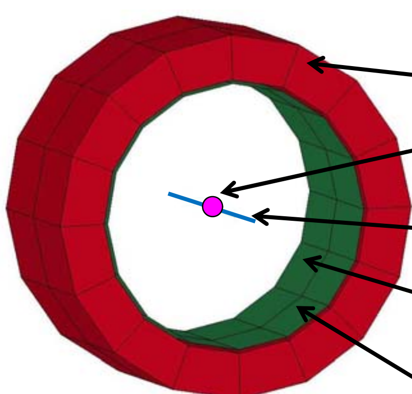
EID LCID MDOT

$$\dot{m} = \rho A V$$

BULKFLOW – modeling flow through a pipe

5 entities are required

LSTC



Five entities are required

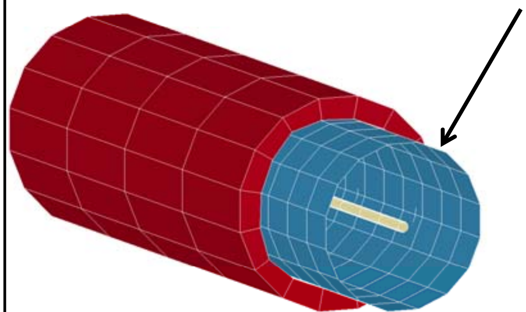
1. **Pipe / Die** – solid elements.
2. **BULKNODE**– defines fluid properties, fluid volume and heat transfer to pipe wall.
3. **BULKFLOW**– beam elements define the flow path (centerline of the pipe).
4. **Surface layer** – shell elements define the outer boundary surface of the fluid.
5. **Contact** - used for fluid to structure thermal interaction.

LS-PrePost can create these entities

BULKFLOW – modeling flow through a pipe

fluid – structure interaction

LSTC



The shell elements define the outer boundary surface of the fluid. However, give them the same material properties as the pipe because we don't want a temperature drop through the thickness of the shell. These elements also provide a method to connect a different fluid mesh to the pipe mesh by defining contact. Use a large h so that there is no temperature drop across the contact surface. The heat transfer between the fluid and wall is controlled by h defined on the BULKNODE keyword

Chapter 10 - 23

BULKFLOW – modeling flow through a pipe

Required keywords

LSTC

*PART
*ELEMENT_SOLID
} 1. define pipe / die

*PART
*BOUNDARY_THERMAL_BULKNODE
} 2. define bulk node

*PART
*ELEMENT_BEAM
*BOUNDARY_THERMAL_BULKFLOW_ELEMENT
} 3. define bulk flow

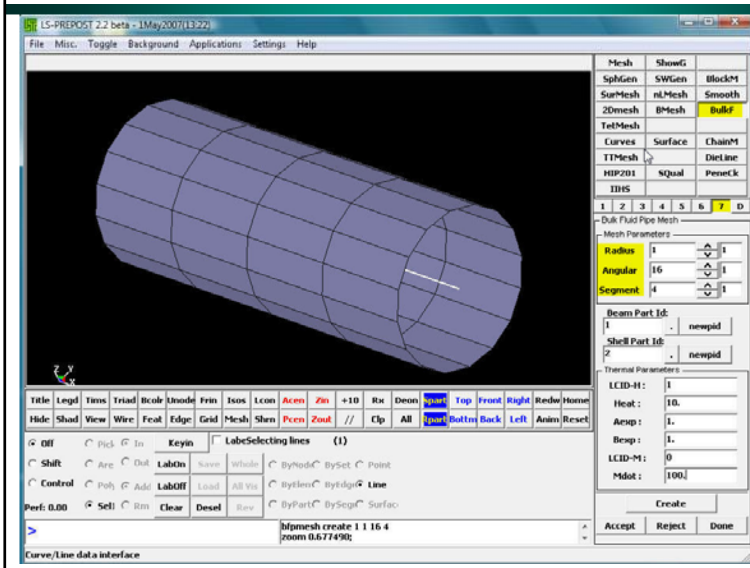
*PART
*ELEMENT_SHELL
} 4. define surface layer

*CONTACT_SURFACE_TO_SURFACE
} 5. Fluid structure interaction

Chapter 10 - 24

Using LS-PrePost to create BULKNODE & BULKFLOW keywords

LSTC



Bulk flow button

Screen 7

Mesh

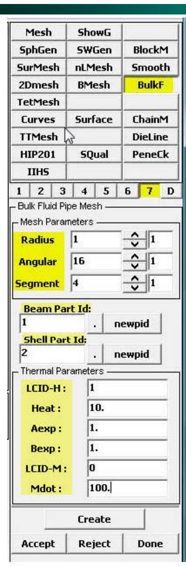
Part definitions

Fluid structure interaction

Chapter 10 - 25

Using LS-PrePost to create BULKNODE & BULKFLOW keywords

LSTC



Bulk flow button

Screen 7

Generate mesh
- **radius of pipe**
- **number of angular segments**
- **number of axial segments**

Beam PID - used to associate fluid material properties to the BULKFLOW elements.

Shell PID - used to associate fluid material properties to the BULKNODES and shell outer surface layer.

Fluid structure interaction $\dot{q}'' = h\left(T_s^a - T_b^a\right)^b$

Fluid mass flow rate

Chapter 10 - 26

Using LS-PrePost to create BULKNODE & BULKFLOW keywords

LSTC

Shown is a serpentine flow channel passing through a die

Curve defining flow path

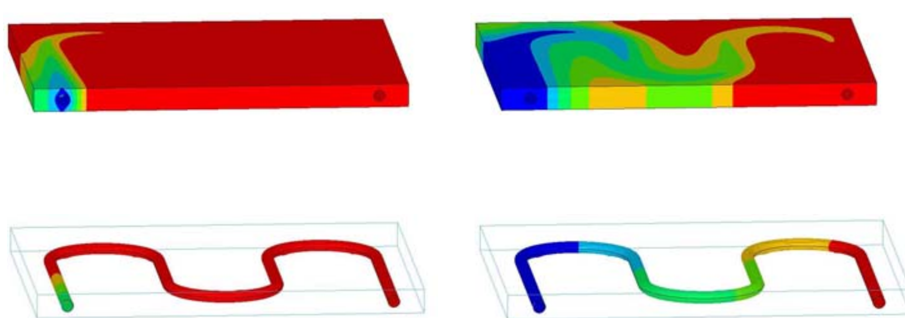
Radial 5
Axial 20

Radial 16
Axial 200

Application – die cooling

LSTC

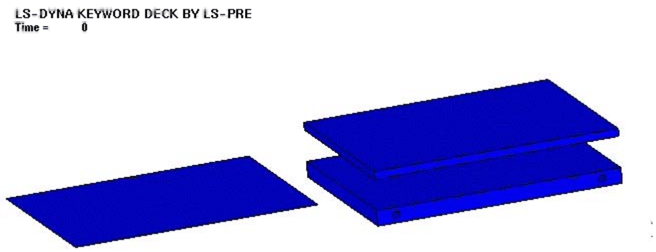
A Bulk Fluid Flow algorithm is used to model the energy exchange between the cold fluid flowing through the die cooling channels.



Application – die cooling

hot_forming_serpantine_cooling_channel.k

LSTC



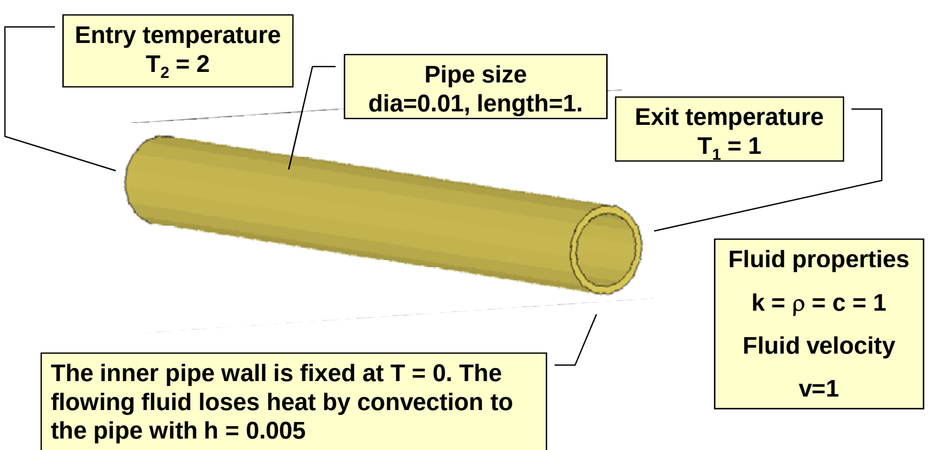
Chapter 10 - 29

Workshop problem: Advection – Diffusion

pipe_round.k

LSTC

Consider steady state 1-dimensional bulk fluid flow through a pipe



Chapter 10 - 30

Workshop problem: Advection – Diffusion

pipe_round.k

LSTC

Pipe geometry

- x = half length = 0.5
- d = diameter = 0.01
- p = perimeter = pd =0.0314
- A = cross sectional area = pd^2 /4= 7.85*10^{-5}

Fluid data

- r = density = 1.
- k = thermal conductivity = 1.
- c = heat capacity = 1.
- a = thermal diffusivity = k/pc = 1.
- V = velocity = 1.
- m = mass flow rate = r*A*v = 7.85*10^{-5}

Boundary conditions

- h = convection coefficient = 0.005
- T_0 = pipe wall temperature = 0.
- T_1 = inlet (x=0.) temperature = 2.
- T_2 = exit (x=2l) temperature = 1.

Workshop problem: Advection – Diffusion

Carslaw & Yaeger, Conduction of Heat in Solids, 2nd ed., p148

LSTC

$$T = \frac{T_2 e^{-V(L-x)/2\alpha} \sinh \xi x + T_1 e^{Vx/2\alpha} \sinh \xi (L-x)}{\sinh \xi L}$$

$$\xi = \sqrt{\frac{V^2}{4\alpha^2} + \frac{hp}{Ak}}$$

$$\alpha = \frac{k}{\rho c}$$

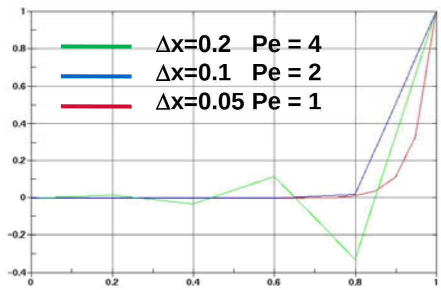
Three analytical solutions to benchmark against:	T at x = 0.5
1) pure conduction	T = 1.500
2) conduction + advection (h=0)	T = 1.622
3) conduction + advection + convection	T = 1.293

BOUNDARY_THERMAL_BULKFLOW_UPWIND

Advanced feature

LSTC

For many flow problems, dissipative mechanisms are only significant in a narrow layer typically adjacent to a boundary. Computational solutions obtained with grids appropriate to the main flow region are often oscillatory when the true solution changes rapidly across the boundary layer.



1D steady advection diffusion problem

$$V\frac{dT}{dx} - \alpha\frac{d^2T}{dx^2} = 0$$

with T(0)=0. and T(1)=1.

'Wiggles' occur at cell Peclet numbers greater than 1.

$$Pe = \mathrm{Re}\,\mathrm{Pr} = \left(\frac{\rho V\Delta x}{\mu}\right)\left(\frac{\mu c}{k}\right) = \frac{V\Delta x}{\alpha}$$

Chapter 10 - 33

C.A.J. Fletcher, Computational techniques for Fluid Dynamics 1, Springer Verlag, 2nd ed., p293.

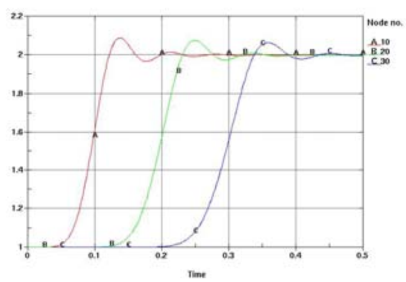
BOUNDARY_THERMAL_BULKFLOW_UPWIND

Advanced feature, upwind_transient.k

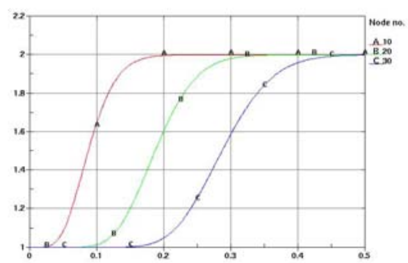
LSTC

UPWIND adds a term (sometimes called **artificial viscosity**) to the element stiffness matrix. This eliminates the 'wiggles' but also makes the solution more diffusive. Note that the curves are now not as steep and their shape is more spread out over time. Wiggles are gone but the solution is less accurate.

UPWIND off



UPWIND on



Transient 1D flow with a step change in entering fluid temperature. Shown is the temperature history at 3 locations down the pipe. Initial and boundary conditions: T(x,0)=1, T(0,t)=2.

Chapter 10 - 34

ALE fluid material models

LSTC

Incompressible inviscid flow

***MAT_ELASTIC_FLUID**
- Density
- Bulk modulus

Incompressible viscous laminar flow

***MAT NULL**
- Density
- Viscosity
- Include an EOS

You can not model **turbulent** flow using ALE

```
*EOS_GRUNEISEN
*EOS_LINEAR_POLYNOMIAL
```


ALE fluid material models

only consider the normal (pressure) stresses

LSTC

MAT_ELASTIC_FLUID and MAT_NULL

These materials allow equations of state to be considered without computing deviatoric stresses.

$$\sigma_{ij} = \overbrace{\underbrace{\sigma^{e}_{ii}}_{\text{Dilatational (normal)}} + \sigma^{e}_{ij}}^{\text{Elastic}} + \overbrace{\sigma^{p}_{ij}}^{\text{Plastic}}$$

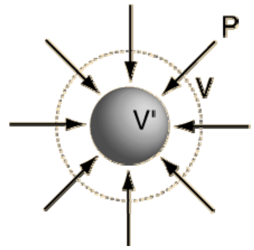
$$\underbrace{\sigma^{e}_{ij} + \sigma^{p}_{ij}}_{\text{Deviatoric (shear)}}$$

Bulk modulus

modulus of compressibility

LSTC

The bulk elastic properties of a material determine how much it will compress under a given amount of external pressure. The ratio of the pressure to the fractional change in volume is called the bulk modulus of the material.



$$B = \frac{P}{-\Delta V / V}$$

Bulk modulus values:

Steel	B = 160e+09 N/m^2
Aluminum	B = 71.3e+09 N/m^2
Water	B = 2.2e+09 N/m^2

For an elastic solid

$$B = \frac{E}{3(1-2\upsilon)}$$

Chapter 10 - 37

Speed of sound

D.S. Drumheller, Introduction to Wave Propagation in Nonlinear Fluids and Solids, 1998, ISBN 0521587468

LSTC

The propagation speed of traveling waves are characteristic of the media in which they travel. The speed of sound in air and other gases, liquids, and solids is predictable from their density and elastic properties of the media (bulk modulus, B).

λ, μ → Lame constants
μ=0 (shear) for gas & liq.

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\lambda + \frac{2}{3}\mu}{\rho}}$$

Used in
*EOS_GRUNEISEN

bulk velocity

$$v = \sqrt{\frac{2.2e+09\ \ N/m^2}{1000\ \ kg/m^3}}$$ = 1483 m/s for water

elastic velocity

Note $B = \rho c^2$

NOTE – the elastic wave (speed of sound) in a solid is $v = \sqrt{\frac{\lambda + 2\mu}{\rho}}$

Chapter 10 - 38

EOS_LINEAR_POLYNOMIAL

LSTC

```
*EOS_LINEAR_POLYNOMIAL
EOSID    C0    C1    C2    C3    C4    C5    C6
```

$$P = C_0 + C_1\mu + C_2\mu^2 + C_3\mu^3 + \left(C_4 + C_5\mu + C_6\mu^2\right)E \qquad \begin{cases} \mu = \dfrac{\rho}{\rho_0} - 1 \\ E = \text{internal energy} \end{cases}$$

for a Gas

$C_0 = C_1 = C_2 = C_3 = C_6 = 0$

$C_4 = C_5 = \gamma - 1$

$\gamma = Cp / Cv$

$$P = (\gamma - 1)\frac{\rho}{\rho_0}E$$

$C_0 = C_2 = C_3 = C_4 = C_5 = C_6 = 0$

C_1 = bulk modulus B

Chapter 10 - 39

EOS_GRUNEISEN

LSTC

```
*EOS_GRUNEISEN
EOSID   C   S1   S2   S3   GAMA0   A   E0
```

$$P = \frac{\rho_0 C^2 \mu\left[1 + \left(1 - \frac{\gamma_0}{2}\right)\mu - \frac{a}{2}\mu^2\right]}{\left[1 - (S_1 - 1)\mu - S_2\frac{\mu^2}{\mu + 1} - S_3\frac{\mu^3}{(\mu + 1)^2}\right]^2} + (\gamma_0 + a\mu)E$$

For an incompressible liquid near atmospheric pressure → set all parameters = 0, except for C = sound speed ⇒ $P = \rho_0 C^2 \mu$

D. Steinberg, Equation of State and Strength Properties of Selected materials, LLNL, UCRL-MA-106439, 1996.

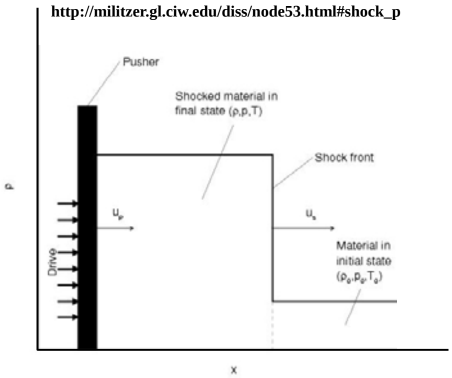
Chapter 10 - 40

EOS_GRUNEISEN

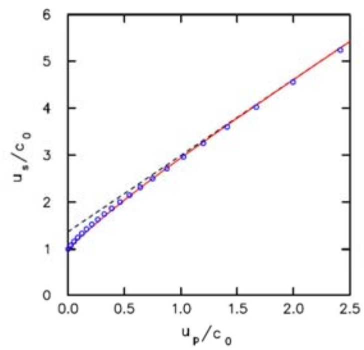
Gruneisen parameters are obtained from a curve fit of the shock velocity vs. particle velocity curve

LSTC

http://militzer.gl.ciw.edu/diss/node53.html#shock_p



http://t14web.lanl.gov/Staff/rsm/Talks/EOStalk.pdf



$$\boxed{u_s = C_0 + S_1 u_p + S_2\left(\frac{u_p}{u_s}\right)u_p + S_3\left(\frac{u_p}{u_s}\right)^2 u_p}$$

EOS_GRUNEISEN

Parameters for water

LSTC

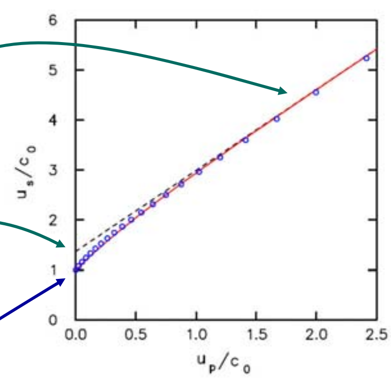
Gruneisen Parameters for water

C_0	1.65 mm/µs
S_1	1.92
γ_0	0.1

Warning – most Gruneisen parameters are obtained from experiments in the 20 to 150 kBar range (i.e., the linear portion of the curve).

The intercept is not necessarily the speed of sound

$$u_s = C_0 + S_1 u_p + S_2\left(\frac{u_p}{u_s}\right)u_p + S_3\left(\frac{u_p}{u_s}\right)^2 u_p$$



R.W. Woolfolk, "A Universal Hugoniot for Liquids", Thermochimica Acta, 5 (1973) 409-

$$u_s = C_0 + 1.62u_p + 0.37C_0\left[1 - \exp\left(-2u_p / C_0\right)\right]$$

LS-PrePost

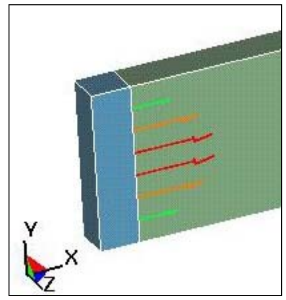
displaying velocity vectors

LSTC

1. click Vector

2. scroll to Velocity

3. Apply



LS-PrePost

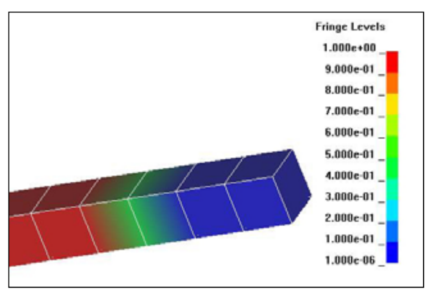
displaying volume fraction

LSTC

1. click Fcomp

2. click Misc

3. history var #2 → volume fraction
history var #1 → density

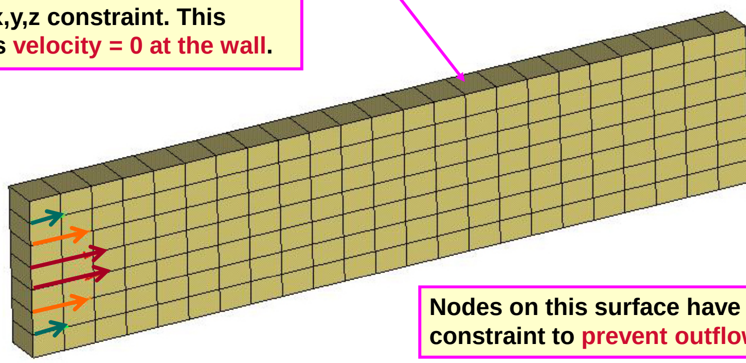


Workshop Problem, channel.k

1-dimensional laminar fluid flow

LSTC

Top and bottom surface nodes have a x,y,z constraint. This imposes **velocity = 0 at the wall**.



Nodes on this surface have a z constraint to **prevent outflow**.

Nodes on this surface are defined with a **parabolic x-velocity** and y,z displacement constraints to create 1D flow.

Chapter 10 - 45

Workshop Problem, channel.k

1-dimensional laminar fluid flow

LSTC

```
*PART
fluid channel
$#       pid      secid         mid      eosid        hgid
           1          1           1          1           1
*SECTION_SOLID
$#     secid     elform
           1          6
*MAT_NULL
$#       mid         ro          pc         mu
           1  1000.0000     -10.000  0.0400000
*EOS_GRUNEISEN
           1  1.5000000
 0.0000000
*HOURGLASS
$#      hgid        ihq          qm
           1          1      0.0001
```

Euler element

*MAT_NULL models viscous laminar flow.
mu = viscosity [Kg/m sec]

Scaled to make the problem run quicker (for water = 1500.)

Turn off the hourglass when dealing with gases and fluids. (see *MAT_NULL in keyword Users Manual)

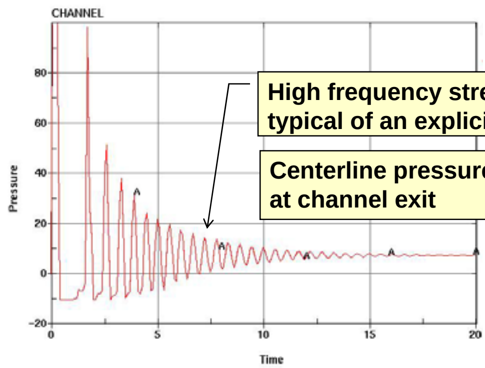
Note: mu = 0.001 for water, but the problem will have to run longer to damp out pressure oscillations

Chapter 10 - 46

Workshop Problem, channel.k

1- dimensional laminar fluid flow

LSTC



Workshop

1. run channel.k
2. using LS-POST select the element at the entrance on the centerline. Plot the pressure history of the element. Compare with the analytical answer below

U_{max} = centerline velocity

μ = viscosity

L = channel length

l = channel height

$$\frac{Kg}{m\ \sec} * \frac{m\ \sec}{m\ \sec} = \frac{N\ \sec}{m\ m} = Pa\ \sec$$

$$\Delta P = 8U_{\max}\mu\frac{L}{l^2} = (8)(0.2)(0.04)\frac{0.19}{0.04^2} = 7.6\ \ Pa$$

R.W. Fox, Introduction to Fluid Mechanics, 1973, p 318, eq. 8.6e

Chapter 10 - 47

Workshop Problem, channel.k

1- dimensional laminar fluid flow

LSTC

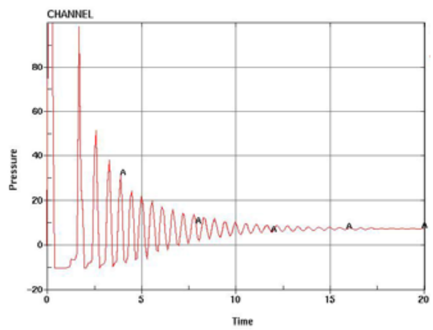
3. Display density fringes or plot the density history.
   - a) Is the flow incompressible? __________
4. Change the Gruneisen parameter from C=1.5 to C=15.
   - a) Observe the execution time.
   - a) Look at density. Is the flow incompressible? __________
   - b) Look at centerline pressure and compare with the analytical answer.
5. Change the Gruneisen parameter from C=1.5 to C=0.15
   - a) Observe the execution time.
   - a) Look at density. Is the flow incompressible? _________
   - b) Look at centerline pressure and compare with the analytical answer.

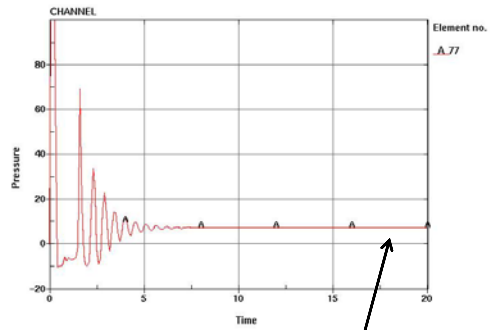
Chapter 10 - 48

Workshop problem with ambient element

Using an ambient element will attenuate the pressure oscillations.

LSTC





An Ambient Element acts as a reservoir

```
*SECTION_SOLID
$ SECID    ELFORM    AET
              7         N
```

3 = pressure outflow
4 = pressure inflow

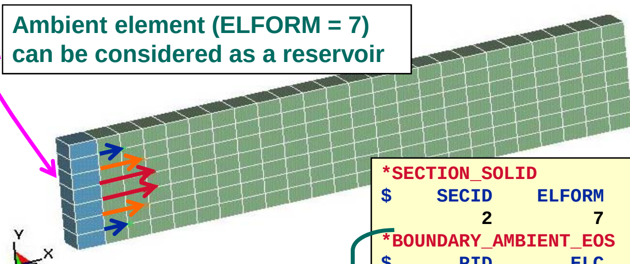
Chapter 10 - 49

Workshop problem with ambient element

channel_ambient.k

LSTC

Ambient element (ELFORM = 7) can be considered as a reservoir



Ambient Element Type 4: pressure inflow (default)

```
*SECTION_SOLID
$     SECID    ELFORM        AET
          2         7          4
*BOUNDARY_AMBIENT_EOS
$       PID       ELC       VFLC
          2         2          3
*DEFINE_CURVE
          2
      0.00000000E+00     0.00000000E+00
      1.00000000E+06     0.00000000E+00
*DEFINE_CURVE
          3
      0.00000000E+00     1.00000000E+00
      1.00000000E+06     1.00000000E+00
```

This defines the ambient element pressure, $P(e, v_f)$, based on the EOS used

For EOS_GRUNEISEN

$$P = \rho C^2 \left(\frac{1}{v_f} - 1 \right)$$

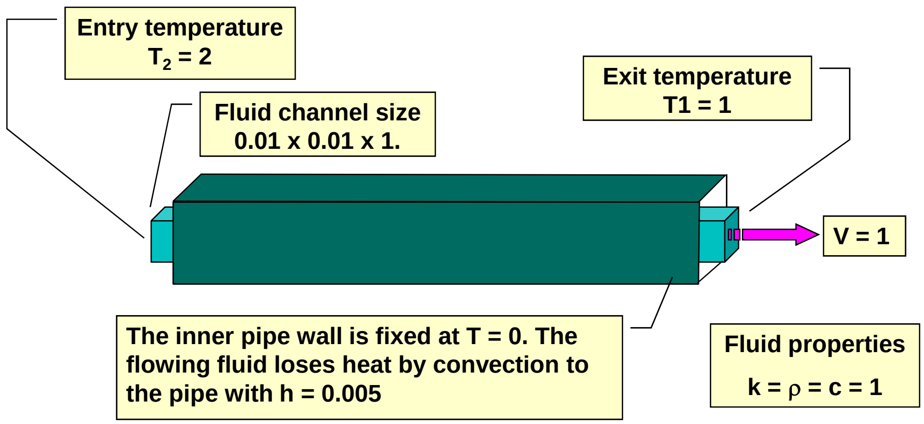
Set P = 0, since we are setting inlet velocity

Chapter 10 - 50

Workshop problem with advection – diffusion

LSTC

Consider 1-dimensional bulk fluid flow through a square channel



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Workshop problem with advection – diffusion

Analytical solution: Carslaw & Yaeger, <u>Conduction of Heat in Solids</u>, 2nd ed., p148

LSTC

Pipe length, L

$$T = \frac{T_2 e^{-V(L-x)/2\alpha} \sinh \xi x + T_1 e^{Vx/2\alpha} \sinh \xi (L-x)}{\sinh \xi L}$$

$$\xi = \sqrt{\frac{V^2}{4\alpha^2} + \frac{hp}{Ak}}$$

pipe perimeter
p = 4 * 0.01

$$\alpha = \frac{k}{\rho c}$$

Convection heat transfer area
A = 4 * 0.01 * 1.

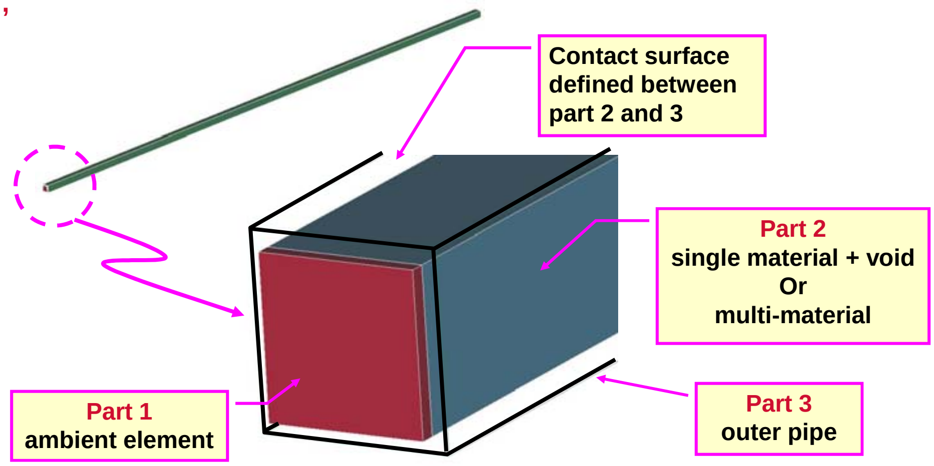
Three analytical solutions to benchmark against:	<u>T at x = 0.5</u>
1) pure conduction	T = 1.500
2) conduction + advection (h=0)	T = 1.622
3) conduction + advection + convection	T = 1.293

Chapter 10 - 52

Workshop problem with advection – diffusion

channel_sgl_matl+void.k, channel_multi_matl.k, channel_multi_matl+contact.k

LSTC



Workshop problem with advection – diffusion

channel_sgl_matl+void.k

LSTC

```
*PART
Fluid channel
$       pid      secid         mid      eosid
          2          2           2          2
*SECTION_SOLID
$     secid     elform
          2         12
*INITIAL_VOID_PART
$       pid
          2
*MAT_NULL
$       mid        rho
          2 1000.0000
*EOS_LINEAR_POLYNOMIAL
$     eosid         c0          c1
          2         0.     2.0e+09
```

A void region must be defined when using elform=12, single material plus void.

Use properties for the material that will fill the void region

Workshop problem with advection – diffusion

channel_sgl_matl+void.k

LSTC

Run channel_sgl_matl+void.k

Using LS-Post

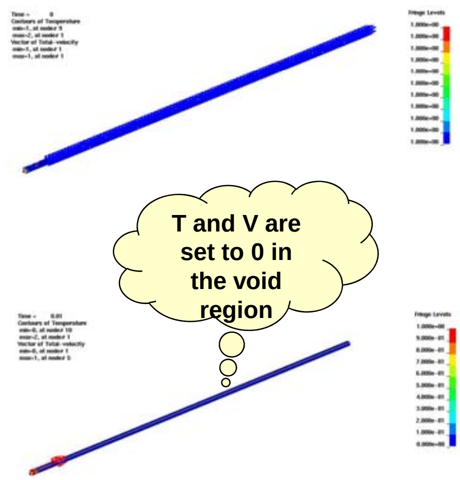
1. display Part 2, the fluid channel.

2. display fringes of temperature and velocity vectors at state 1.

What is T_{max} 2. What is T_{min} 1.
What is V_{max} 1. What is V_{min} 1.

3. display fringes of temperature and velocity vectors at state 2. Note that in a void region, T and V are set to 0.

What is T_{max} 2. What is T_{min} 0.
What is V_{max} 1. What is V_{min} 0.



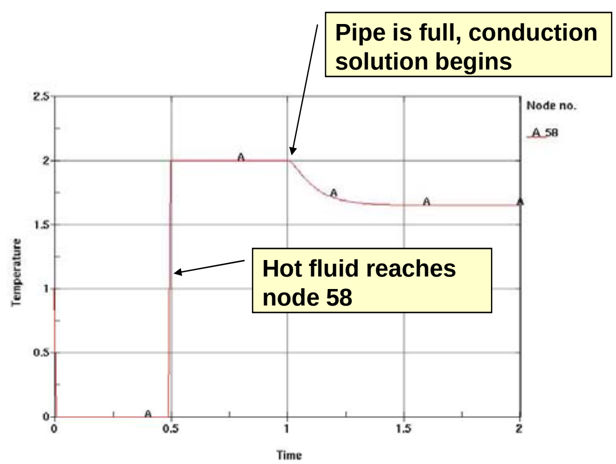
Workshop problem with advection – diffusion

channel_sgl_matl+void.k

LSTC

4. run an animation. Note the fluid (or temperature) front as it moves down the channel. When the last element is filled, the boundary condition T=1 at x=1 becomes active. Note the development of the conduction solution.

5. use the find command to locate node 58. Plot the temperature time history for this node.



Workshop problem with advection – diffusion

channel_multi_matl.k - The initial void part (elform=12) in the previous problem is defined as initially containing material for this problem (elform=11).

LSTC

```
*PART
fluid inflow ambient
$       pid      secid        mid      eosid
          1          1          1          1
*SECTION_SOLID
$     secid     elform        aet
          1         11          4
*PART
Fluid channel
$       pid      secid        mid      eosid
          2          2          2          2
*SECTION_SOLID
$     secid     elform        aet
          2         11
*ALE_MULTI_MATERIAL_GROUP
$       pid     idtype
          1          1
          2          1
```

Part 1: ambient element

Part 2: fluid channel

A multi-material group must be defined

Workshop problem with advection – diffusion

channel_multi_matl.k

LSTC

Run channel_multi_matl.k

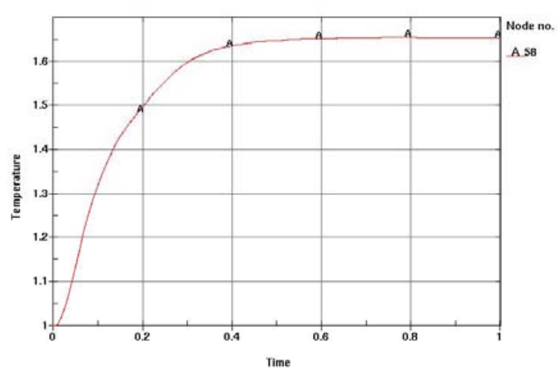
Using LS-Post

1. display part 2, the fluid channel

2. display fringes of temperature and velocity vectors at state 1 and 2. Because the pipe is filled with material, T and V exist throughout the pipe.

3. use the find command to locate node 58. Plot the temperature time history for this node.

The advection and conduction solution develop simultaneously because the entire pipe has fluid in it from time 0.



Contact between an ALE mesh and a Lagrangian mesh

channel_multi_matl+contact.k- Contact between the fluid channel (part 2) and the surrounding pipe (part 3) is included.

LSTC

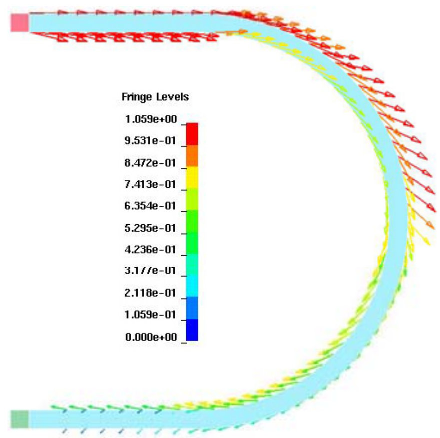
WARNING

You must define the perimeter surface of an ALE material as Lagrangian using
*ALE_REFERENCE_SYSTEM_GROUP
for contact with another Lagrangian surface.

I did not do the above for channel_multi_matl+contact.k because the flow is 1-dimensional. This is a dangerous approach because leakage can occur.

Notice the velocity vectors decreasing due to material leakage out of the mesh as we go around the curve.

channel_curved.k



Chapter 10 - 59

Contact between an ALE mesh and a Lagrangian mesh

LSTC

The following 3 methods are recommended for coupling between an ALE mesh and a Lagrangian mesh. Other methods may seem to work but they do not guarantee that fluid is not leaking out of the mesh. Such as perfectly aligning the perimeter of the ALE mesh with the Lagrangian mesh and then defining a contact surface. With these alternate methods you are playing around with algorithm epsilons that may work in one instance but not in another. The following methods are designed for this problem.

1) Merge the nodes on the interface between the ALE and Lagrangian mesh

2) Use the *CONSTRAINED_LAGRANGE_IN_SOLID keyword do define the coupling. Note → the Lagrangian solid must be surrounded by the ALE mesh.

3) Use *ALE_REFERENCE_SYSTEM_GROUP to define the perimeter of the ALE mesh as Lagrangian and then use tied contact.

Chapter 10 - 60

Plate Casting

plate_casting.k

LSTC

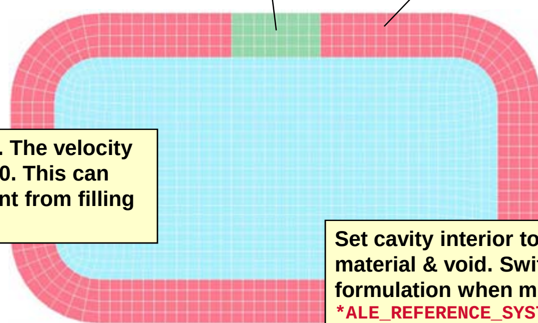


Plate Casting

contact

LSTC

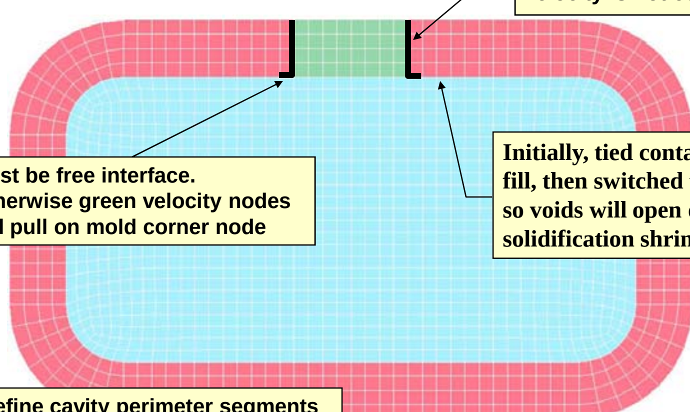


Plate Casting

Modeling issues

LSTC

Time	Action
0.0	Mold filling, green ambient elements set to T=1000 and v=10
	T_{inf}=800 to keep the metal mold hot. Otherwise, liquid metal will solidify on contact which will drastically increase strength in the Eulerian fluid elements. This will tear things apart due to high velocity. This approach is not necessary for a sand mold due to decreased heat transfer between the liquid metal and sand.
0.22	The mold is full, turn inflow off. Keep the elements as Eulerian to allow velocity to decay as we transition from fluid behavior to solid behavior during solidification.
0.30	Change the environment from 800 to 25C to start cool down.
0.32	Switch Eulerian formulation to Lagrangian formulation. Fluid velocities are very small and we are below the solidus temperature.
0.33	Switch contact between the cast part and mold from tied to sliding to allow shrinkage gaps to form.

Plate Casting

Modeling issues

LSTC

1. **Mold Filling** – the filling time was chosen such that all elements filled but not enough pressure was built up in the cavity to significantly deform the elastic mold. Some elements along the perimeter, especially in the curves, are just barely filled and cycle between filled and void on subsequent time steps. 90^o corners should be avoided.
2. **Mold heating** – When using a cold mold, the hot liquid metal would contact the mold surface and immediately cool down due to very good metal-to-metal heat conduction and tied contact. The fluid material properties would instantaneously increase in strength to solid properties. The nodes would retain the ALE fluid velocity. However, now due to increased strength, these nodes would shoot out and tear things apart. Therefore, it is necessary to keep the mold at a high temperature in relation with the temperature dependent mechanical material properties. This may not be as much of problem when using a sand mold due to its lower thermal conductivity.
3. **Switch to Lagrange Formulation** – Allow a settling time after inflow is turned off to allow the fluid velocity to decrease in magnitude. This switch is also needed for the contact (see below).
4. **Switch contact** – The coupling between an ALE and a Langrangian mesh can only be done through tied contact. However, now we wish to model shrinkage. This can be accomplished by switching the ALE mesh to Lagrangian and then switching from tied contact to sliding contact.

Plate Casting

Temperature fringes and velocity vectors

LSTC

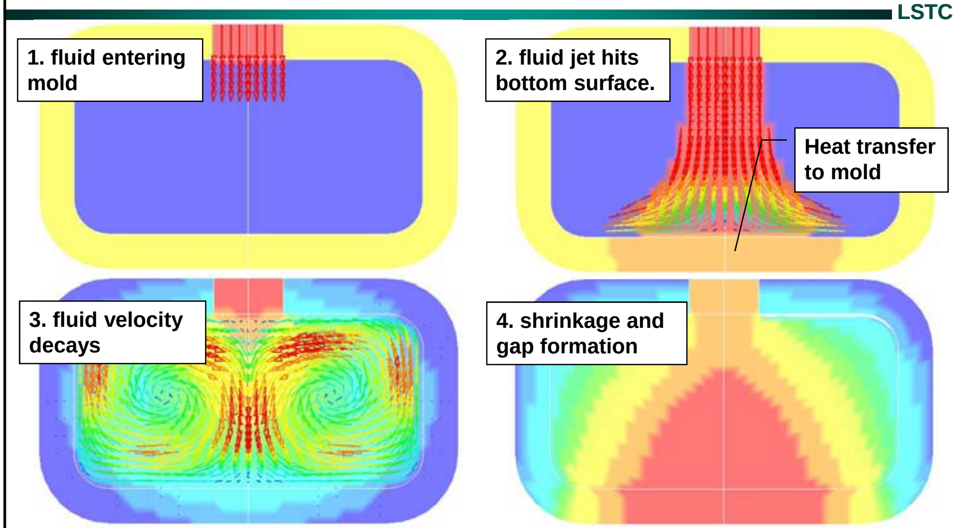
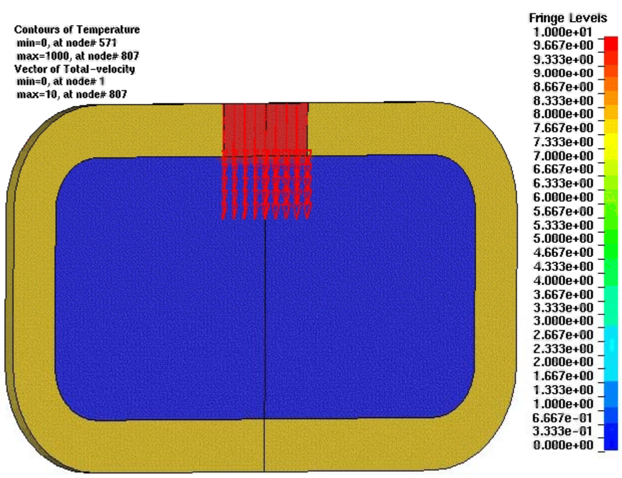


Plate Casting

LSTC



movie file – double click

LSTC

LSTC

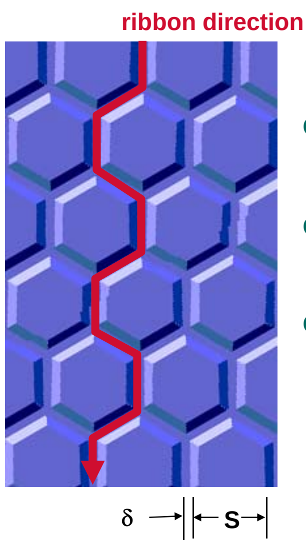
Chapter 11 – miscellaneous modeling methods

LSTC

Thermal conductivity of Honeycombs

D. Gilmore, Satellite Thermal Control Handbook, Aerospace Corp., p. C-12.

LSTC



Conduction in ribbon direction $\bar{k} = 1.5\dfrac{k\delta}{S}$

Conduction $\perp$ to ribbon direction $\bar{k} = \dfrac{k\delta}{S}$

Conduction through thickness $\bar{k} = 2.66\dfrac{k\delta}{S}$

S = face-to-face cell size

δ = ribbon thickness

Thermal conductivity of powders

LSTC

$$k_{powder} = 4k_{gas} \ln\left(\frac{k_{metal}}{k_{gas}}\right)$$

Equation was developed for spheres with $k_{metal}/k_{gas} >> 1$.

Authors state that "values for the effective thermal conductivity for packed beds of particles of different shape are not expected to be greatly different".

G.K. Batchelor, et. al., "Thermal or electrical Conduction Through a Granular Material", Proc. R. Soc. Lond. A. 335, 313-333 (1977).

Thermal conductivity of thin films (0.05 μm to 10 μm)

LSTC

k values for thin films may be 1 to 2 orders of magnitude lower than those for the corresponding bulk material

Material	Bulk k [W/mK}	Film k [W/mK]
Al_2O_3	38.5	18.3
HfO_2	1.70	0.05
SiO_2	1.38	0.45
TiO_2	10.5	0.48
ZrO_2	1.55	0.04

1. J.C. Lambropoulos, "Thermal Conductivity of Dielectric Thin Films", J. Appl. Phys., Vol. 66, No. 9, November 1989.
2. W.D. Nix, "Mechanical Properties of Thin Films", Metallurgical Transactions A, Vol. 20A, November 1989.

Macro-to-Micro scale transition

LSTC

The boundary between the microscale to macroscale regimes is

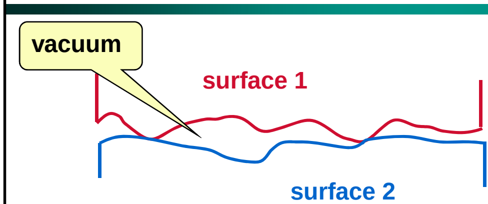
$$d = 7\Lambda \quad \begin{cases} d = \text{layer thickness} \\ \Lambda = \text{mean free path} \end{cases}$$

	Copper	silicon
d μm at 300K	0.2	0.3
d μm at 50K	3.0	60.0

M.I. Flick, "Heat Transfer Regimes in Microstructures", DSC-Vol. 32, Micromechanical Sensors, Actuators and Systems, ASME 1991.

Contact Conductance in vacuum

LSTC



$$\overline{E} = \frac{E_1 E_2}{E_2\left(1-\upsilon_1^2\right)+E_1\left(1-\upsilon_2^2\right)}$$

- σ = surface roughness (typical, 10 to 125 μin)
- tan θ = asperity slope (typical, 0.10 to 0.15)
- E = elastic modulus
- ν = Poisson's ratio
- k = metal thermal conductivity
- P = interface pressure

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$$

$$h = 1.55 \frac{k \tan\theta}{\sigma} \left(\frac{P / \sqrt{2}}{\overline{E} \tan\theta} \right)$$

D. Gilmore, Satellite Thermal Control Handbook, Aerospace Corp., p. 4-23.

Determining optical path difference (OPD)

LSTC

When an optically isotropic material is subject to mechanical stresses it becomes anisotropic. The change of the refractive index is called the stress birefringence or the photo-elastic effect.

$$dOPD(r) = \underbrace{\frac{\partial n}{\partial T} T(r)dz}_{\text{temperature diffusion}} + \underbrace{(n_0 - 1)\frac{\partial u(r)}{\partial z} dz}_{\text{axial elongation}} + \underbrace{\sum_{i,j=1}^{3} \frac{\partial n}{\partial \varepsilon_{ij}} \varepsilon_{ij}(r)dz}_{\text{strain induced birefringence}}$$

If the curve of OPD plotted against the radius r is parabolic, then another lens can be inserted to focus the light.

C. Pfistner, “Thermal Beam Distortions in End-Pumped ND:YAG Rods”, The Journal of Quantum Electronics, Vol. 30, No. 3, 1994.

Induction heating

W.H. Hayt, Jr., Engineering Electromagnetics, McGraw Hill, 4th ed., p. 400.

LSTC

Induction heating is the use of time-varying magnetic fields to induce current in a material for the purpose of heating the material. The heating is very rapid because the heat is generated in the piece itself without having to be conducted in. Induction heating is used for melting, case hardening, heating for forging, brazing, and soldering.

This can be modeled using either heat generation by material or `*LOAD_HEAT_GENERATION_SET`. We can calculate the current penetration depth (skin depth, heat effected zone) using

$$\delta = \frac{1}{2\pi}\sqrt{\frac{\rho \cdot 10^9}{\mu f}}$$

δ = penetration depth [cm]

ρ = resistivity [ohm-cm]

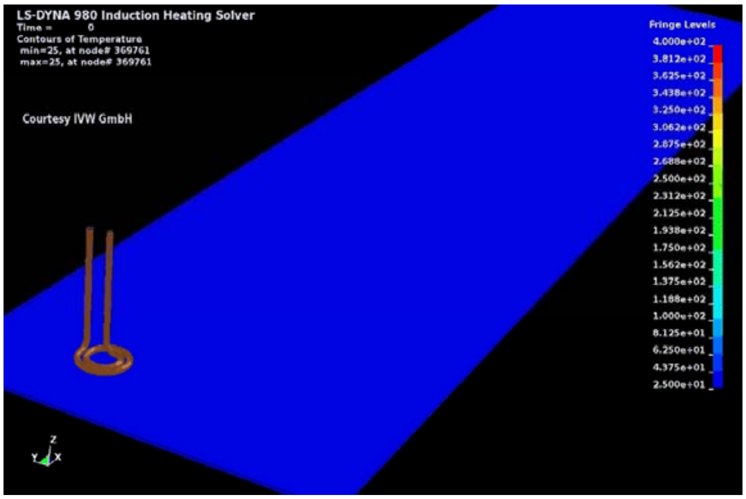
μ = relative permeability

f = frequency [cps]

Induction heating

LS-980

LSTC



Bio-heat equation

LSTC

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot k \nabla T + \dot{Q}_g''' + W_b c_b \left(T - T \right)$$

```
*LOAD_HEAT_GENERATION_SOLID
sid     lcid      mult     Wb      cb      Tb
sid     = solid element set ID
lcid    = load curve ID for Q
mult    = curve multiplier for Q
Wb      = load curve id for Wb(t); [kg/m³ s]
cb      = load curve id for cb(Tb); [J/kg C]
Tb      = load curve id for Tb(t); [C]
```

Ref: H.H. Pennes, “Analysis of Tissue and Arterial Blood Temperature in the Resting Human Forearm”, J. Appl. Physiol., V1, N2, pp 93-122, August 1948.

Hyperbolic heat transfer

LSTC

The Fourier heat equation is **parabolic** – this means that every location in the object will respond instantaneously to a heat source disturbance.

The **hyperbolic** form models the heat source disturbance as traveling with a finite speed of propagation through the medium.

$$\tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

- cryogenic temperatures
- pico-second time scales
- μm length scales

	metals	superconductors	semiconductors
τ [sec] at 298K	1.e-11	na	1.e-10
τ [sec] at 50K	1.e-06	1.e-08	1.e-07

A. Vedavarz, “Significance on Non-Fourier Heat Waves in Microscale Conduction DSC-Vol. 32, Micromechanical Sensors, Actuators and Systems, ASME 1991.

Thermostat controller

LOAD_HEAT_CONTOLLER

LSTC

$$Q_{cont} = Q_0 + \underbrace{G_p\left(T_{set} - T\right)}_{\text{proportional}} + \underbrace{G_i \int_{t=0}^{t}\left(T_{set} - T\right)dt}_{\text{integral}}$$

Q_{cont}	volumetric heating rate
Q_0	constant volumetric heating rate
G_p	proportional gain
G_i	integral gain
T_{set}	set point temperature
T	measured temperature

Thermostat controller

LOAD_HEAT_CONTROLLER keyword input

LSTC

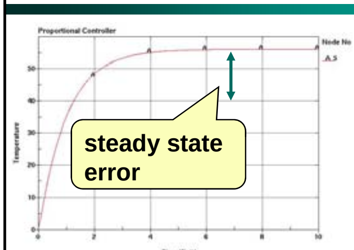
NODE	PID	LOAD	TSET	TYPE	GP	GI

NODE	sensor is located at this node
PID	heater (or cooler) part id being controlled
LOAD	heater output Q_0 [W/m^3]
TSET	set point temperature @ NODE
TYPE	1 = on off 2 = proportional + integral
GP	proportional gain
GI	integral gain

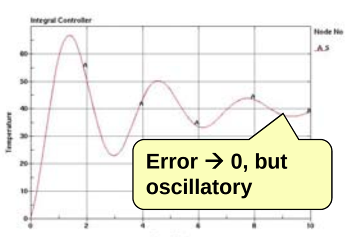
Thermostat controller

Set point is T_{set}=40

LSTC



Proportional Control – corrective action is taken which is proportional to the error. Should a sustained correction (brought about by a sustained disturbance) be required, an accompanying steady state error will exist.

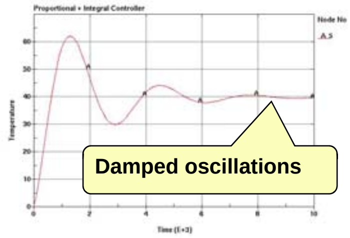


Integral Control – corrective action is made which is proportional to the time integral of the error. An integral controller will continue to correct until the error is zero (eliminating any steady state system error). But, there is also a weakness. Integral control tends to overshoot, thereby producing an oscillatory response and, in some cases, instability.

Thermostat controller

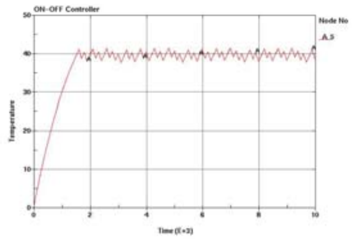
Set point is T_{set}=40

LSTC



Proportional + Integral Control – the oscillations will be damped and the set point error → 0.

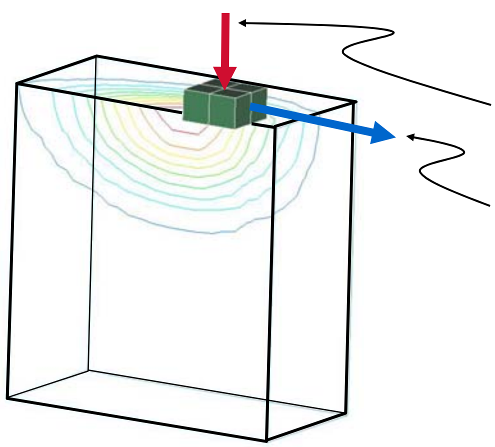
Example: controller_p_i.k



On – Off Control can also be activated.

Weld Modeling

LSTC



A convenient way to model welding is to define a small block to represent the "torch". The torch weld power can be defined using `*BOUNDARY_THERMAL_WELD`

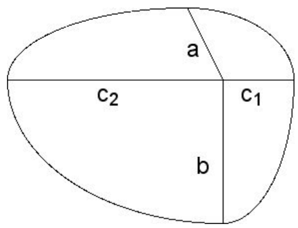
The torch motion can be defined using `*BOUNDARY_PRESCRIBED_MOTION`

It is not necessary to define a contact surface between the torch and the workpiece. The torch is a fictitious part and does not enter into the thermal-mechanical problem.

Weld Modeling

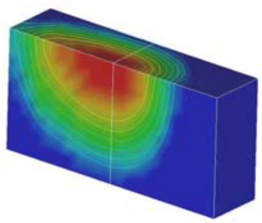
J. Goldak, "A New Finite Element Model for Welding Heat Sources", Metallurgical Trans. B., V15B, June 1984, pp 299-305.

LSTC

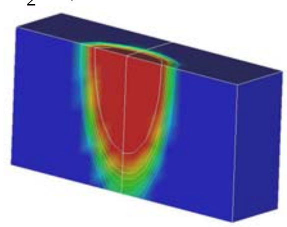


$$\dot{q}_f''' = \frac{6\sqrt{3}F_f Q}{abc_1\pi\sqrt{\pi}} e^{-\frac{3x^2}{a^2}} e^{-\frac{3y^2}{b^2}} e^{-\frac{3(z+vt)^2}{c_1^2}}$$

$$\dot{q}_r''' = \frac{6\sqrt{3}F_r Q}{abc_2\pi\sqrt{\pi}} e^{-\frac{3x^2}{a^2}} e^{-\frac{3y^2}{b^2}} e^{-\frac{3(z+vt)^2}{c_2^2}}$$



TIG a=1, b=2, c_1=1, c_2=4



Laser a=1, b=5, c_1=1, c_2=1

Chapter 11 - 17

Weld Modeling

BOUNDARY_THERMAL_WELD_KEYWORD (weld.k)

LSTC

Part set being welded

Eq. 1: source moves with NID
Eq. 2: source fixed in space

Node location of weld source

Coordinates of weld source if NID=0

```
*BOUNDARY_THERMAL_WELD
PID     PTYP    NID     NFLAG     X0     Y0     Z0     N2ID
a       b       c1      c2        LCID   Q      ff     fr
tx      ty      tz
```

Power deposition parameters

Beam direction is from N2ID to NID or by vector

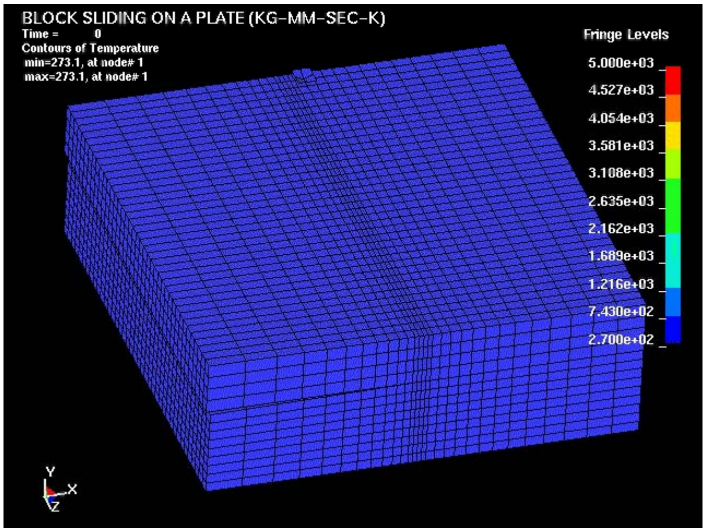
Q has units of watts

Chapter 11 - 18

Weld Modeling

Example problem

LSTC



Courtesy:
Scott Perfect
LLNL

Water spray cooling

J. Wendelstorf, "Spray Cooling heat transfer and Calculation of Water Impact Density for Cooling of Steel Sheet materials by Inverse Process Modeling", Steel Research International, V80, September, 2009, pp. 639-644.

LSTC

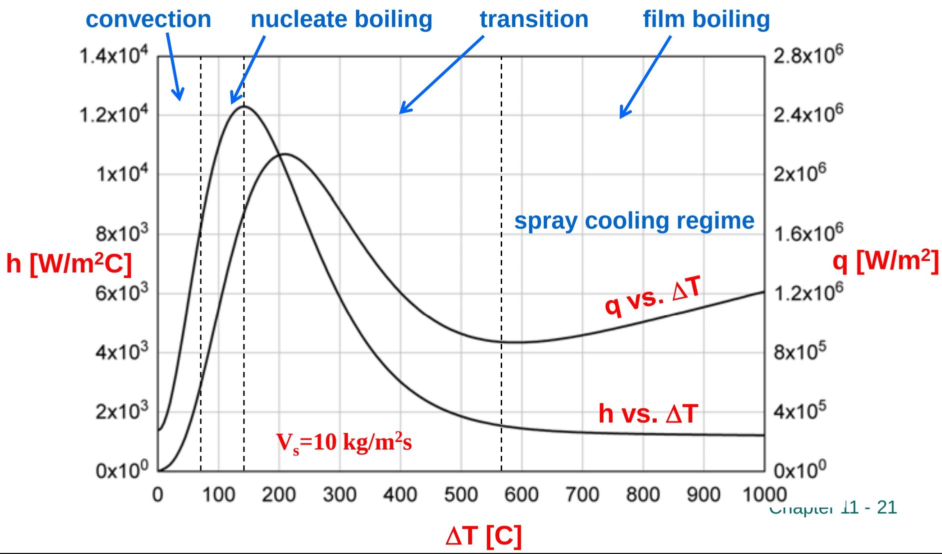
$$h = 190 + \tanh\left(\frac{V_s}{8}\right) * \left\{140 V_s \left(1 - \frac{V_s \Delta T}{72000}\right) + 3.26 \Delta T^2 \left[1 - \tanh\left(\frac{\Delta T}{128}\right)\right]\right\}$$

$$\dot{q}'' = h \Delta T$$

$$\Delta T = T_{surface} - T_{liquid}$$

Water spray cooling

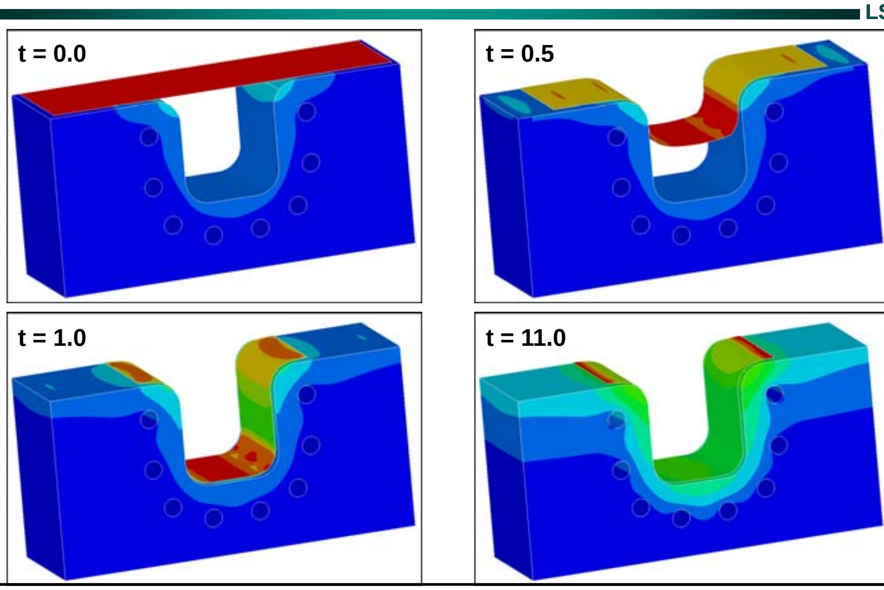
LSTC



Chapter 11 - 21

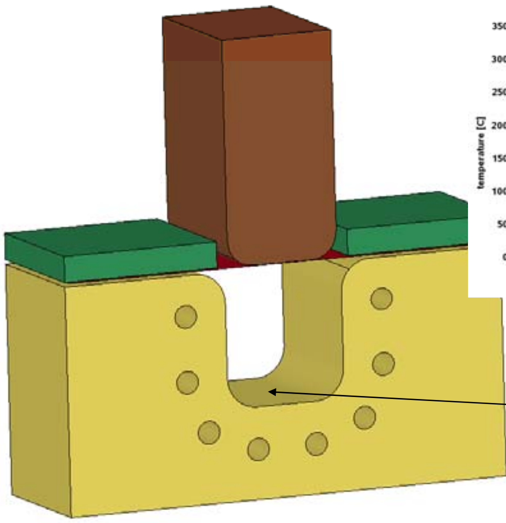
Process start-up

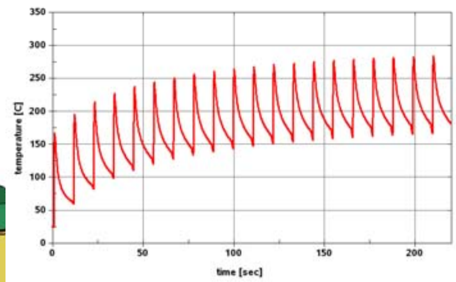
LSTC



Process start-up

LSTC

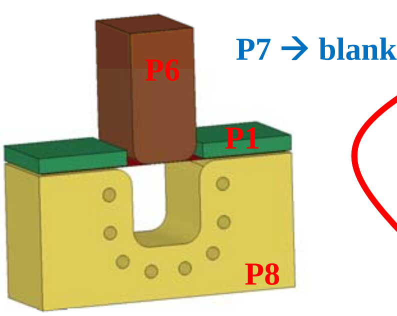




Shown is the tempeature history for this location during 20 stamping cycles.

Process start-up

LSTC




```
#!/bin/csh -f
set i=1
while ( $i <= 20 )
./ls971 i=stamping.k g=d3plot$i"_"
@ i = $i + 1
end
```

```
*KEYWORD
*INITIAL_TEMPERATURE_SET
$    nsid     temp
        1      25.
        6      25.
        8      25.
*END
```

loop

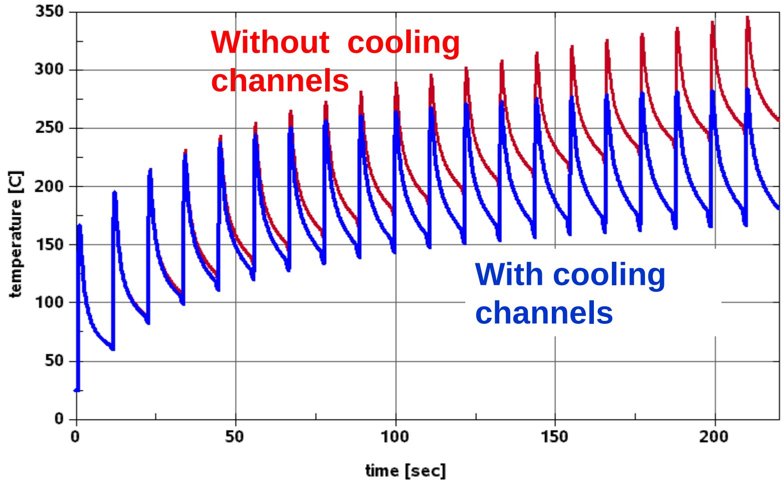
```
*INITIAL_TEMPERATURE_SET
        7     800.
*INCLUDE
new_temp_ic.inc

*INTERFACE_SPRINGBACK_LSDYNA
$    psid
        1
*SET_PART_LIST
$    psid
        1
$    pid1     pid2     pid3
        1        6        8
```

Process start-up

Tool temperature after 20 stampings

LSTC



LSTC

LSTC

Chapter 11 - 27

LSTC

Chapter 11 - 28

Appendix A – Using LS-PrePost

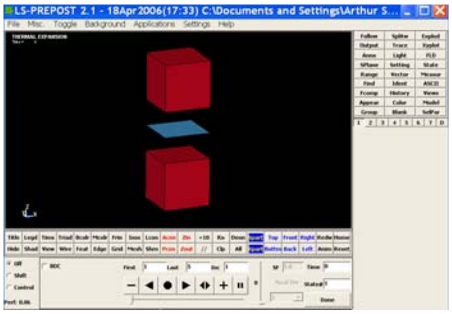
LSTC

LS-PrePost rotate, translate, zoom

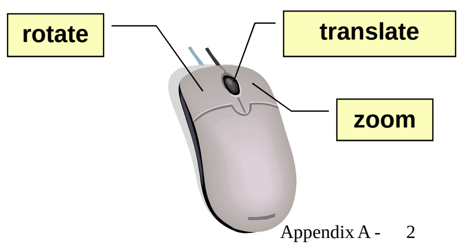
LSTC

```
*DATABASE_EXTENT_BINARY
Card 1
Card 2
V1  V2  V3  V4  V5  V6
```

V6:
1 Temperature (default)
2 temperature + flux
3 shell middle, top, bottom temperature +flux

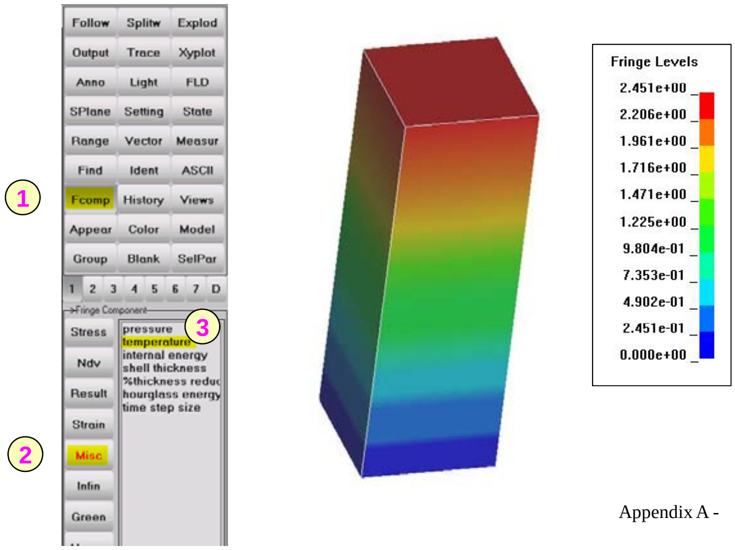


Use shift or ctrl in combination with



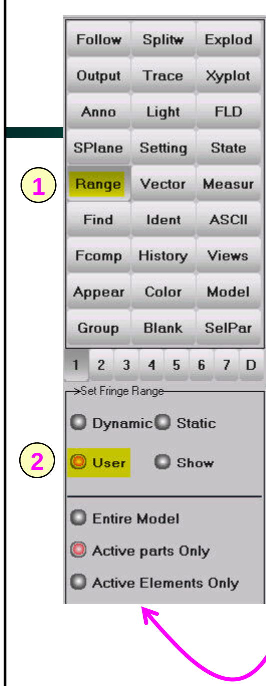
Displaying fringes of temperature

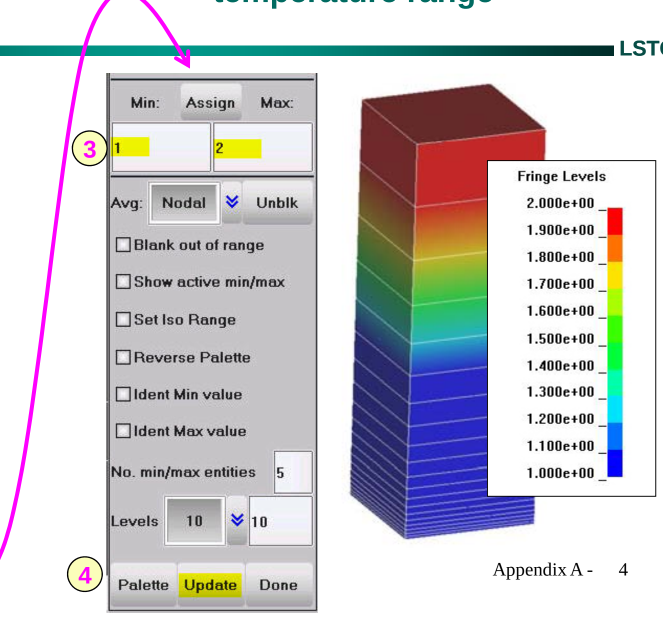
LSTC



Changing the temperature range

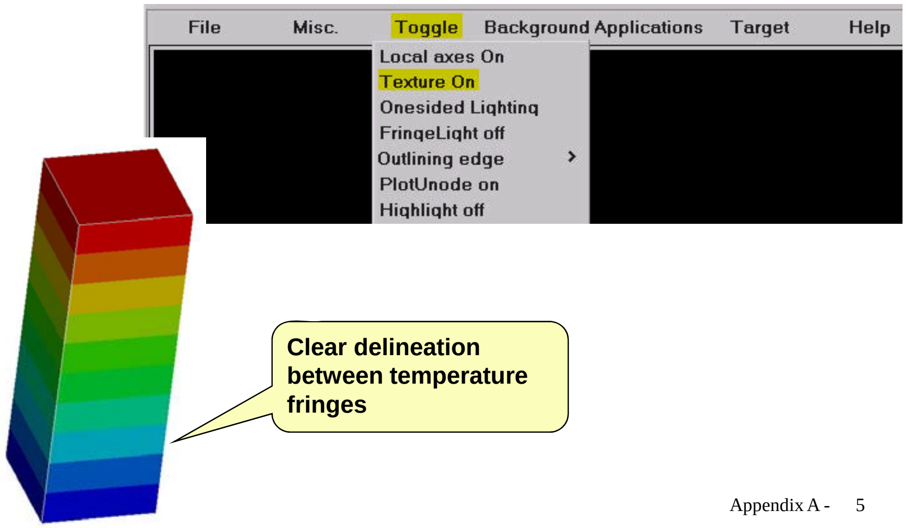
LSTC





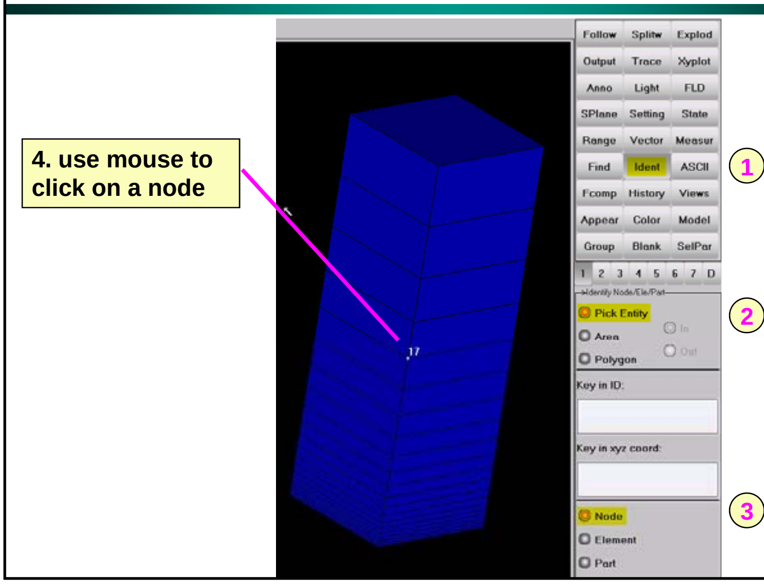
Temperature fringes without texture

LSTC



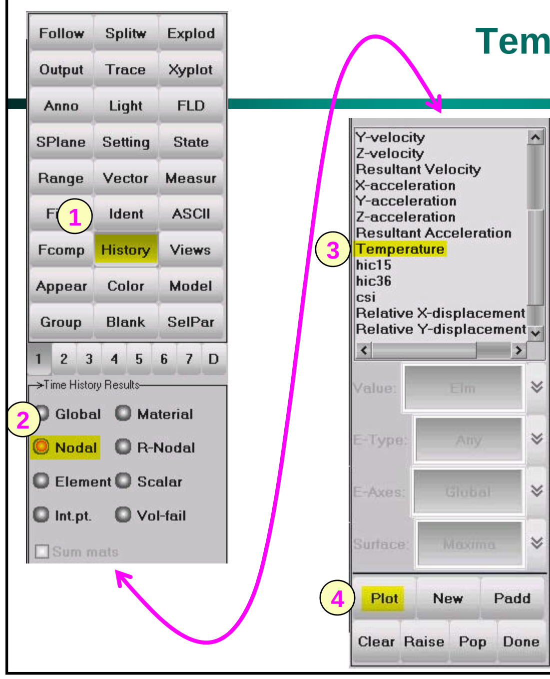
Identifying a node for history plotting

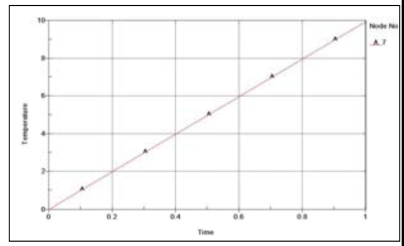
LSTC



Temperature history plot

LSTC

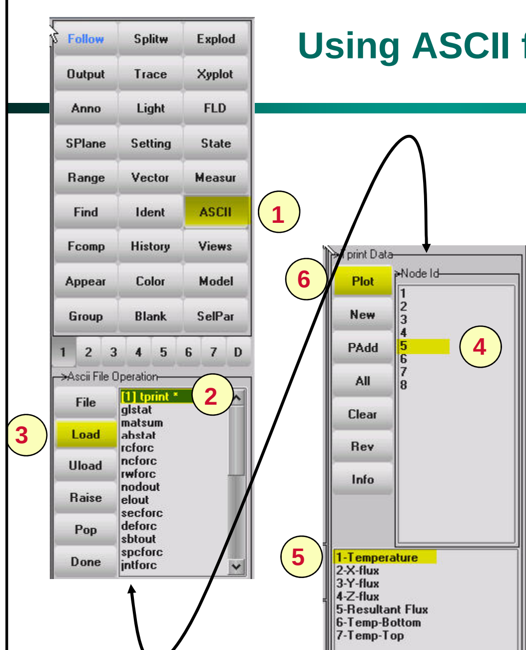


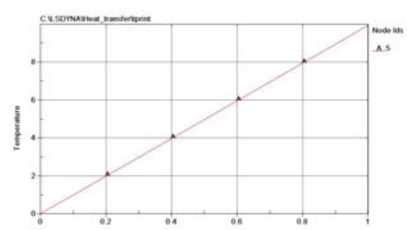


Appendix A - 7

Using ASCII file tprint

LSTC

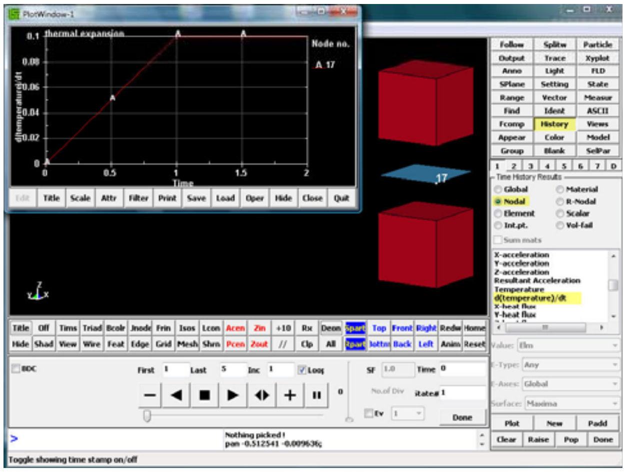




Appendix A - 8

Plot node dT/dt history

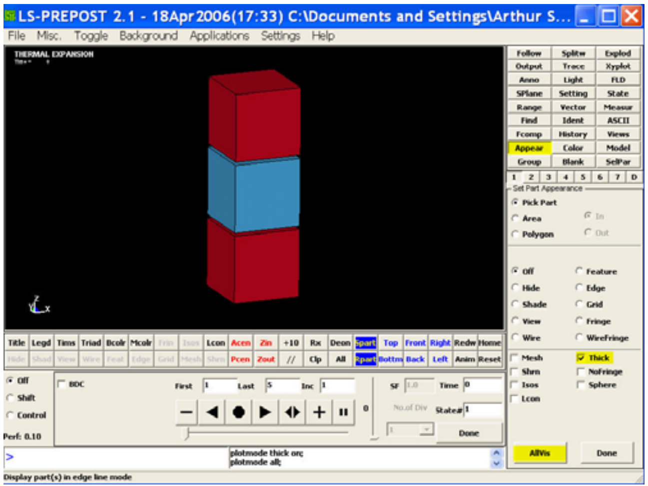
LSTC



Appendix A - 9

Display thick shell

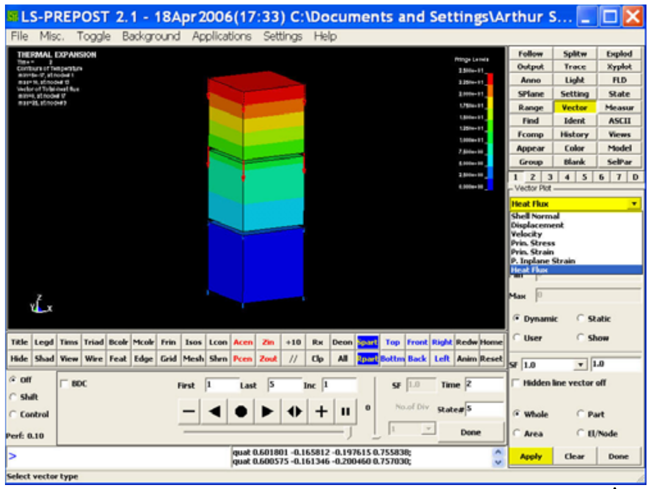
LSTC



Appendix A - 10

Display flux vectors

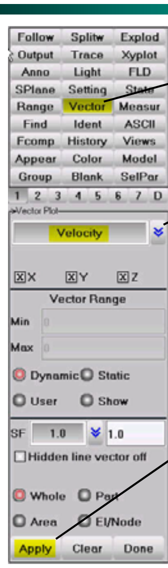
LSTC



Appendix A - 11

Display velocity vectors

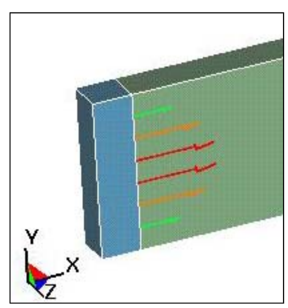
LSTC



1. click Vector

2. scroll to Velocity

3. Apply



Appendix A - 12

Mysteries behind the Coefficient of Thermal Expansion (CTE) Revealed

Art Shapiro (LSTC)

Which calculation is correct?

Your job is to calculate the final length of a 1 meter long metal rod heated from 20C to 1020C. The coefficient of thermal expansion is α = 5.e-04 m/mC.

You use the formula $\frac{L - L_0}{L_0} = \alpha(T - T_0)$

And calculate $L = 1 + (1)(5.e - 04)(1020 - 20) = 1 + 0.5 = 1.5 \ \ m$

In checking your work, a colleague uses the thermodynamic definition for the coefficient of thermal expansion

$$\alpha = \frac{1}{L}\left(\frac{\partial L}{\partial T}\right)_P \quad \text{or} \quad \frac{dL}{L} = \alpha dT$$

Integrating $\int_1^L \frac{dL}{L} = 5.e - 04 \int_{20}^{1020} dT$

He gets $\ln(L) - \ln(1) = (5.e - 04)(1020 - 100) = 0.5$

$$L = \exp[\ln(1) + 0.5] = 1.65 \ \ m$$

There is a large difference between the two answers. Which one is correct?

Both are correct plus 1 other is also correct

The difference is related to the definition of the CTE – there are 3. When using a CTE from a reference publication, you must determine how the CTE is defined. The 3 definitions for CTE are:

1. Tangent CTE using current length $\alpha_t = \frac{1}{L}\left(\frac{\partial L}{\partial T}\right)_P$ (eq. 1)
2. Tangent CTE using reference length $\alpha_{t,r} = \frac{1}{L_r}\left(\frac{\partial L}{\partial T}\right)_P$ (eq. 2)
3. Secant (or, mean) CTE $\alpha_s = \frac{L - L_r}{L_r(T - T_r)}$ (eq. 3)

These are shown graphically in figure 1. The thermodynamic defined coefficient, α_t, is the slope of the tangent to the curve at a specific temperature, T. The secant coefficient, α_s, is the slope of the line between two points on the curve. One point is taken as the reference state (T_r, L_r). The subscript, r, means reference state. The thermal strain is zero at the reference temperature, T_r , and reference length, L_r. The reference temperature is usually 20C.

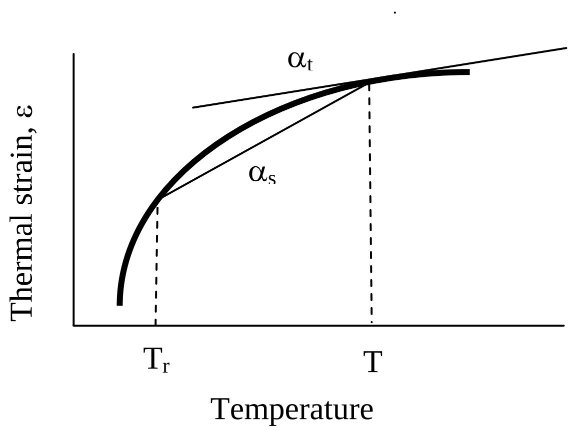


Figure 1: Shown is the graphical representation of the tangent, α_t, and secant, α_s, coefficient of thermal expansion.

The CTEs can also be interpreted as representing the:

- "natural (logarithmic)" strain $\bar{\varepsilon} = \frac{dL}{L} = \alpha_t dT$ using eq. 1.
- "engineering (linear)" strain $\varepsilon = \frac{L - L_r}{L_r} = \alpha_s (T - T_r)$ using eq. 3.

Integrating $\int_{L_r}^{L} \frac{dL}{L}$, we obtain the logarithmic strain $\bar{\varepsilon} = \ln\left(\frac{L}{L_r}\right)$.

This can also be expressed as $\bar{\varepsilon} = \ln\left(1 + \frac{L - L_r}{L_r}\right) = \ln(1 + \varepsilon)$

If $\alpha_s (T - T_r) << 1$, then $\bar{\varepsilon} \cong \varepsilon$. CTE values for metals and alloys are in the range of $10\text{x}10^{-6}$ to $30\text{x}10^{-6}$ /K. The difference in the thermal strain calculation if you use a tangent or secant CTE for reasonable temperature changes is very small for metals. However, using the correct CTE definition becomes more important for plastics with a typical CTE around $1\text{x}10^{-4}$ K.

The tangent coefficient of thermal expansion (default CTE definition in LS-DYNA)

The tangent coefficient of thermal expansion, $\alpha_t = \frac{1}{L}\left(\frac{\partial L}{\partial T}\right)_P$, is a very convenient value to use in an explicit finite element code. An explicit analysis is an incremental method where calculations are based on the instantaneous properties of the material. A reference state (e.g., reference temperature, reference length) is not required. The same tangent CTE values are applicable for heating an object up from room temperature or, cooling it down from an elevated temperature (e.g., hot stamping, casting). This is not true for secant CTEs which have different values for heating and cooling because the secant CTE is a function of a strain free reference state. The secant CTE values depend on whether the strain free reference state is at room temperature or at the elevated temperature.

A thermodynamic relation defines the tangent CTE. It is therefore compatible with thermodynamic defined equations of state. This makes solid-solid and liquid-solid volume changes during phase transition easy to calculate.

A main difficulty is finding values for tangent CTEs. They are much less reported in the literature than secant values. However, they can be easily calculated. An excellent source for CTE values is the reference book, Thermophysical Properties of Matter, Thermal Expansion, Vol. 12., ed. Y.S. Touloukin. Thermal expansion data is presented as polynomial curve fits, such as:

$$\frac{L - L_{293}}{L_{293}} = a + bT + cT^2 + dT^3 \qquad \text{(eq. 4)}$$

To obtain the tangent CTE, we can simply take the derivative of this polynomial, dL/dT, and then divide by the current length, L, at a specific temperature, T.

The tangent coefficient of thermal expansion using a reference length

The tangent coefficient of thermal expansion using a reference length, $\alpha_{t,r} = \frac{1}{L_r}\left(\frac{\partial L}{\partial T}\right)_P$, is presented as tabulated data in the reference book, Thermophysical Properties of Matter, Thermal Expansion, Vol. 12., ed. Y.S. Touloukin. I'm not aware of any FE codes that use this definition. However, this definition gets confused with the tangent coefficient, α_t, in the literature. You will find the statement in the literature that, "if the CTE is not a function of temperature, then the tangent CTE equals the secant CTE". This is only true for $\alpha_{t,r}$ and not for α_t.

Integrating eq. 2 $$\int_{L_r}^{L} \frac{dL}{L_r} = \int_{T_r}^{T} \alpha_{t,r}\, dT \qquad \text{(eq. 5)}$$

We obtain $$\frac{L-L_r}{L_r}=\int_{Tr}^{T}\alpha_{t,r}\,dT \qquad \text{(eq. 6)}$$

Equation 3 can be written as $$\frac{L-L_r}{L_r}=\alpha_s\left(T-T_r\right) \qquad \text{(eq. 7)}$$

Then, equating equations 6 and 7 we obtain: $$\alpha_s=\frac{1}{\left(T-T_r\right)}\int_{T_r}^{T}\alpha_{t,r}\,dT \qquad \text{(eq. 8)}$$

We can see from equation 8 that if $\alpha_{t,r}$ is not a function of temperature, then $\alpha_s=\alpha_{t,r}$. Equation 8 also shows that α_s is the mean value of $\alpha_{t,r}$ over the temperature interval.

The secant coefficient of thermal expansion (optional CTE definition for MAT_106 in LS-DYNA)

Another definition for the coefficient of expansion exists which is called the secant value, α_s. These are easily obtained in the laboratory and I'm sure many of you performed this experiment in a college physics lab. Take a rod at room temperature (i.e. reference temperature), T_r, and measure its length, L_r. Then uniformly (usually by an electric current) heat the rod and measure its new length, L, (or, change in length) and temperature, T. Then,

$$\alpha_s=\frac{L-L_r}{L_r\left(T-T_r\right)} \qquad \text{(eq. 9)}$$

Note that a reference temperature, T_r, must be specified when using the secant value of thermal expansion. α_s is also referred to as the "mean" or "effective" coefficient of thermal expansion.

Historically, the specification of α_s as a function of temperature allowed modeling the nonlinear influence of temperature on thermal strain in linear finite element codes. This specification carried over to many of the current nonlinear codes. For linear and nonlinear incremental material analysis, the increment in thermal strain can be calculated by

$$\Delta\varepsilon=\frac{\Delta L}{L_r}=\alpha_{s,T+\Delta T}\left(T+\Delta T-T_r\right)-\alpha_{s,T}\left(T-T_r\right) \qquad \text{(eq. 10)}$$

This is an exact calculation and therefore is not dependent on the incremental time step size. This expression is used in implicit finite element calculations.

The main disadvantage in using α_s is the requirement of a reference state. If the part initial temperature is different from the material reference state temperature, then the α_s values are no longer valid. They must be adjusted to account for the new strain free condition at the part initial temperature.

How do you adjust α_s values at a new reference state?

If the part initial temperature state (P_1), is different from the material reference temperature state (P_0), then the α_s values must be modified. The subscript RM means ‘reference material’, and the subscript RP means ‘reference part’. Figure 2 graphically depicts the computational method.

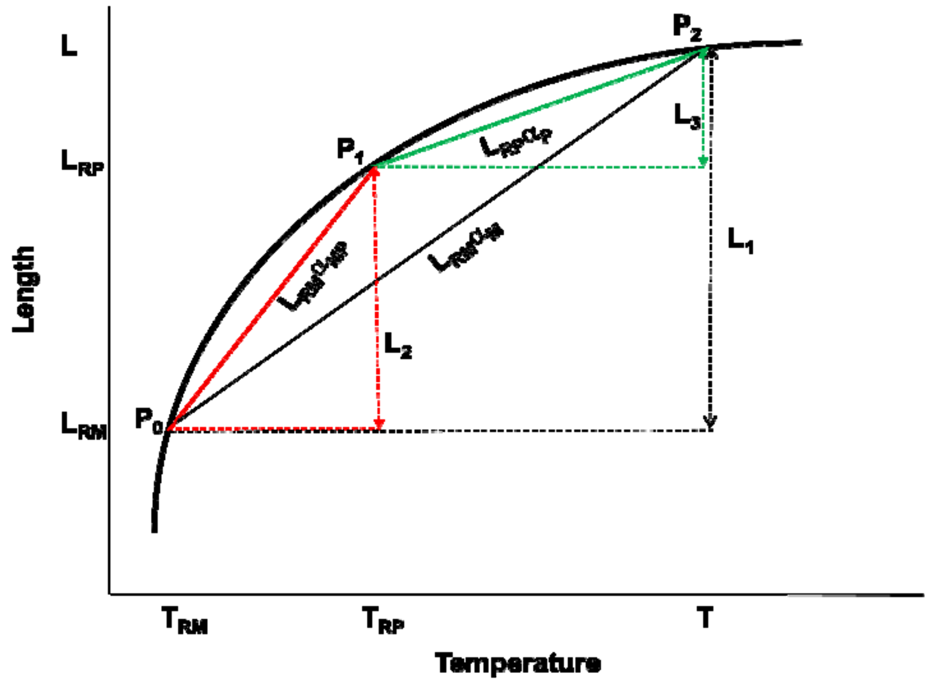


Figure 2: This figure shows graphically the parameters used in eq. 11 to shift the CTE reference state.

(T_{RM}, L_{RM}) is the material reference state at point P_0. This is the reference state for the material data. α_M is the secant CTE at the temperature, T. α_{MP} is the secant CTE at the temperature T_{RP}. α_M and α_{MP} are obtained from the literature.

(T_{RP}, L_{RP}) is the part reference state at point P_1. This is the initial temperature for the part at which the thermal strain is 0. We want to calculate α_P.

$L_{RM}\alpha_M$ is slope of the line from point P_0 to P_2.

$L_{RM}\alpha_{MP}$ is the slope of the line from point P_0 to P_1.

$L_{RP}\alpha_P$ is the slope of the line from point P_1 to P_2.

$$L_1 = L - L_{RM} = L_{RM}\alpha_M\left(T - T_{RM}\right)$$

$$L_2 = L_{RP} - L_{RM} = L_{RM}\alpha_{MP}\left(T_{RP} - T_{RM}\right)$$

$$L_3 = L - L_{RP} = L_{RP}\alpha_P\left(T - T_{RP}\right)$$

$$L_3 = L_1 - L_2$$

$$\alpha_P = \left(\frac{L_{RM}}{L_{RP}}\right)\frac{\alpha_M\left(T - T_{RM}\right) - \alpha_{MP}\left(T_{RP} - T_{RM}\right)}{\left(T - T_{RP}\right)} \qquad \text{(eq. 11)}$$

How do you calculate α_t from α_s?

The secant lines can be used to approximate the tangent. The slope of a secant line (e.g., α_p in figure 2) approaches the slope of the tangent line as the secants' 2nd point (i.e., P_2) approaches the 1st point (i.e., P_1). The problem of finding the tangent line to a graph was one of the main problems that originated calculus. In calculus this problem is solved using Newton's difference quotient.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

This equation is similar to eq. 11. Also, by observation, the secant α_P in figure 2 appears to represent the slope of the tangent to the curve at the point $(T+T_{RP})/2$. The input to most FE codes is by piecewise linear data tables. If we have 2 data pairs (α_{s1},T_1) and (α_{s2}, T_2), we can calculate α_t at the midpoint temperature (T1+T2)/2 using eq. 11. Remember that this calculation is approximate and becomes more accurate in the limit as $\lim(T-T_{RP}) \rightarrow 0$.

How do you calculate α_s from α_t?

We can use equation (1) to calculate the thermal strain, $\epsilon = \frac{dL}{L} = \int_{T_r}^{T} \alpha_t dT = \propto_t (T - T_r)$ for the case α_t = constant. The input to most FE codes is by piecewise linear data tables in which the tangent coefficient of thermal expansion is considered constant over the temperature increment. For example, $\alpha_{t,1}$ is constant between T_r and T_1, $\alpha_{t,2}$ is constant between T_1 and T_2, etc. Then

$$\varepsilon_1 = \alpha_{t,1}(T_1 - T_r)$$

$$\varepsilon_2 = \varepsilon_1 + \alpha_{t,2}(T_2 - T_1)$$

$$\varepsilon_3 = \varepsilon_2 + \alpha_{t,3}(T_3 - T_2)$$

Secant values of the coefficient of thermal expansion can then be calculated from:

$$\propto_{s,1} = \boldsymbol{\varepsilon_1}/(\boldsymbol{T_1} - \boldsymbol{T_r})$$

$$\propto_{s,2} = \boldsymbol{\varepsilon_2}/(\boldsymbol{T_2} - \boldsymbol{T_r})$$

$$\propto_{s,3} = \boldsymbol{\varepsilon_3}/(\boldsymbol{T_3} - \boldsymbol{T_r})$$

CTE Data for Aluminum

Temperature [K]	Length [m]	α_{tan} [µm/m K]	α_{tan_r} [µm/m K]	α_{sec} [µm/m K]
293	1.000000	23.59	23.59	23.59
300	1.000165	23.63	23.64	23.61
325	1.000759	23.84	23.86	23.72
350	1.001359	24.08	24.11	23.84
375	1.001965	24.35	24.40	23.97
400	1.002579	24.65	24.71	24.10
425	1.00321	24.98	25.06	24.25
450	1.003833	25.34	25.44	24.41
475	1.004474	25.73	25.85	24.58
500	1.005126	26.15	26.29	24.76
525	1.005789	26.61	26.76	24.95
550	1.006465	27.09	27.26	25.15
575	1.007153	27.60	27.80	25.37
600	1.007855	28.14	28.36	25.59
625	1.008572	28.71	28.95	25.82
650	1.009304	29.31	29.58	26.06
675	1.010052	29.94	30.24	26.31
700	1.010817	30.60	30.93	26.58
725	1.011599	31.28	31.65	26.85
750	1.012400	32.00	32.40	27.13
775	1.013220	32.74	33.18	27.43
800	1.014060	33.52	33.99	27.73

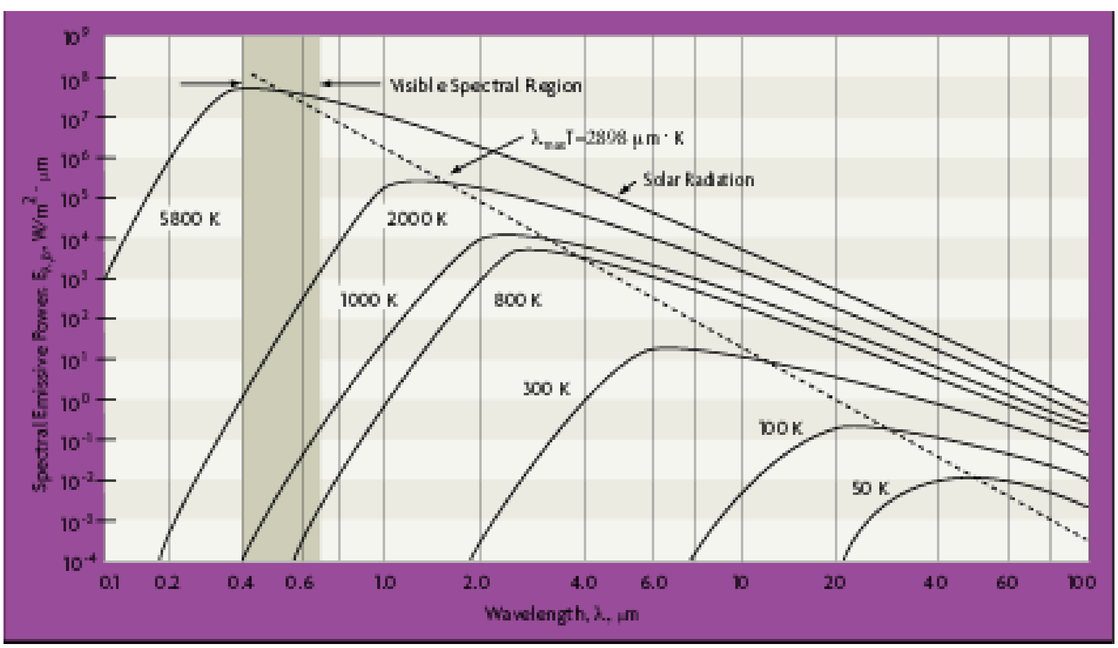
Appendix B

Walking on Fire with LS-Dyna

The bed of wooden coals was meticulously prepared and simmered all day. By nightfall, the coals were glowing red and posed an intimidating path to cross. The spiritual leader tossed a steak on the coals and it immediately sizzled. The steak was removed from the coals and had a seared surface. Then, miraculously a person walked across the coals without being burnt (or cooked).

Firewalking has been practiced for thousands of years. Abundant information can be found by typing “firewalking” into a web search engine. Many articles are by faith healers who claim successful firewalking is an exercise in connecting the mind and body. A plethora of classes are offered to get you in touch with your inner self by walking on fire. However, as an alternative, you can use LS-DYNA to model the process and avoid chanting mantras all day to get your mind and body in sync – let your computer do the walking.

First, we need to know how hot the coals are. We can use the hemispherical spectral emissive chart [1] below to determine this. The visible wavelength is between 0.4μ (violet) to 0.7μ (red). The 1000K (727C) curve below has an amount of radiant energy sufficient to be observed by the human eye between wavelengths of 0.4 to 0.7 microns. Since a larger percentage of the radiant energy is toward the longer wavelength of 0.7μ, an object at that temperature glows with a dull-red color. Now you also know how hot your hair dryer is.



Second, we need to know the thermal physical properties of skin, fat, muscle, and wood coals. This information can be found in textbooks. Values are presented in Table 1.

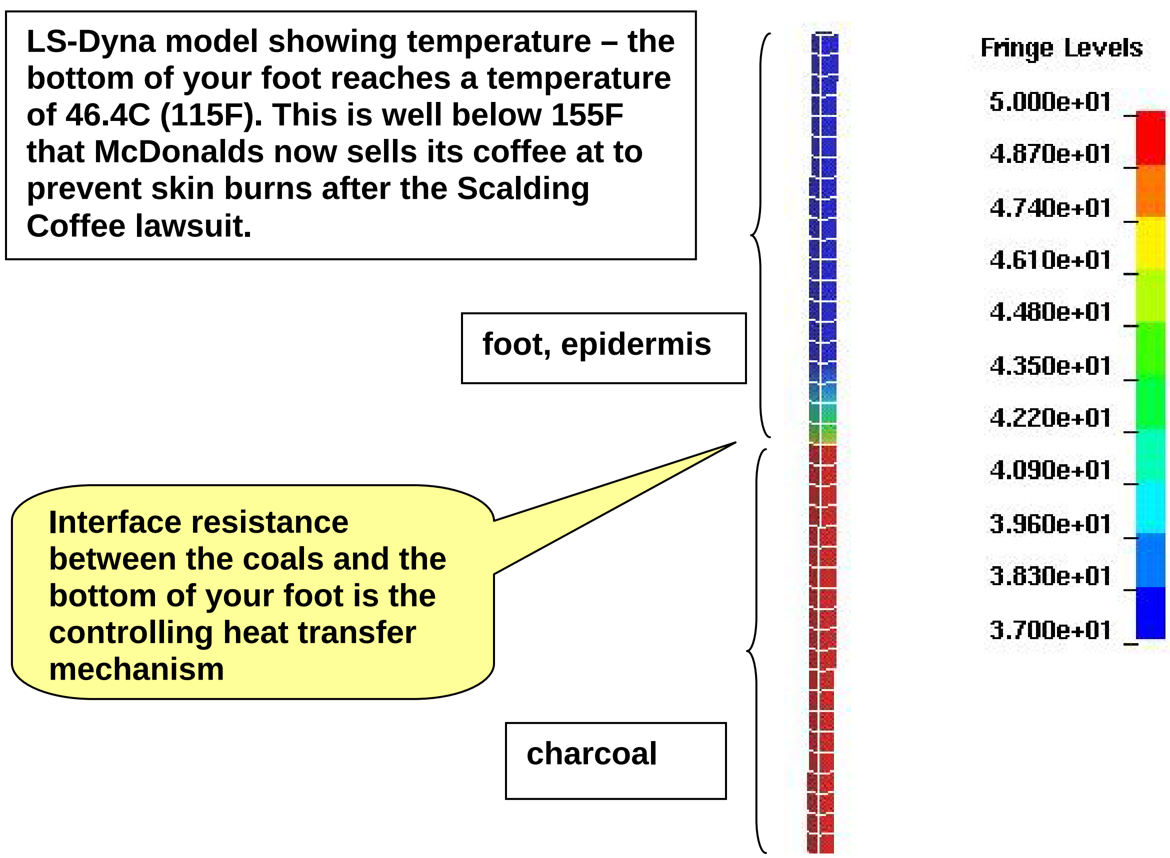
Table 1 Thermal Properties of Selected Materials

material	density, ρ [kg / m3]	heat capacity, c [J / kg C]	conductivity, k [W / m C]
epidermis	1200.	3440.	0.34
muscle / fat	1060.	3350.	1.60
wood charcoal	240.	838	0.052

Third we need to solve the bio-heat equation

$$\rho C\frac{\partial T}{\partial t} = \nabla \cdot k\nabla T - W_b C_b \left(T - T_b\right)$$

The second term on the right models heat removal from tissue due to blood flow (W_b=1.8 kg/m^3 sec). This term can be represented as a temperature dependent volumetric heat sink in the thermal material constitutive model. However, in the end, the heat removed by this term is negligible. It doesn't make any difference on the final temperature if you are relaxed or completely tense up and stop the blood flow in your foot. Other heat removal terms that would also work in our favor in reducing foot temperature have been omitted. This includes energy loss due to mass diffusion of water through the skin (i.e., sweating).



The problem can be modeled in 1-dimension with a contact surface between the bottom of the foot and the top of the coals. The coal bed thickness is modeled as 3cm with a temperature initial condition of 727C. The epidermis thickness is 0.2cm with a temperature initial condition of 37C (i.e., normal body temperature). The contact resistance between the coals and your foot is the governing heat transfer mechanism. Because we waited so long before walking on the coals, a grey powdery ash covered them. We will assume the ash is 1mm thick with a thermal conductivity half that of the charcoal. If you time yourself walking, you will find that your foot is in contact with the coals for 0.5 seconds.

LS-Dyna predicts a foot surface temperature of 46.4C (115F) after 0.5 seconds. This is considerably below the temperature of 155F that McDonalds now sells its coffee at to prevent skin burns after the Scalding Coffee lawsuit [2]. The only real chance for a burn is if an ember gets stuck between your toes so walk flat footed. However, walk fast because the time constant for heat transfer is in an exponential term and does mater.

What About the Steak – The thermal diffusivity is defined as $\alpha=k/\rho c$. The reciprocal of the thermal diffusivity is a measure of the time required to heat a material to some temperature level. Using the values given in Table 1., the reciprocal thermal diffusivity for the epidermis is 1.2e+07. Steak is muscle and fat. Its reciprocal thermal diffusivity is 2.2e+06. Steak will heat up 5 times faster than your foot. This presents problems in cooking steak. When meat is heated to temperatures in the range of 125F to 150F, the connective tissue sheaths collapse and shrink. Free water in the muscle cells flow out the ends of the muscle fibers presenting the appetizing appearance of a juicy steak. However, once the “juices” are gone (driven by a much too high energy input), the steak becomes very dry. In a future article, I will discuss using LS-DYNA to cook the perfect steak.

References:

[1] Hemispherical Spectral Emissive Chart from
http://www.omega.com/literature/transactions/volume1/theoretical2.html

[2] McDonald Scalding Coffee
http://www.lectlaw.com/files/cur78.html

Appendix C

LSTC

Using LS-DYNA To Model Hot Stamping

Arthur Shapiro
shapiro@lstc.com

A. Shapiro, "Finite Element Modeling of Hot Stamping", Steel research International, p. 658, Vol. 80, September 2009.

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LSTC

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LSTC

Katana: how to make a Japanese sword

LSTC

The sword smiths of China during the Tang Dynasty (618-907) are often credited with the forging technologies that the Japanese used in later centuries. These technologies include folding, inserted alloys, and quenching of the edge. Okazaki-san is recognized as Japan's greatest sword smith creating such weapons as the katana (14^{th} century).

Heat the steel to the color of the moon in February

Transfer the blank to the anvil

Form the blank with a hammer and lots of muscle

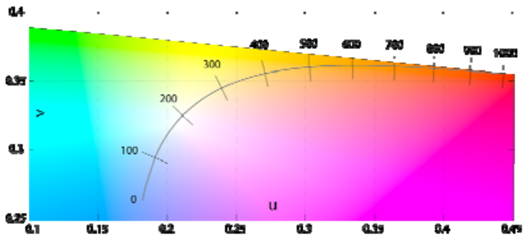
Quench the blade.

Katana: how to make a Japanese sword

Temperature of the Moon in February

LSTC

A color triangle is an arrangement of colors within a triangle, based on the additive combination of 3 primary colors (RGB) at its corners. The correlated color temperature is the temperature of the Planckian radiator whose perceived color most closely resembles that of a given stimulus. Shown is the Plankian locus (in mired) overlaid on the color triangle.



NASA 2/23/09

$$T_{moon} = \frac{1*10^6}{M} \approx \frac{1*10^6}{900} = 1111\ K = 838\ C$$

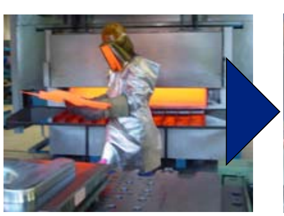
http://en.wikipedia.org/wiki/Color_temperature

B-pillar: how to make Car parts

Courtesy of Mercedes Car Group, Sindelfingen , Germany

LSTC

1. Heat



2. Transfer

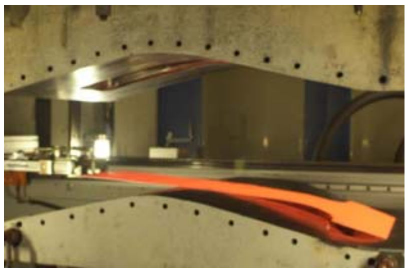
3. Form

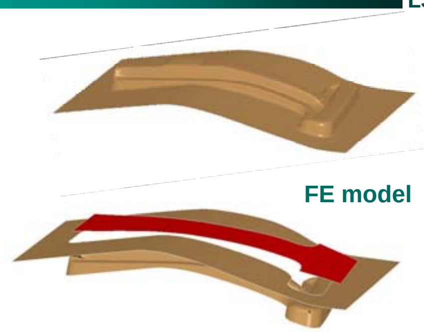
4. Quench

Numisheet 2008 Benchmark BM03

proposed by Audi

LSTC





Benchmark process specification

1. Heating of the blank to 940C.
2. Transport from the oven into the tool 6.5 sec.
3. Temperature of the blank at the beginning of the die movement 810C.
4. Forming process time 1.6 sec.
5. Quench hold time in the tool 20 sec.
6. Cool down to room temperature 25C

Appendix D - 7

Numisheet 2008 Benchmark BM03

proposed by Audi

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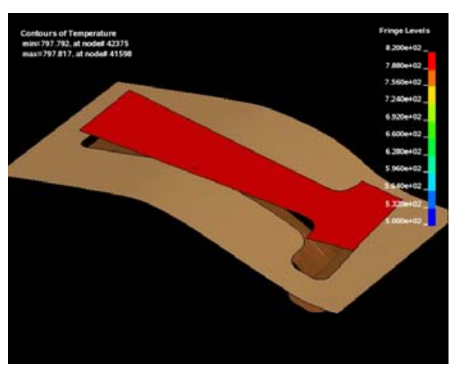

Benchmark process specification

1. Heating of the blank to 940C.
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4. Forming process time 1.6 sec.
5. Quench hold time in the tool 20 sec.
6. Cool down to room temperature 25C

Appendix D - 8

Symbols and values
metal

LSTC

Blank	
material	22MnB5
dimensions	
l, thickness	0.00195 m
length	1m
width	0.25 m
properties	
ρ, density kg/m^3	7830.
Cp, heat capacity J/kgK	650.
k, thermal conductivity W/mK	32.
λ, latent heat, kJ/kg	58.5
α, linear expansion, 1/C	1.3e-05
E, Young's modulus, Gpa	100.
μ, Poisson's ratio	0.30

Symbols and values
air at 483C

LSTC

$$T_{film} = \frac{940+25}{2} = 483.$$

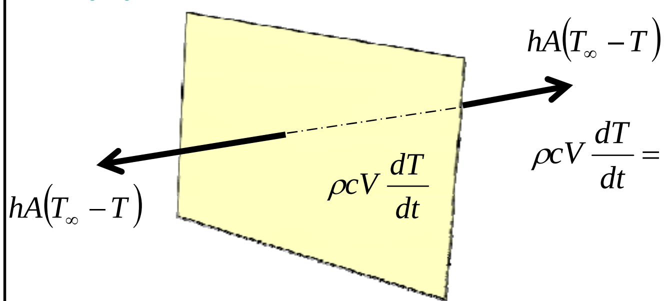
Air properties at 483 C	
ρ, density, kg/m^3	0.471
Cp, heat capacity, J/kg C	1087.
k, thermal conductivity, W/m C	0.055
μ, viscosity, kg/m s	3.48e-05
β, volumetric expansion, 1/C	1.32e-03

Newtonian heating or cooling

convection lumped parameter model

LSTC

Consider an object being heated from some uniform initial temperature, T_i. If the object is of high thermal conductivity, then its internal resistance can be ignored, and we can regard the heat transfer process as being controlled solely by surface convection.



$$\rho c V \frac{dT}{dt} = 2hA(T_\infty - T)$$

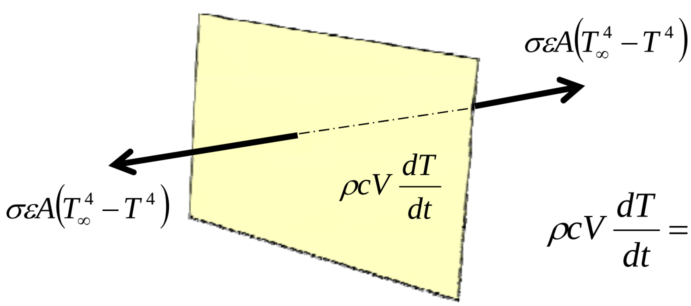
The solution is

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\left(\frac{2hA}{\rho c V}\right)t}$$

Newtonian heating or cooling

radiation lumped parameter model

LSTC



$$\rho c V \frac{dT}{dt} = 2\sigma\varepsilon A\left(T_\infty^4 - T^4\right)$$

The solution to this differential equation between the limits ($T=T_i$ @ t=0) and ($T=T_f$ at t), is

$$t = \frac{\rho c V}{2A\sigma\varepsilon}\left[\frac{1}{4T_\infty^3}\ln\frac{(T_f + T_\infty)/(T_f - T_\infty)}{(T_i + T_\infty)/(T_i - T_\infty)} + \frac{1}{2T_\infty^3}\left(\tan^{-1}\frac{T_f}{T_\infty} - \tan^{-1}\frac{T_i}{T_\infty}\right)\right]$$

Blank heating and transport into tools

LSTC

Our starting point for the FE analysis was Process Step Specification 3. However, we performed a hand calculation to verify steps 1 and 2.

The following analytical equation can be used to calculate the time for the blank to cool by radiation from T_i=940C to T_f=810C during the transport operation from the oven into the tool. The surroundings are at T_∞=25C.

$$time = \frac{\rho C_p l}{2\sigma\varepsilon}\left[\frac{1}{4T_\infty^3}\ln\frac{(T_f + T_\infty)/(T_f - T_\infty)}{(T_i + T_\infty)/(T_i - T_\infty)} + \frac{1}{2T_\infty^3}\left(\tan^{-1}\frac{T_f}{T_\infty} - \tan^{-1}\frac{T_i}{T_\infty}\right)\right]$$

$$time = 6.68$$

The calculated time is in agreement with the benchmark specification of 6.5 sec.

σ = 5.67e-08 W/m2 K4	ε = 1.
ρ = 7870 kg/m3	C_p = 650 J/kg C
l = 1.95 mm	

Use degrees Kelvin in above equation

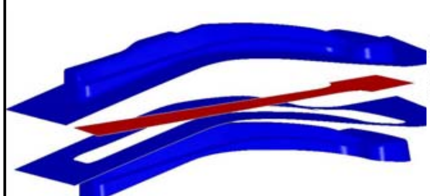
Blank heating and transport into tools

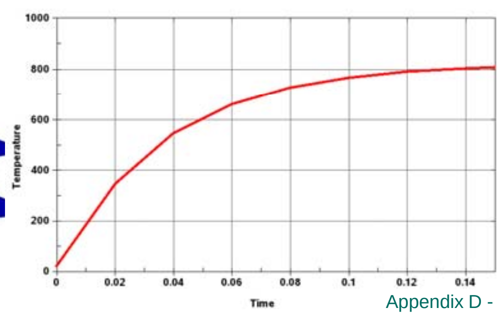
Blank T=810C at beginning of die movement

LSTC

The easiest modeling technique is to define the initial temperature of the blank to be 810C. However, doing this will not calculate the thermal expansion of the blank between 25C and 810C. Therefore, the blank is heated in the FE model resulting in a thickness increase from1.95mm to 1.97mm.

The time to heat the blank is not a critical parameter for this analysis. All we want is the blank to be at 810C and have the correct thickness at the beginning of the die movement.





Blank heating and transport into tools

How do you chose an h for heating the blank

LSTC

It takes 0.4 sec for the upper tool to touch the blank according to the specified tool displacement curve. Therefore, select 0.15 seconds for heating.

$$h = -\frac{\rho C_p l}{t} \ln\left(\frac{T - T_\infty}{T_i - T_\infty}\right) = -\frac{(7830)(486)(0.00195)}{(0.15)} \ln\left(\frac{809.9 - 810}{25 - 810}\right) = 444{,}000 \ \frac{W}{m^2 C}$$

h=444,000 is a ridiculously high number and is not physically possible. But, remember that the time to heat the blank is not a critical parameter for this analysis. All we want is the blank to be at 810C and have the correct thickness at the beginning of the die movement.

Radiation & convection heat loss during transfer and forming

LSTC

After heating the blank, it is transferred to the tools. The blank cools by convection and radiation to the environment.

1. Heat

2. Transfer

The heat loss is calculated by: $\dot{q}'' = h_{eff} A (T_s - T_\infty)$

How do you determine $h_{eff} = h_{conv} + h_{rad}$

Radiation & convection heat loss

How to calculate coefficients

LSTC

Convection

$$T_{film} = \frac{T_{surf} + T\infty}{2} = \frac{940 + 25}{2} = 483\ C$$

$$L = \frac{2(length * width)}{lenght + width} = \frac{2(1*0.25)}{1+0.25} 0.4\ m$$

$$Gr = \frac{g\beta\rho^2 L^3 (T_{surf} - T_\infty)}{\mu^2} = \frac{(9.8)(1.32*10^{-3})(0.471)^2(.4)^3(940-25)}{(3.48*10^{-5})^2} = 1.39*10^8$$

$$\Pr = \frac{C_p \mu}{k} = \frac{(1087)(3.48*10^{-5})}{0.055} = 0.687$$

$$h_{conv} = 0.14\frac{k}{L}(Gr * \Pr)^{0.33} = .14\frac{0.055}{.4}(1.39*10^8 * 0.687)^{0.33} = 8.3\ \frac{W}{m^2 C}$$

940C + 273C = 1213K

Radiation

$$h_{rad} = \frac{\sigma\varepsilon(T_{surf}^4 - T_\infty^4)}{(T_{surf} - T\infty)} = \frac{(5.67*10^{-8})(0.8)(1213^4 - 298\ )}{(1213-298)} = 107\ \frac{W}{m^2 K}$$

Radiation & convection heat transfer coefficients

LSTC

h_{conv} + h_{rad} = h_{eff}

T [C]	h_{conv}	h_{rad}	h_{eff} [W/m^{2}C]
50	5.68	5.31	11.0
100	6.80	6.8	13.6
200	7.80	10.8	18.6
300	8.23	16.3	24.5
400	8.43	23.6	32.0
500	8.51	33.0	41.5
600	8.52	44.8	53.3
700	8.50	59.3	67.8
800	8.46	76.6	85.1
900	8.39	97.2	106.
1000	8.32	121.	129.

Note:

a) h_{rad} dominates

b) h_{conv} @ T>400 uncertain

Temperature of the blank at tool contact

LSTC

After the blank is positioned within the tools, it continues to lose heat by convection and radiation to the environment. The benchmark specifies a heat transfer coefficient of h_{air}=160 W/m²K. We feel that this value is too high and h_{air}=115 is more appropriate. However, a hand calculation reveals that the blank only drops by 10C before the tools make contact. Therefore, knowing h_{air} precisely is not important. We ignored modeling this energy loss (i.e., temperature and thickness change) in our FE model.

t = time until top tool contacts blank

$$T = T_\infty + (T_i - T_\infty)e^{-\left(\frac{2ht}{\rho C_p l}\right)}$$

$$800 = 25 + (810 - 25)e^{-\left(\frac{2(160)(0.4)}{(7870)(650)(0.00195)}\right)}$$

Heat transfer to air and to dies

LSTC

The top surface loses heat to the environment by convection and radiation.

The bottom surface loses heat to the tool. The contact heat transfer to the tool is 10x greater than conv. + rad. loss.

There will be a through thickness temperature gradient in the blank due to the large difference in heat loss rates from the top and bottom surfaces. This is calculated using the 12 node thick thermal shell formulation developed by G. Bergman & M. Oldenburg at Lulea University.

What is h contact ?

Heat transfer to air and to dies

LSTC

Top blank surface

Convection + radiation heat loss to the environment, use:

- *BOUNDARY_CONVECTION
- *BOUNDARY_RADIATION

Bottom blank surface

Turn off thermal boundary conditions when parts are in contact.

*CONTACT_(option)_THERMAL

parameter BC_FLAG = 1

There will be a through thickness temperature gradient in the blank caused by the different heat loss rates from the surfaces.

*CONTROL_SHELL

ISTUPD = 1 → calculate shell thickness change

TSHELL= 1 → 12 node thick thermal shell, T gradient through thickness

Contact parameters

(1) Friction function of T (2) heat transfer function of P

LSTC

```
*CONTACT_(option)_THERMAL_FRICTION
lcfst   lcfdt   formula   a   b   c   d   lch
```

Mechanical friction coefficients vs. temperature

Static → $\mu_s = \mu_s$ * lcfst(T)

Dynamic→ $\mu_d = \mu_d$ * lcfdt(T)

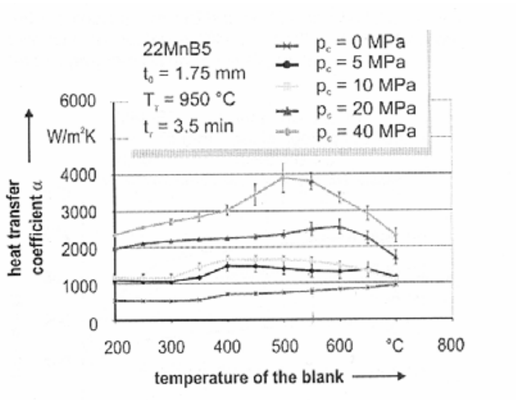
1	h(P) is defined by load curve “a”	such as GE data
2	h(P) = a + bP + cP2 + dP3	polynomial curve fit
3	$h(P)=\frac{\pi k_{gas}}{4\lambda}\left[1.+85\left(\frac{P}{\sigma}\right)^{0.8}\right]=\frac{a}{b}\left[1.+85\left(\frac{P}{c}\right)^{0.8}\right]$	I.T. Shvets, “Contact Heat Transfer between Plane Metal Surfaces”, Int. Chem. Eng., Vol4, No. 4, p621, 1964.
4	$h(P)=a\left[1-\exp\left(-b\frac{P}{c}\right)\right]^{d}$	Li & Sellers, Proc. Of 2nd Int. Conf. Modeling of Metals Rolling Processes, The Institute of Materials, London, 1996.

Contact parameters

Contact conductance function of pressure

LSTC

M. Merklein and J. Lechler, "Determination of Material and process Characteristics for Hot Stamping Processes of Quenchable Ultra High Strength Steels with Respect to a FE_based Process design", SAE Technical Paper 2008-01-0853, April, 2008.



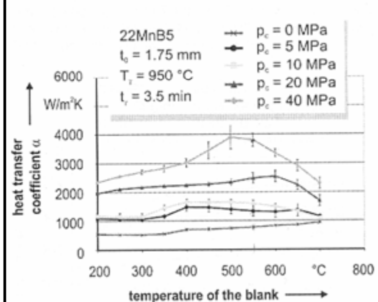
Numisheet BM03 data

P [MPa]	h [W/m²K]
0	1300
20	4000
35	4500

Contact parameters

How do you calculate h(P) at the interface

LSTC



P	h @ 550C (curve)	h calculated
0	750	750
5	1330	1330
10	1750	1770
20	2500	2520
40	3830	3830

$$h = \frac{k\pi}{4\lambda}\left[1 + 85\left(\frac{P}{\sigma_r}\right)^{0.8}\right]$$

h = contact conductance [W/m²C]
k = air thermal conductivity
0.059 W/mC at 550 C
λ = surface roughness [m]
P = interface pressure [MPa]
σ_r = rupture stress [MPa]

M. Merklein and J. Lechler, "Determination of Material and process Characteristics for Hot Stamping Processes of Quenchable Ultra High Strength Steels with Respect to a FE_based Process design", SAE Technical Paper 2008-01-0853, April, 2008.

I.T. Shvets, "Contact Heat Transfer Between Plane Metal Surfaces", Int. Chem. Eng, Vol 4, No 4, p621, 1964.

Contact parameters

How do you calculate h(P) at the interface

LSTC

1. Using curve data, solve the equation for λ at (P, h) = (0, 750).

$$750 = \frac{(0.059)\pi}{4\lambda}\left[1+85\left(\frac{0}{\sigma_r}\right)^{0.8}\right]$$

λ = 61.8e-05

2. Using curve data and the above value for λ, solve the equation for σ_r at (P, h) = (40, 3830).

$$3830 = \frac{(0.059)\pi}{4\left(6.18*10^{-5}\right)}\left[1+85\left(\frac{40}{\sigma_r}\right)^{0.8}\right]$$

σ_r = 1765

3. Now use the equation to calculate h(P)

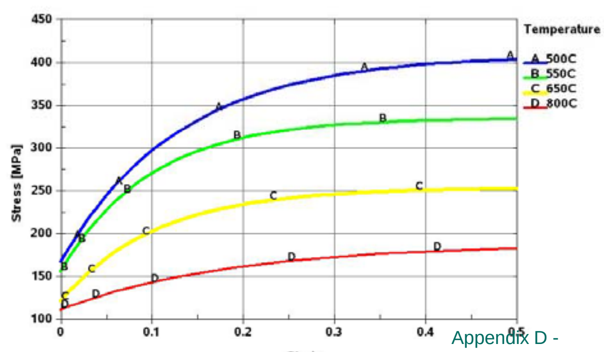
$$h = \frac{(0.059)\pi}{4\left(6.18*10^{-5}\right)}\left[1+85\left(\frac{p}{1765}\right)^{0.8}\right]$$

Numisheet 2008 data for 22MnB5

LSTC

de/dt [s^{-1}]	T [°C]				
	500	550	650	700	800
0.01					
0.1					
1.0					

A material model (MAT_106) was used that allowed interpolation of the σ vs. ε data as a function of temperature at a specified strain rate.



MAT_106 : Elastic Viscoplastic Thermal

LSTC

1	2	3	4	5	6	7	8
MID	RO	E	PR	SIGY	ALPHA	LCSS	
C	P	LCE	LCPR	LCSIGY			LCALPH
LCC	LCP						

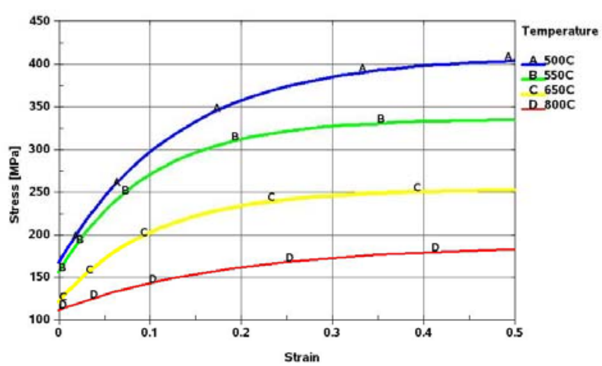
MAT_106 : Elastic Viscoplastic Thermal

How to enter σ vs. ε vs. T

LSTC

***DEFINE_TABLE**
500
550
650
800
***DEFINE_CURVE**
(stress,strain) at T=500
.
***DEFINE_CURVE**
(stress,strain) at T=550
.
***DEFINE_CURVE**
(stress,strain) at T=650
.
***DEFINE_CURVE**
(stress,strain) at T=800
.

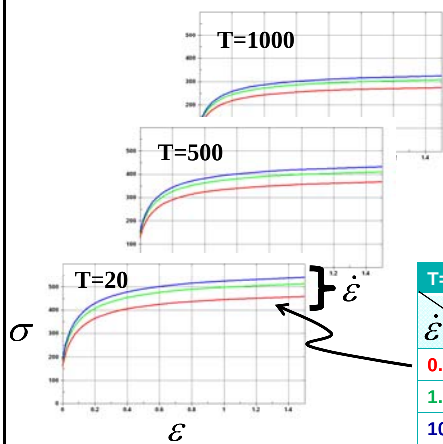
Material: 22MnB5 ($d\varepsilon/dt=0.1\ s^{-1}$)
Data from University of Erlangen



DEFINE_TABLE_3D → $\sigma = f(\varepsilon, \dot{\varepsilon}, T)$

For each temperature, T, we have a table of hardening curves of σ vs ε at 3 strain rates, $\dot{\varepsilon}$

LSTC



T=1000

$\dot{\varepsilon}$ \ ε	0.0	0.1	0.4	1.0
0.1	98	190	244	268

T=500

$\dot{\varepsilon}$ \ ε	0.0	0.1	0.4	1.0
0.1	130	253	325	357

T=20

$\dot{\varepsilon}$ \ ε	0.0	0.1	0.4	1.0
0.1	σ = 162	σ = 316	σ = 406	σ = 446
1.0	182	354	455	500
10.	192	373	479	527

DEFINE_TABLE_3D → $\sigma = f(\varepsilon, \dot{\varepsilon}, T)$

Keyword input

LSTC

```
*DEFINE_TABLE_3D
$      tbid
       2000
$          temperature   tbid
                   20.    100
                  500.    200
                 1000.    300
*DEFINE_TABLE
$      tbid
        100
$          strain_rate   lcid
                   0.1    101
                   1.0    102
                  10.0    103
*DEFINE_CURVE
$      lcid
        101
$               strain  stress
                   0.0    162
                   1.0    446
```

For each temperature, we specify a table with 3 strain rates

For each strain rate, we specify a curve of σ vs ε

MAT_106 : Elastic Viscoplastic Thermal

Cowper and Symonds model

LSTC

Viscous effects are accounted for using the Cowper and Symonds model, which scales the yield stress with the factor

$$1+\left(\frac{\dot{\varepsilon}_{eff}^{P}}{C}\right)^{1/P}$$

Temp [C]	20	100	200	300	400	500	600	700	800	900	1000
E [MPa]	212	207	199	193	166	158	150	142	134	126	118
v	0.284	0.286	0.289	0.293	0.298	0.303	0.310	0.317	0.325	0.334	0.343
p	4.28	4.21	4.10	3.97	3.83	3.69	3.53	3.37	3.21	3.04	2.87
c	6.2e9	8.4e5	1.5e4	1.4e3	258.	78.4	35.4	23.3	22.2	30.3	55.2

Courtesy of David Lorenz, Dynamore, Stuttgart, Germany.

MAT_244 : Ultra High Strength Steel

LSTC

MAT_244
MAT_UHS_STEEL

This material model is based on the Ph.D thesis by Paul Akerstrom and implemented by Tobias Olsson (ERAB)

Input includes:

1. 15 element constituents
2. Latent heat
3. Expansion coefficients
4. Phase hardening curves
5. Phase kinetic parameters
6. Cowper-Symonds parameters

Output includes:

1. Austenite phase fraction
2. Ferrite phase fraction
3. Pearlite phase fraction
4. Bainite phase fraction
5. Martensite phase fraction
6. Vicker's hardness distribution
7. Yield stress distribution

Paul Akerstrom, "Modelling and Simulation of Hot Stamping", Lulea University of Technology, 2006.

MAT_244 : Ultra High Strength Steel

LSTC

Material model predicts phase fractions and hardness.

% martensite

Vickers hardness

MAT_244 : Ultra High Strength Steel

Boron steel composition, wt%

LSTC

	HAZ	Akerstrom	Naderi	ThyssenKrupp Max. values
B		0.003	0.003	0.005
C	0.168	0.23	0.230	0.250
Co				
Mo	0.036			0.250
Cr	0.255	0.211	0.160	0.250
Ni	0.015			
Mn	1.497	1.25	1.18	1.40
Si	0.473	0.29	0.220	0.400
V	0.026			
W				
Cu	0.025			
P	0.012	0.013	0.015	0.025
Al	0.020			
As				
Ti			0.040	0.05
S		0.003	0.001	0.010

MAT_244 : Ultra High Strength Steel

Phase start temperatures

LSTC

Start temperature calculation algorithm

$T_{ferrite} = A_{e3}$ = 273+912-203C$^{1/2}$-15.2Ni+44.6Si+104Va+31.5Mo-30Mn-11Cr-20Cu+700P+400Al+120As+400Ti

$T_{pearlite} = A_{e1}$ = 273+723-10.7Mn-16.9Ni+29Si+16.9Cr+290As+6.4W

$T_{bainite}$ = 273+ 656-58C-35Mn-75Si-15Ni-34Cr-41Mo

$T_{martensite}$ = 273+561-474C-35Mn-17Ni-17Cr-21Mo

Element wt%

Temperature initial condition must be greater than $T_{ferrite}$

Data printed to D3HSP file and messag file

```
Ferrite    start temperature       = 1.06986E+03
Pearlite   start temperature       = 9.94761E+02
Bainite    start temperature       = 8.43146E+02
Martensite start temperature       = 6.80303E+02
```

http://www.msm.cam.ac.uk/map/kinetics/programs/haz_microstructure.html

MAT_244 : Ultra High Strength Steel

Phase change kinetics

LSTC

austenite to ferrite

$$\frac{dX_f}{dt} = \frac{\exp\left(-\frac{Q_f}{RT}\right)}{C_f} 2^{(G-1)/2} (\Delta T)^3 X_f^{2(1-X_f)/3} (1-X_f)^{2X_f/3}$$

C_f = 59.6Mn + 1.45Ni + 67.7Cr + 24.4Mo + K_fB

austenite to pearlite

$$\frac{dX_p}{dt} = \frac{\exp\left(-\frac{Q_p}{RT}\right)}{C_p} 2^{(G-1)/2} (\Delta T)^3 DX_p^{2(1-X_p)/3} (1-X_p)^{2X_p/3}$$

C_p = 1.79 + 5.42(Cr + Mo + 4MoNi) + K_pB

Input parameters

- Q_f = activation energy
- Q_p = activation energy
- Q_b = activation energy
- G = grain size
- α = material constant
- K_f = boron factor
- K_p = boron factor

MAT_244 : Ultra High Strength Steel

Phase change kinetics

LSTC

austenite to bainite

$$\frac{dX_b}{dt} = \frac{\exp\left(-\frac{Q_b}{RT}\right)}{C_b} 2^{(G-1)/2} (\Delta T)^2 D X_b^{2(1-X_b)/3} (1-X_b)^{2X_b/3}$$

$C_b = 10^{-4}(2.34 + 10.1C + 3.8Cr + 19Mo)Z$

austenite to martensite

$$X_m = X_a\left[1 - e^{-\alpha(T_{ms}-T)}\right]$$

Empirical equation with $\alpha = 0.011$

A.J. Fletcher, Thermal Stress and Strain Generation in Heat Treatment, 1989, ISBN 1-85166-245-6.

Appendix D - 37

MAT_244 : Ultra High Strength Steel

Hardness calculation is empirically based

LSTC

$H = (x_f+x_p)H_{f\text{-}p} + x_bH_b + x_aH_a$

$H_{f\text{-}p} = 42 + 223C + 53Si + 30Mn + 12.6Ni + 7Cr + 19Mo + (10 - 19Si + 4Ni + 8Cr + 130V)\ln(dT/dt)_{973}$

$H_b = -323 + 185C + 330Si + 153Mn + 65Ni + 144Cr + 191Mo + (89 + 53C - 55Si - 22Mn - 10Ni - 20Cr - 33Mo)\ln(dT/dt)_{973}$

$H_a = 127 + 949C + 27Si + 11Mn + 8Ni + 16Cr + 12\ln(dT/dt)_{973}$

Appendix D - 38

MAT_244 : Ultra High Strength Steel
Mechanical & Plasticity Material Model

LSTC

Since the material has 5 phases, the yield stress is represented by a mixture law

$$\sigma_y = x_1\sigma_1\left(\bar{\varepsilon}_1^p\right) + x_2\sigma_2\left(\bar{\varepsilon}_2^p\right) + x_3\sigma_3\left(\bar{\varepsilon}_3^p\right) + x_4\sigma_4\left(\bar{\varepsilon}_4^p\right) + x_5\sigma_5\left(\bar{\varepsilon}_5^p\right)$$

LC1
LC2
LC3
LC4
LC5

Where $\sigma_i\left(\bar{\varepsilon}_i^p\right)$ is the yield stress for phase i at the effective plastic strain for that phase.

References
1. T. Olsson, "An LS-DYNA Material Model for Simulations of Hot Stamping Processes of Ultra High Strength Steels", ERAB, April 2009, tobias.olsson@erab.se
2. P. Akerstrom, Modeling and Simulation of Hot Stamping, Doctoral Thesis, Lulea University of Technology, Lulea, Sweden, 2006.

MAT_244 QA parameter study

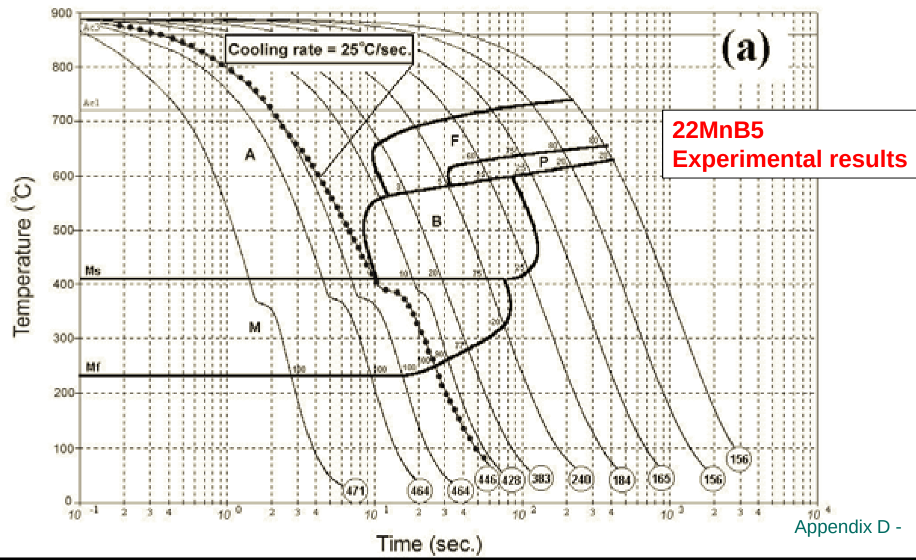
LSTC

	1	2
Q_1/R	11575	13022
Q_2/R	13839	15569
Q_3/R	13588	15287
K_f	1.9e+05	0.
K_p	3.1e+03	0.
a	0.011	0.011
G	8	8

$[Q/R]_2 = 1.125*[Q/R]_1$

MAT_244 QA parameter study

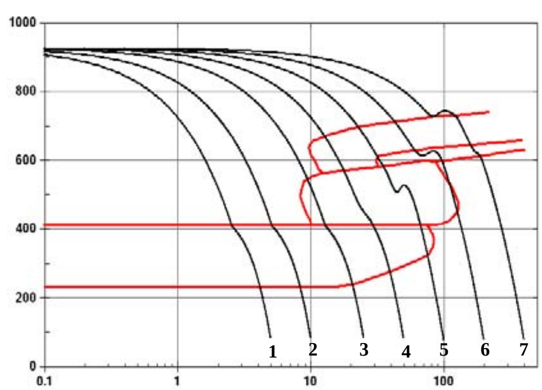
M. Naderi, Thesis 11/2007, Dept. Ferrous Metallurgy, RWTH Aachen University, Germany



MAT_244 QA parameter study

Using data set 2

CCT Diagram for 22MnB5 overlaid with LS-DYNA calculated cooling curves and Vickers hardness using MAT_UHS_STEEL

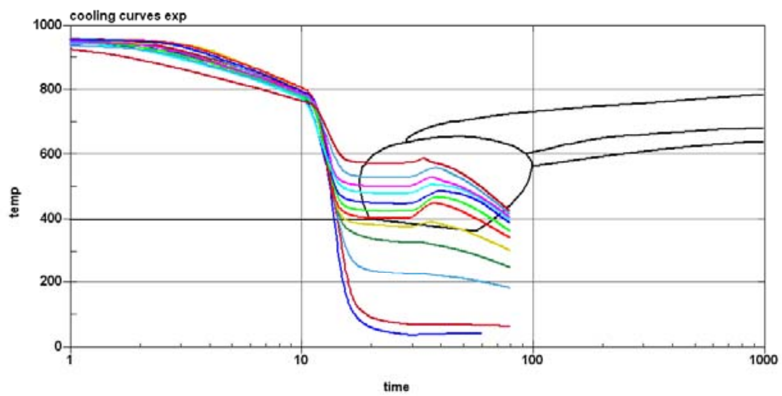


	Rate C/sec	Vickers Hardness	Exp. Naderi
1	200	478	-----
2	100	472	471
3	40	459	428
4	20	376	383
5	10	273	240
6	5	174	175
7	2.5	172	165

MAT_244 QA parameter study

Recent experimental data

LSTC



MAT_244 QA parameter study

LSTC

1

Cooling rate [C/sec]	Vickers Hardness	Ferrite wt%	Pearlite wt%	Bainite wt%	Martensite wt%
200	428	0.0001	0.0010	0.3978	0.5840
100	336	0.0001	0.0031	0.9825	0.0139
40	310	0.0001	0.0188	0.9810	0.0001
20	283	0.0002	0.1193	0.8804	0.0001
10	176	0.0006	0.9993	0.0001	0.0000
5	174	0.0023	0.9976	0.0001	0.0000
2.5	172	0.0125	0.9874	0.0001	0.0000

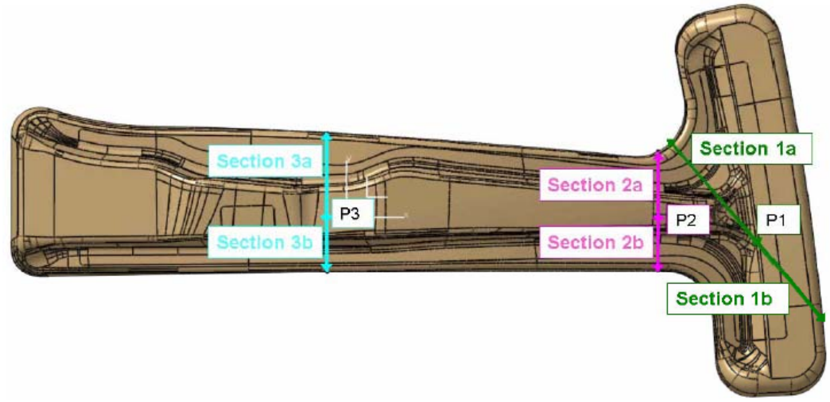
2

Cooling rate [C/sec]	Vickers Hardness	Ferrite wt%	Pearlite wt%	Bainite wt%	Martensite wt%
200	478	0.0001	0.0004	0.0008	0.9692
100	472	0.0001	0.0009	0.0028	0.9668
40	459	0.0002	0.0040	0.0256	0.9416
20	376	0.0005	0.0154	0.4819	0.4880
10	273	0.0018	0.0852	0.9015	0.0111
5	174	0.0093	0.9906	0.0001	0.0000
2.5	172	0.7023	0.2976	0.0000	0.0000

MAT_244 QA parameter study

Numisheet Benchmark BM03

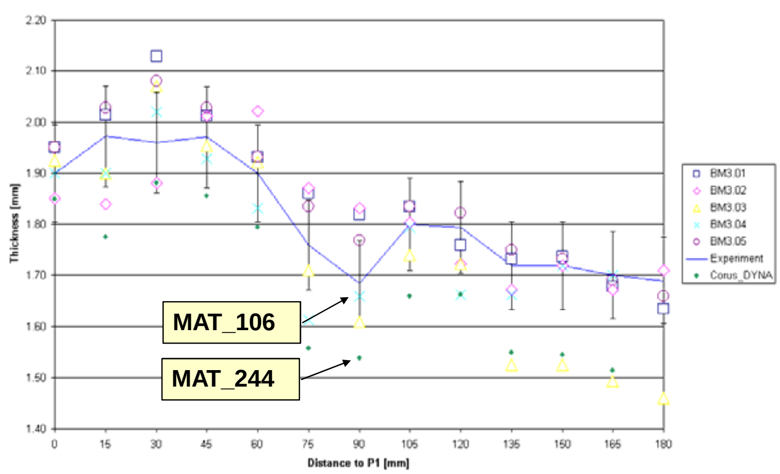
LSTC



MAT_244 QA parameter study

Numisheet Benchmark BM03 section 1a

LSTC

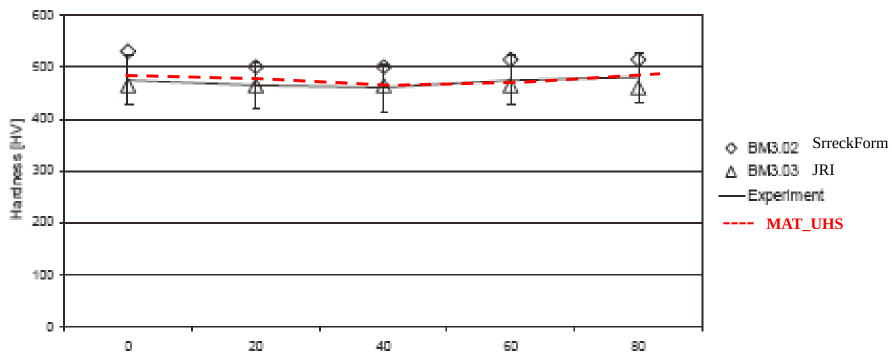


By: Sander van der Hoorn, Corus, The Netherlands

MAT_244 QA parameter study

Vickers hardness for section 2b

LSTC



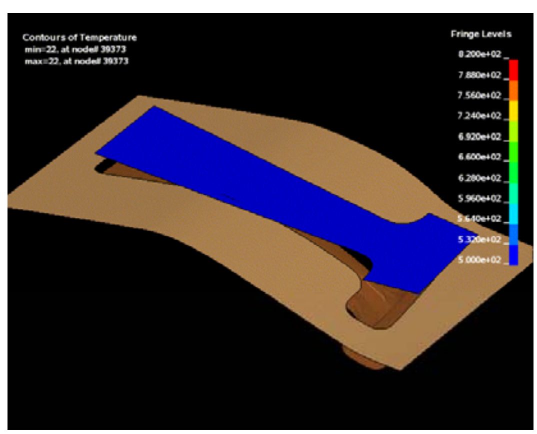
Numisheet 2008 BM03 Model

Forming process

LSTC

FE model

Tools: 68,268 rigid shells
Blank: 3,096 deformable shells increasing to 11,682 after adaptivity



Run time:
INTEL Core Quad CPU @ 2.40GHz

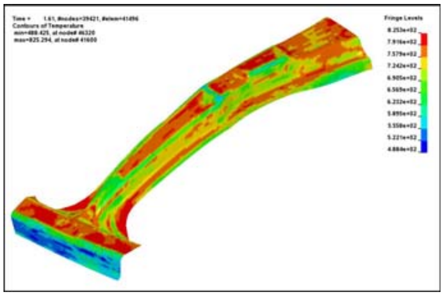
1 cpu → 5.10 hr
2 cpu → 3.96 hr
4 cpu → 2.65 hr

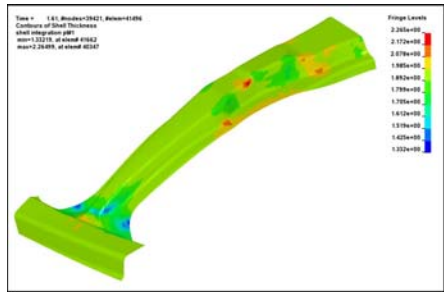
Time step
- **mechanical 1.e-05**
- **thermal 1.e-03**

Numisheet 2008 BM03 Simulation

LSTC

Results after forming





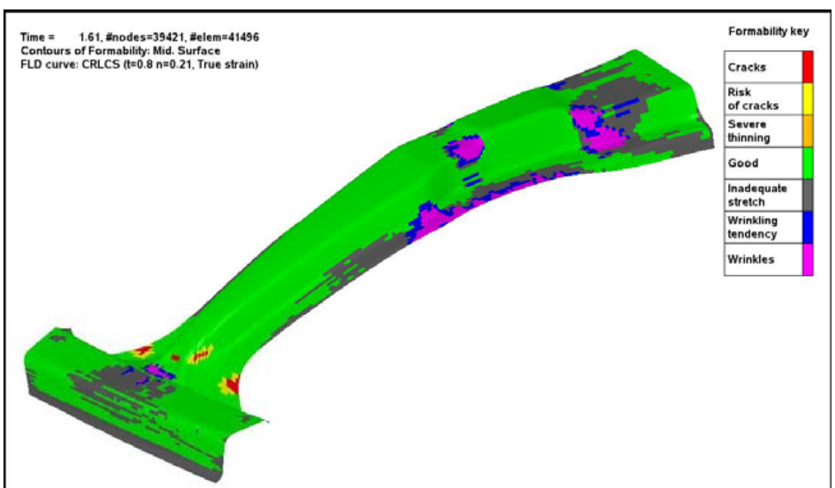
Temperature
- min = 488C
- max = 825C

Thickness
- min = 1.33mm
- max = 2.26mm

Numisheet 2008 BM03 Simulation

FLD

LSTC

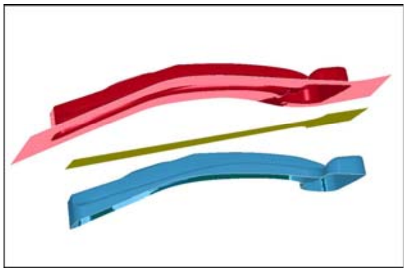


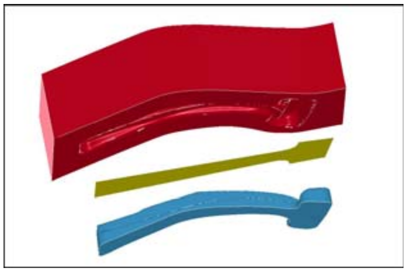
Numisheet 2008 BM03 Simulation

Quench: hold time in the tool 20 sec

LSTC

Modeling the cooling rate correctly is critical in determining the material phase composition and the material hardness. The local cooling rate is affected by the heat transfer between the blank and tools. The tools must be modeled using solid elements as shown in the figure below for an accurate calculation. We did not do this for the benchmark. Our FE model used shells for the tools fixed at the specified tool temperature of 75C.





shell model dT/dt > solid model dT/dt

Numisheet 2008 BM03 Simulation

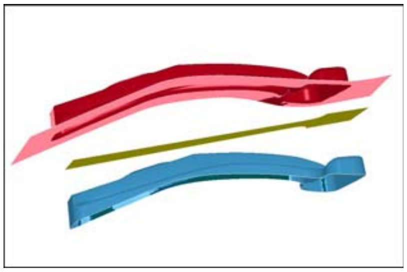
LSTC

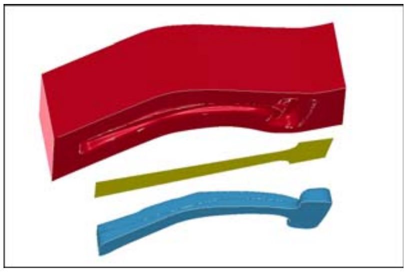
Shell geometry (5.0hr run time)
- 68,268 rigid shells
- 3,096 deformable shells

- 11,682 shells after adaptivity

Solid geometry (5.9hr run time)
- 532,927 solids (punch & die)
- 6,692 shells (holder)
- 3,096 deformable shells (blank)

- 11,682 shells af*ter* adaptivity



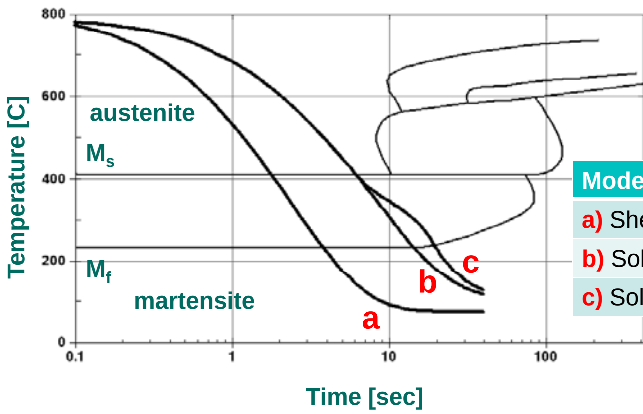


Numisheet 2008 BM03 Simulation

Cool down to room temperature

LSTC

CCT diagram for 22MnB5 steel



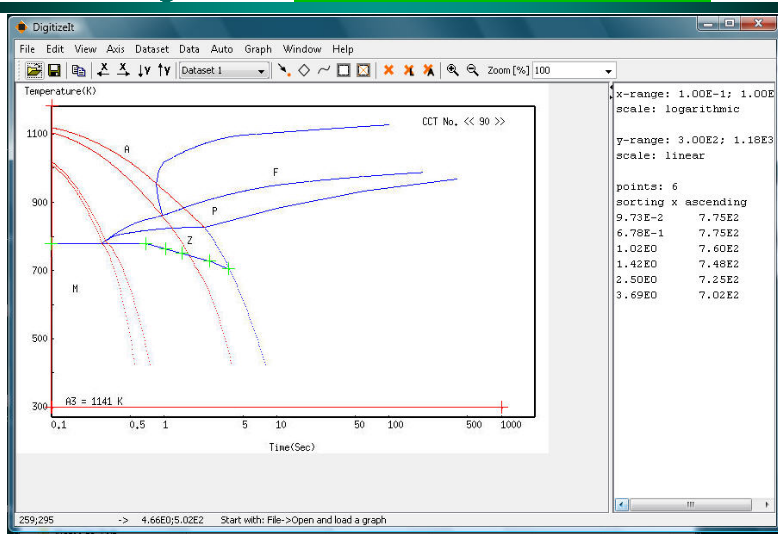
Model
a) Shell tools
b) Solid tools
c) Solid tools + phase change

Our benchmark results are depicted by curve (a). Subsequently, we looked at the affect when using solid tools (b) and including phase change (c). The cooling rate is much slower.

Creating a CCT diagram

DigitizeIt, http://www.digitizeit.de/

LSTC



Creating a CCT diagram

LSTC

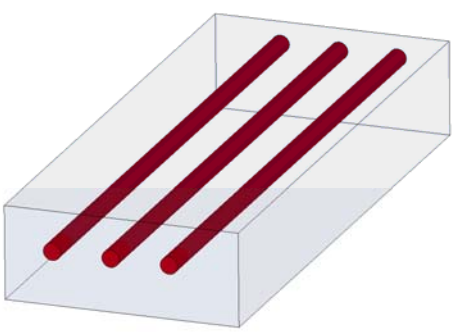
1. Obtain an image of a CCT diagram (e.g., from https://inaba.nims.go.jp/Weld/cct/
2. Use software to digitize curves (e.g, DigitizeIt, http://www.digitizeit.de/) and save as xy-data
3. Using LS-PrePost, plot temperature history of one or more nodes and save as xy-data
4. Import xy-data into LS-PrePost and display curves on a single plot

Modeling tool cooling

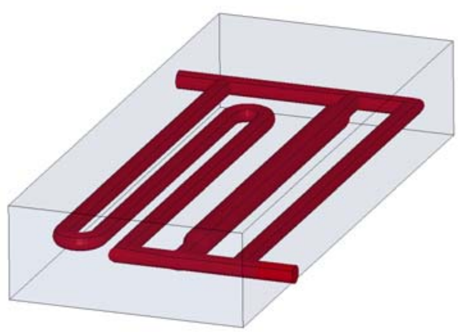
There are 2 methods to model fluid flow

LSTC

BULKFLOW



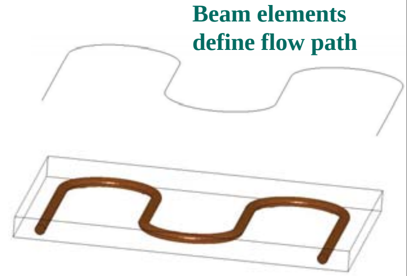
BOUNDARY_CONVECTION using network analyzer



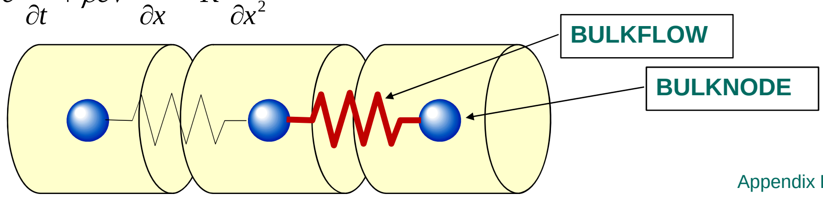
BULKNODE and BULKFLOW method

LSTC

BULK FLOW is a lumped parameter approach to model fluid flow in a pipe. The flow path is defined with a contiguous set of beam elements. The beam node points are called **BULK NODES** and represents a homogeneous slug of fluid. Using the **BULKFLOW** keyword we define a mass flow rate for the beams. We then solve the advection-diffusion equation.



$$\rho c \frac{\partial T}{\partial t} + \rho c V \frac{\partial T}{\partial x} = K \frac{\partial^2 T}{\partial x^2}$$

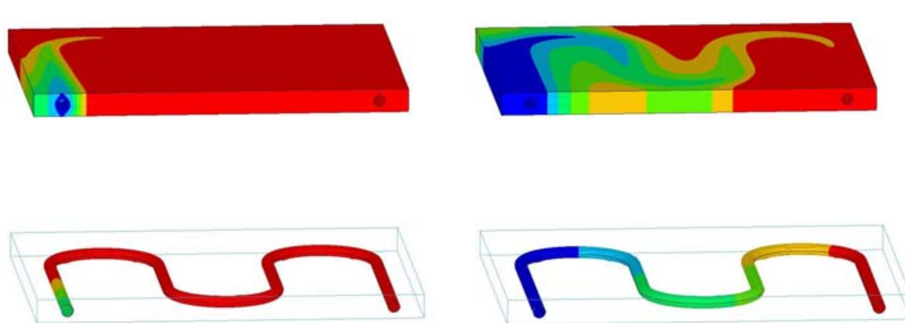


BULKNODE and BULKFLOW method

Die cooling

LSTC

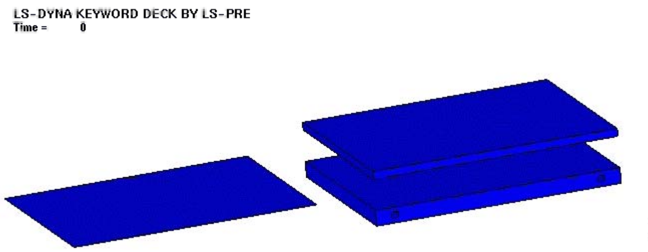
A Bulk Fluid Flow algorithm is used to model the energy exchange between the cold fluid flowing through the die cooling channels.



BULKNODE and BULKFLOW method

Die cooling

LSTC



Appendix D - 59

Water properties

LSTC

T [C]	ρ [kg/m^3]	C_p [J/kg C]	μ [kg/m s]	k [W/m C]
20	998.	4182.	1.002e-03	0.603
40	992.	4179.	0.651e-03	0.632
60	983.	4185.	0.462e-03	0.653
80	972.	4197.	0.350e-03	0.670
100	958.	4216.	0.278e-03	0.681

Appendix D - 60

How do you determine a pipe flow convection coefficient

Problem definition

LSTC

Pipe diameter = D = 15mm = 0.015 m

Pipe cross section area = A = $\pi D^2/4 = \pi(0.015)^2/4$ = 1.77e-04 m^2

Volumetric flow rate = G = 20 l/min = 0.02 m^3/min = 3.33e-04 m^3/sec

Flow velocity = G/A = 1.89 m/sec

Pipe wall temperature = T_{wall} =100C

Water temperature = T_{fluid} = 20C

How do you determine a pipe flow convection coefficient

Some preliminaries

LSTC

Fully developed – the effect of entrance conditions (e.g., pipe from a header) on h are negligible.

$$\frac{L}{D} > 40$$

Fluid properties are evaluated at the film temperature, T_{film}

$$T_{film} = \frac{T_{wall} + T_{fluid}}{2} = \frac{100+20}{2} = 60$$

Reynolds number

$$\mathrm{Re} = \frac{V\rho D}{\mu} = \frac{(1.89)(983)(0.015)}{0.462*10^{-3}} = 6.03*10^{4}$$

Prandtl number

$$\mathrm{Pr} = \frac{c_p \mu}{k} = \frac{(4185.)(0.462*10^{-3})}{0.653} = 2.96$$

How do you determine a pipe flow convection coefficient

Classical empirical correlations

LSTC

Dittus-Boelter equation

$$h = 0.023\frac{k}{D}\mathrm{Re}^{0.8}\,\mathrm{Pr}^{n}$$

n=0.3 for cooling of the fluid
n=0.4 for heating of the fluid

$$= 0.023\frac{0.653}{0.015}\left(6.03*10^{4}\right)^{0.8}\left(2.96\right)^{0.4} = 10{,}300\ \frac{W}{m^{2}C}$$

Sieder-Tate equation

$$h = 0.023\frac{k}{D}\mathrm{Re}^{0.8}\,\mathrm{Pr}^{n}\left(\frac{\mu_{bulk}}{\mu_{wall}}\right)^{0.14}$$

μ(T) correction factor

What do you do if the pipe is not perfectly smooth

How do you determine a pipe flow convection coefficient

Gnielinski correlation

LSTC

$$h = \left(\frac{k}{D}\right)\left[\frac{(f/8)(\mathrm{Re}-1000)\mathrm{Pr}}{1+12.7(f/8)^{0.5}\left(\mathrm{Pr}^{2/3}-1\right)}\right] = 11{,}400\ W/m^{2}C$$

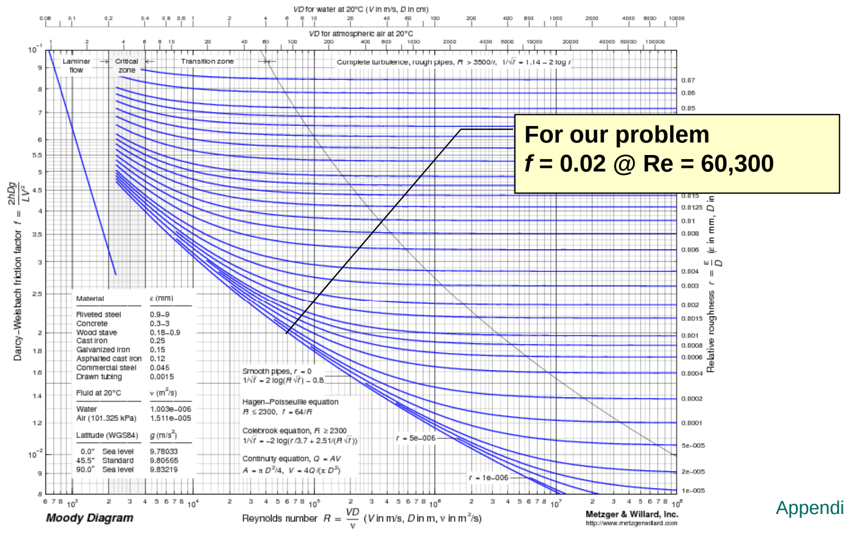
f **= Darcy–Weisbach friction factor (see next vu-graph for value)**

There are 2 definitions for *f*. The Darcy–Weisbach friction factor is 4 times larger than the Fanning friction factor, so attention must be paid to note which one of these is meant in any "friction factor" chart or equation being used. The Darcy–Weisbach factor is more commonly used by civil and mechanical engineers, and the Fanning factor by chemical engineers, but care should be taken to identify the correct factor regardless of the source of the chart or formula.

How do you determine a pipe flow friction factor

http://www.mathworks.com/matlabcentral/fx_files/7747/1/moody.png

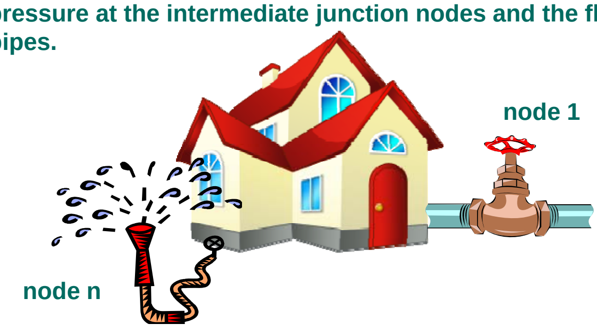
LSTC



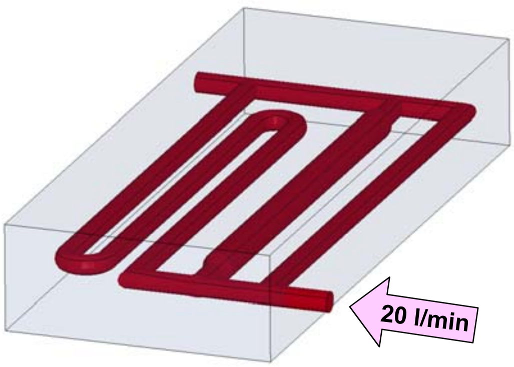
Pipe Network

LSTC

Think about pipes in your house. The starting point is the valve on the pipe entering your house. We will call this NODE 1. Node 1 is special and has a boundary condition specified. The BC is the pressure you would read on a pressure gauge at this location. The water enters your house and passes through several pipe junctions before it exits through your garden hose. Every junction is represented by a NODE. The last node also needs a BC specified. This BC is the mass flow rate. The pipe flow code will calculate the pressure at the intermediate junction nodes and the flow rate through the pipes.



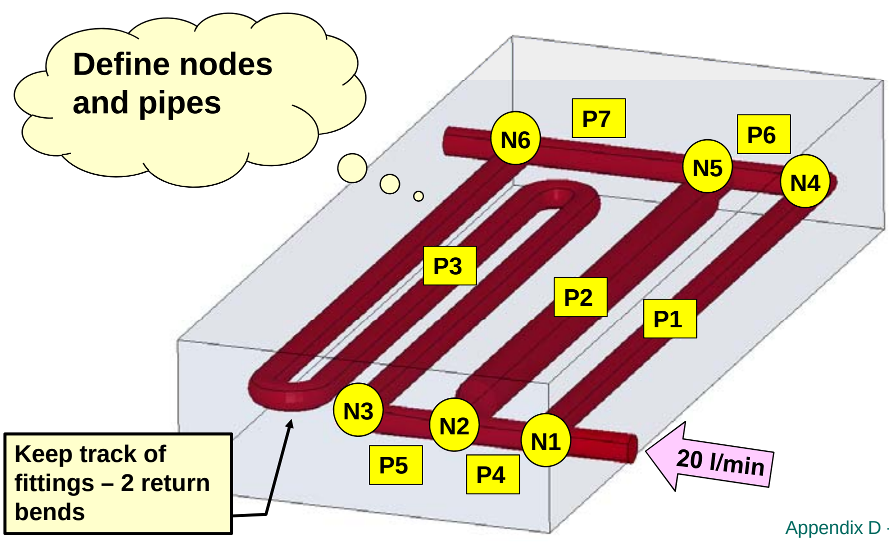
Pipe Network



Given an entering flow rate, calculate the flow in each pipe and the convection heat transfer coefficient

Pipe Network

Define nodes and pipes



Pipe Network

LSTC

	input						output	
Pipe	N1	N2	Length [m]	Dia. [mm]	Rough [mm]	Ftg. [L_e/D]	Q [l/min]	h [W/m²C]
1	1	4	1	10	0.05		5.7	5600
2	2	5	1	20	0.05		9.7	2400
3	3	6	3	10	0.05	100	4.5	4600
4	1	2	0.2	10	0.05		14.2	11000
5	2	3	0.2	10	0.05		4.5	4600
6	4	5	0.2	10	0.05		5.7	5600
7	5	6	0.4	10	0.05		15.5	12000

Pipe Network

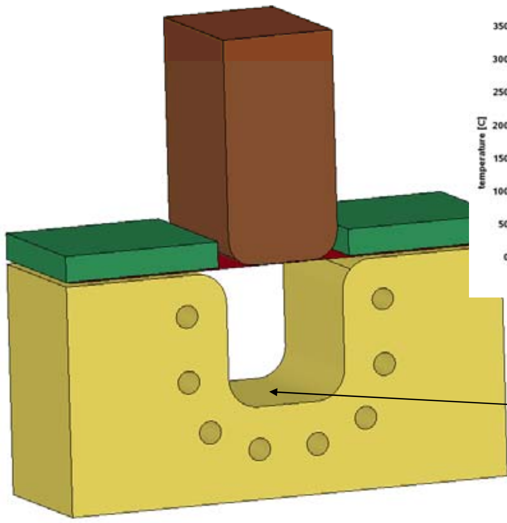
LSTC

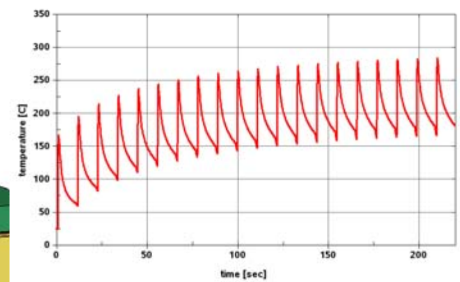
Pipe type	Roughness, e [mm]
Cast iron	0.25
Galvanized iron	0.15
Steel or wrought iron	0.046
Drawn tubing	0.0015

Fitting type	Equivalent length L_e/D
Globe valve	350
Gate valve	13
Check valve	30
90º std. elbow	30
90º long radius	20
90º street elbow	50
45º elbow	16
Tee flow through run	20
Tee flow through branch	60
Return bend	50

Process start-up time

LSTC

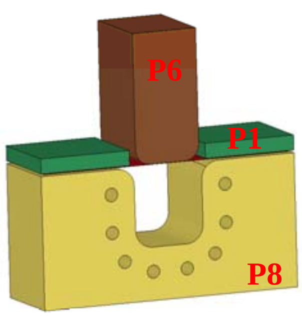




Shown is the tempeature history for this location during 20 stamping cycles.

Process start-up time

LSTC




```
#!/bin/csh -f
set i=1
while ( $i <= 20 )
./ls971 i=stamping.k g=d3plot$i"_"
@ i = $i + 1
end
```

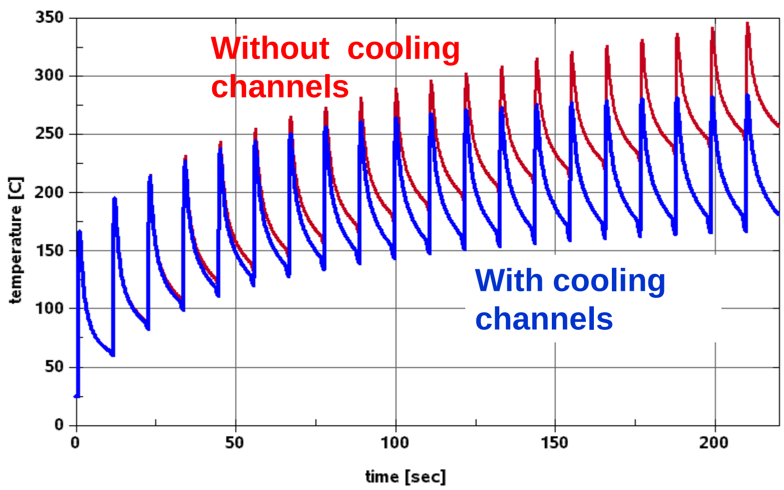
```
*KEYWORD
*INITIAL_TEMPERATURE_SET
$     nsid      temp
         1       25.
         6       25.
         8       25.
*END
```

```
*INCLUDE

new_temp_ic.inc

*INTERFACE_SPRINGBACK_LSDYNA
$     psid
         1
*SET_PART_LIST
$     psid
         1
$     pid1     pid2     pid3
         1        6        8
```

Process start-up time

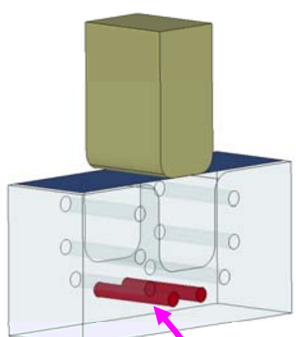
Tool temperature after 20 stampings

LSTC



Thermostat feature adjusts the heating rate to keep the sensor temperature at the set point.

LSTC



***LOAD_HEAT_CONTROLLER**

Q_{cont}	volumetric heating rate
Q_0	constant volumetric heating rate
G_p	proportional gain
G_i	integral gain
T_{set}	set point temperature
T	sensor temperature (at a node)

$$Q_{cont} = Q_0 + \underbrace{G_p\left(T_{set} - T\right)}_{\text{proportional}} + \underbrace{G_i \int_{t=0}^{t} \left(T_{set} - T\right) dt}_{\text{integral}}$$

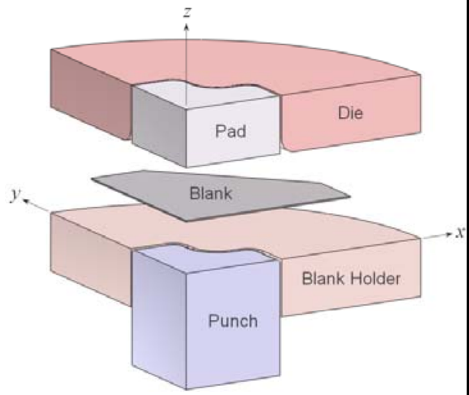
LSTC

LSTC

Warm Stamping (Numisheet bm02, 2011)

by: Art Shapiro, LSTC

LSTC



Magnesium alloys exhibit very poor workability and formability at room temperature. Using warm or hot press forming technology, the forming limit can be considerably increased.

Process Specification

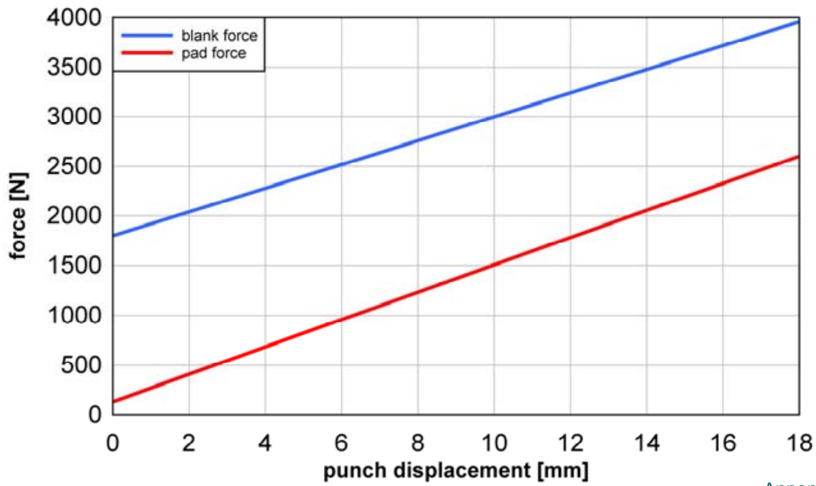
LSTC

In order to maximize the deep drawability of the blank material, the die and the blank holder are heated, while the punch and pad are cooled. Process parameters are as follows:

1. Tool material: hardened tool steel SKD11
2. Blank material: magnesium alloy AZ31B
3. Surface temperature of the die and blank holder: 250C
4. Surface temperature of punch and pad: 100C
5. Punch velocity: 0.15 mm/sec
6. Blank holding force: 1.8 – 3.96 kN as shown in slide 3.
7. Pad force: 0.137 – 2.603 kN as shown in slide 3.
8. Drawing depth: 18mm

Process Specification
Pad and Blank Holding Force

LSTC



Process Specification
Magnesium Alloy AZ31B properties

LSTC

Young's modulus	45 GPa
Poisson's ratio	0.35
Density	1,770 kg/m^3
Thermal conductivity	96 W/(m·℃)
Heat capacity	1,000 J/(kg·℃)
Friction coefficient	0.1
Interface heat transfer	4,500 W/m^{2}C

Process Specification
Magnesium anisotropic Lankford R-value data

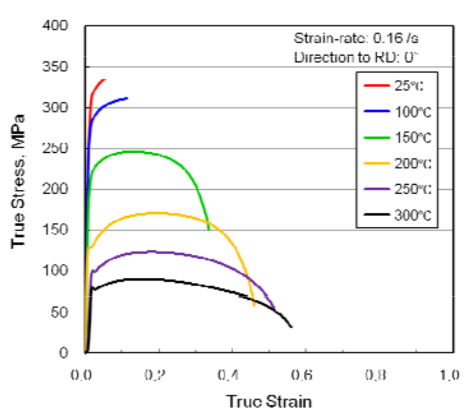
LSTC

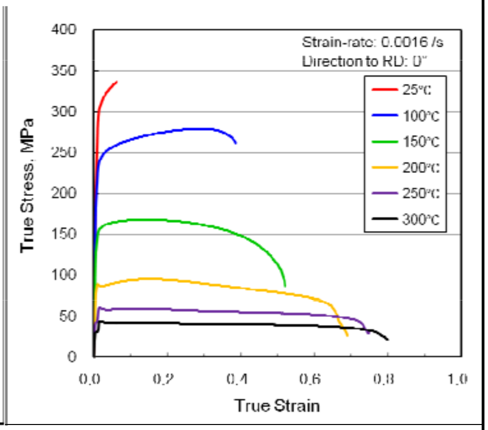
	Strain rate : 0.016 /s						Temperature : 250℃		
Test direction	Temperature [℃]						Strain rate [/s]		
	RT	100	150	200	250	300	0.16	0.016	0.0016
0°	1.347	2.006	1.291	1.621	1.344	1.374	2.002	1.344	0.965
45°	2.793	2.412	1.976	2.118	1.532	1.477	2.648	1.532	1.112
90°	4.109	4.406	3.189	2.672	1.799	1.881	3.082	1.799	1.350
Mean	2.760	2.809	2.108	2.132	1.552	1.552	2.595	1.552	1.135

Appendix E 5

Process Specification
Magnesium true stress vs. true strain data

LSTC





Data for strain rates of 0.00016 and 0.016 are not shown.

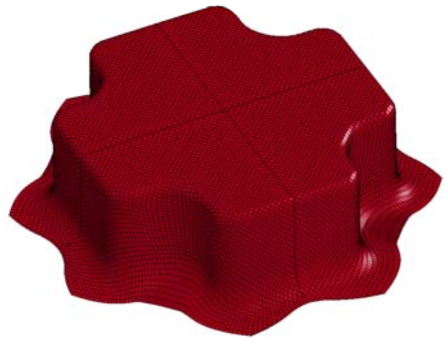
Appendix E 6

LS-DYNA analysis
Deformed shape

LSTC

Experiment

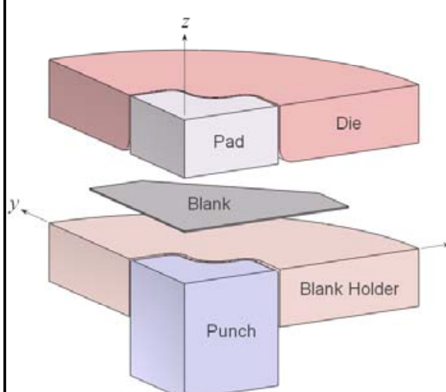
LS-DYNA analysis



The following slides show the LS-DYNA modeling methodology.

Model Geometry
PID and CID

LSTC



Part ID	Contact ID, PID1-PID2
PID 1 → blank	CID 1 → pad → 1-2
PID 2 → pad	CID 2 → punch → 1-3
PID 3 → punch	CID 3 → holder → 1-4
PID 4 → holder	CID 4 → die → 1-5
PID 5 → die	

Material Model

There are 2 material models that can be used

LSTC

MAT_36 → MAT_3-PARAMETER_BARLAT
- set R00, R45, R90 as a function of temperature
- define a 4-dimensional table of (stress, strain, strain rate, temperature)
- slow execution speed due to 4-dimensional table interpolation (~9min)

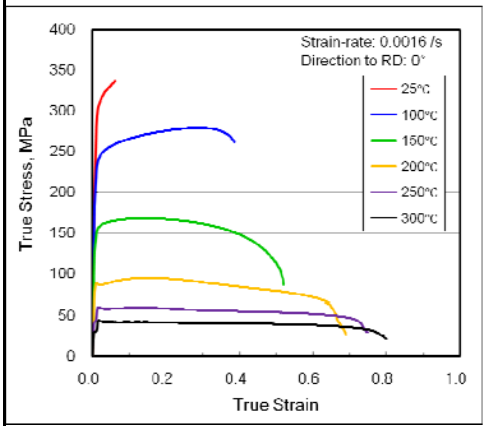
MAT_37 → MAT_TRANSVERSELY_ANISOTROPIC_ELASTIC_PLASTIC
- define a single mean R value
- define a curve of (stress, strain) at a fixed temperature and strain rate
- fast execution speed due to single curve interpolation (~6 min)

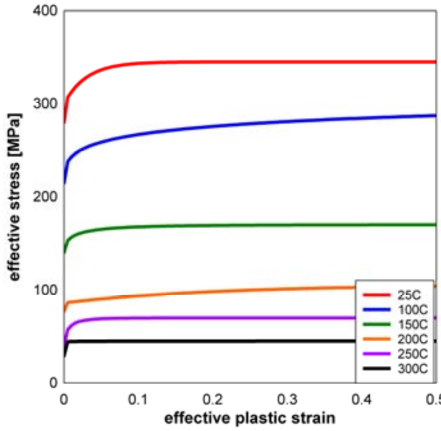
We have to convert the given raw (true σ, true ε) data into (effective σ, effective plastic ε) data required by these numerical material models.

Material Model

Conversion procedure described in:
http://www.dynasupport.com/howtos/material/from-engineering-to-true-strain-true-stress

LSTC

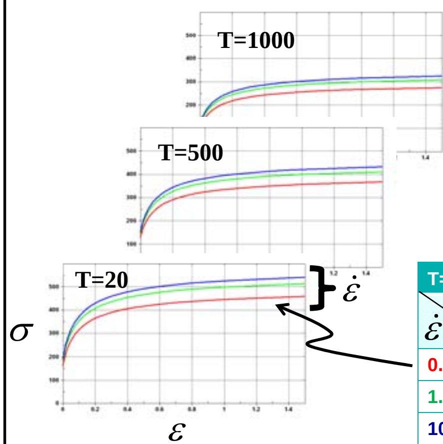




DEFINE_TABLE_3D → $\sigma = f(\varepsilon, \dot{\varepsilon}, T)$

For each temperature, T, we have a table of hardening curves of σ vs ε at 3 strain rates, $\dot{\varepsilon}$

LSTC



T=1000

$\dot{\varepsilon}$ \ ε	0.0	0.1	0.4	1.0
0.1	98	190	244	268

T=500

$\dot{\varepsilon}$ \ ε	0.0	0.1	0.4	1.0
0.1	130	253	325	357

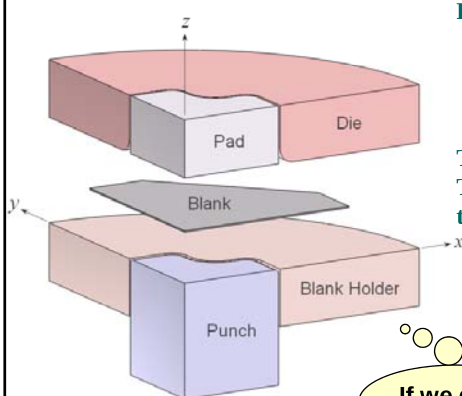
T=20

$\dot{\varepsilon}$ \ ε	0.0	0.1	0.4	1.0
0.1	σ = 162	σ = 316	σ = 406	σ = 446
1.0	182	354	455	500
10.	192	373	479	527

Appendix E 11

Process loads and times

LSTC



Process specification

- **Blank holding force: 1.8 – 3.96 kN**
- **Pad force: 0.137 – 2.603 kN**

The numerical model uses quarter symmetry. Therefore, the forces must be reduced by ¼ in the model.

- **Blank holding force: 0.45 – 0.99 kN**
- **Pad force: 0.034 – 0.65 kN**

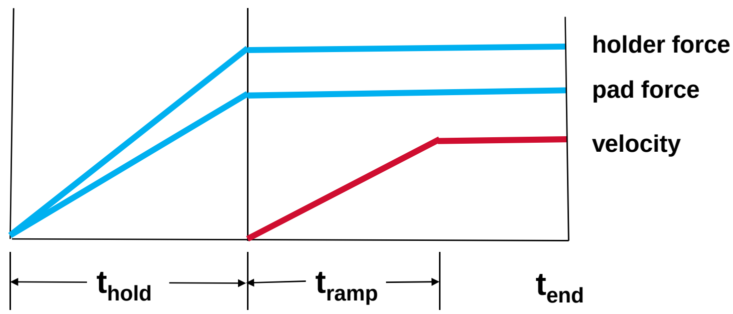
If we don't reduce the forces, the blank is constrained from drawing into the die cavity. The blank tears and wrinkles.

Appendix E 12

Establishing initial conditions

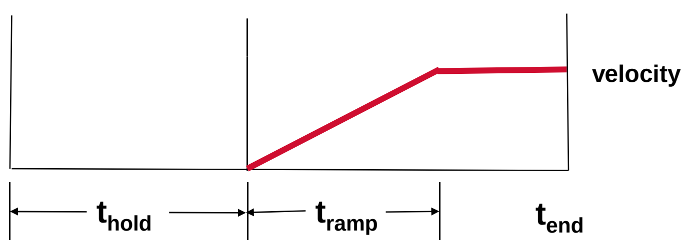
LSTC

A good modeling method is to ramp up initial forces and velocities. Applying them as a step function in a single time step can lead to numerical instability.



Calculating the termination time, t_{end}

LSTC



$$disp = \frac{1}{2} v t_{ramp} + v(t_{end} - t_{ramp} - t_{hold})$$

$$t_{end} = \frac{disp}{v} + t_{hold} + \frac{1}{2} t_{ramp}$$

Velocity & heat transfer time scaling to decrease run time

LSTC

1. **Punch velocities are artificially increased but not to exceed 2 to 5 m/s. The punch velocity should be defined by a smooth curve and not a step function.**

   punch velocity → 0.15 mm/sec

   scaled velocity → 40,000 * 0.15 = 6000 mm/sec

2. **Strain rates must be multiplied by 40,000 in the input.**

3. **Define a mass scaled time step where cycles/mm is in the range of 100 to 1000.**

$$dt2ms = \frac{1.}{(tool\ velocity)(cycles/mm)} = \frac{1.}{(6000)(333)} \approx 5.*10^{-7}\ \text{sec}$$

4. **In the initial configuration, the punch should be in contact with the blank to avoid dynamic impact effects.**
5. **Set TSF=40000 on the CONTROL_THERMAL_SOLVER keyword for heat transfer scaling.**

Mass scaled time step to decrease run time

LSTC

Anytime you add nonphysical mass to increase the time step in a dynamic analysis, you affect the results (think of F = ma). Sometimes the effect is insignificant and in those cases adding nonphysical mass is justifiable where the velocity is low and the kinetic energy is very small relative to the internal energy. In the end, it's up to the judgment of the analyst to gage the affect of mass scaling. You may have to reduce or eliminate mass scaling in a second run to gage the sensitivity of the results to the amount of mass added.

Use the keyword *DATABASE_GLSTAT to create the text file glstat which contains system kinetic and internal energy

Warm forming workshop problem
Run warm_forming.k

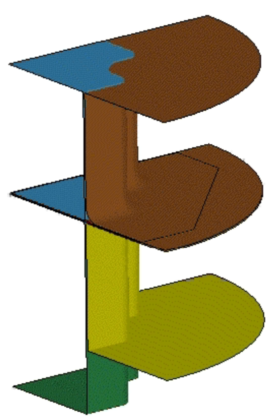
LSTC

Start the problem running, get a cup of coffee, and take a break. It will take about 15 minutes to run.

Look at the top of the input file. Notice the use of the keywords
***PARAMETER**
***PARAMETER_EXPRESION**
and how the parameters are used, such as endtime in the *CONTROL_TERMINATION keyword

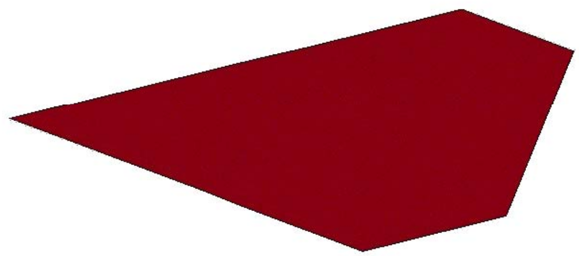
Warm forming workshop problem
¼ symmetry tool motion

LSTC



Warm forming workshop problem
¼ symmetry – blank only

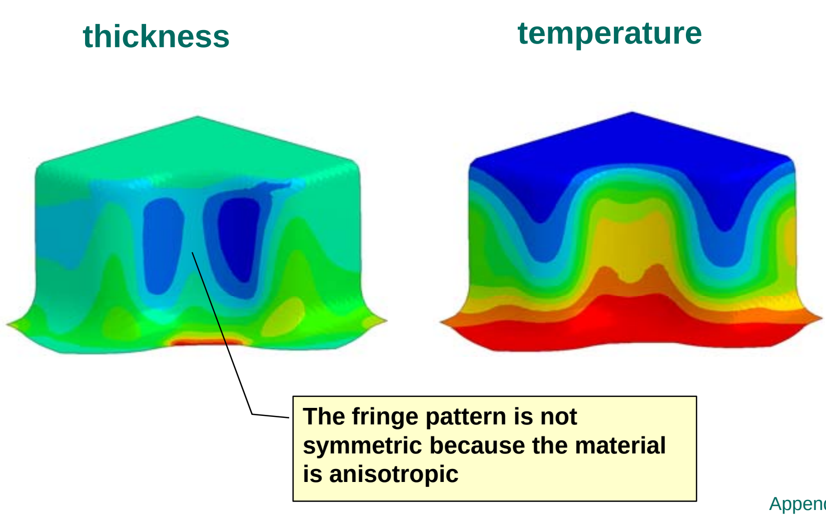
LSTC



Appendix E 19

Warm forming workshop problem

LSTC



Appendix E 20

Justification for using a mass scaled time step dt2ms on slide #15

LSTC

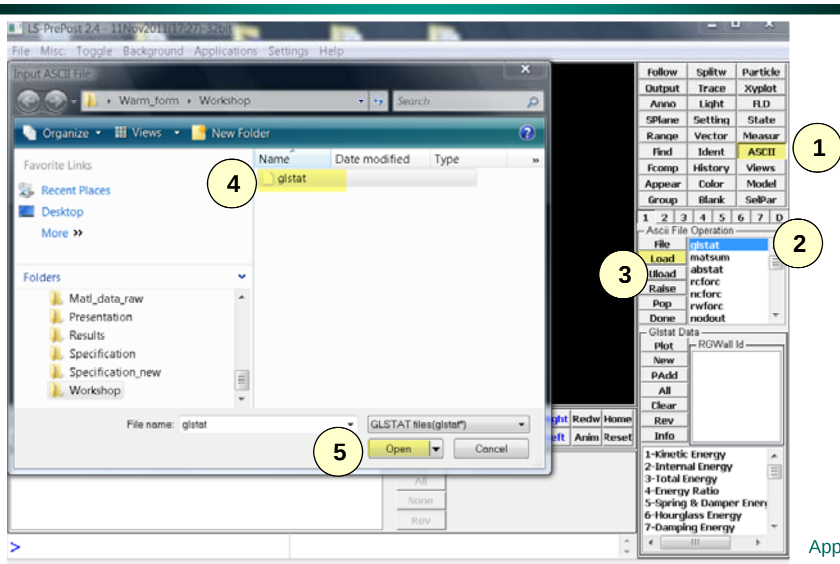
Anytime you add nonphysical mass to increase the time step in a dynamic analysis, you affect the results (think of F = ma). Sometimes the effect is insignificant and in those cases adding nonphysical mass is justifiable where the velocity is low and the kinetic energy is very small relative to the internal energy. In the end, it's up to the judgment of the analyst to gage the affect of mass scaling. You may have to reduce or eliminate mass scaling in a second run to gage the sensitivity of the results to the amount of mass added.

Use the keyword `*DATABASE_GLSTAT` to create the text file glstat which contains system kinetic and internal energy

Appendix E 21

Using LS-PrePost to display KE & IE Load GLSTAT file

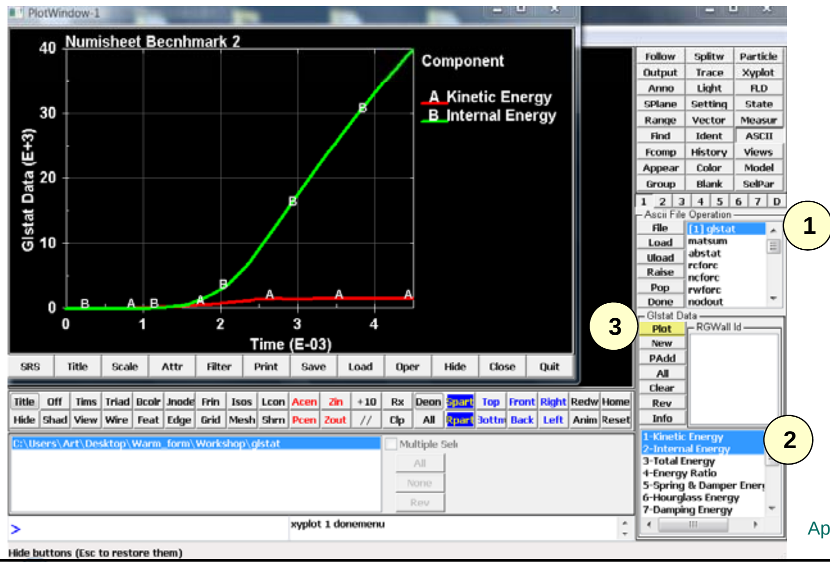
LSTC



Appendix E 22

Using LS-PrePost to display KE & IE
Mass scaling justified because KE << IE

LSTC



Appendix E 23

The GLSTAT file contains energy data useful to check the validity of an analysis

LSTC

The following equation should hold at all times during an analysis

$$\underbrace{E_{ke} + E_{ie} + E_{si} + E_{rw} + E_{d} + E_{hg}}_{E_{tot} = \text{total energy}} = E_{ke,0} + E_{ie,0} + W_{ext}$$

E_{ke} = kinetic
E_{ie} = internal
E_{si} = sliding interface
E_{rw} = rigid wall
E_{d} = damping

E_{hg} = hourglass
$E_{ke,0}$ = initial kinetic
$E_{ie,0}$ = initial internal
W_{ext} = external work

Appendix E 24

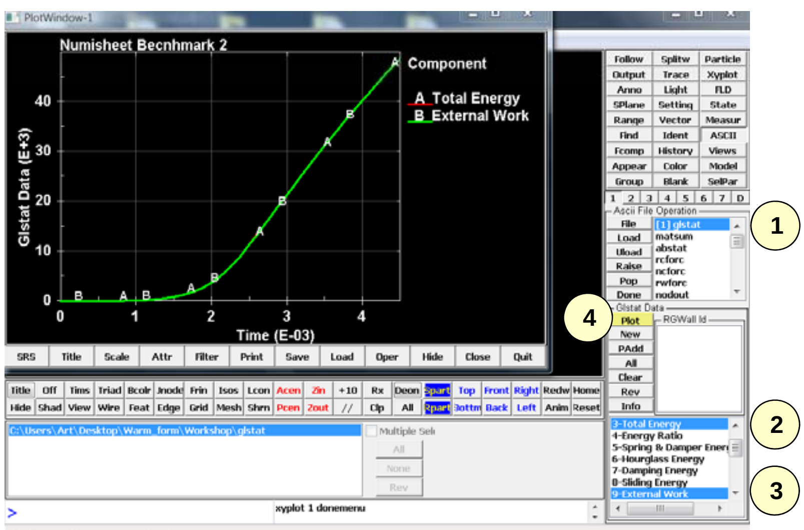
The GLSTAT file contains energy data useful to check the validity of an analysis

LSTC

The terms in the equation can all be plotted using LS-POST and ASCII database GLSTAT. If the equation does not hold the user should suspect an error. If the left hand side of the equation rises above the right hand side, energy is being introduced artificially - for example, by numerical instability, or the sudden detection of artificial penetration through a contact surface. The latter condition is often shown by sudden jumps in the total energy. If the left hand side falls below the right hand side, energy is being absorbed artificially, perhaps by excessive hourglassing or by stonewalls or over-compliant contact surfaces.

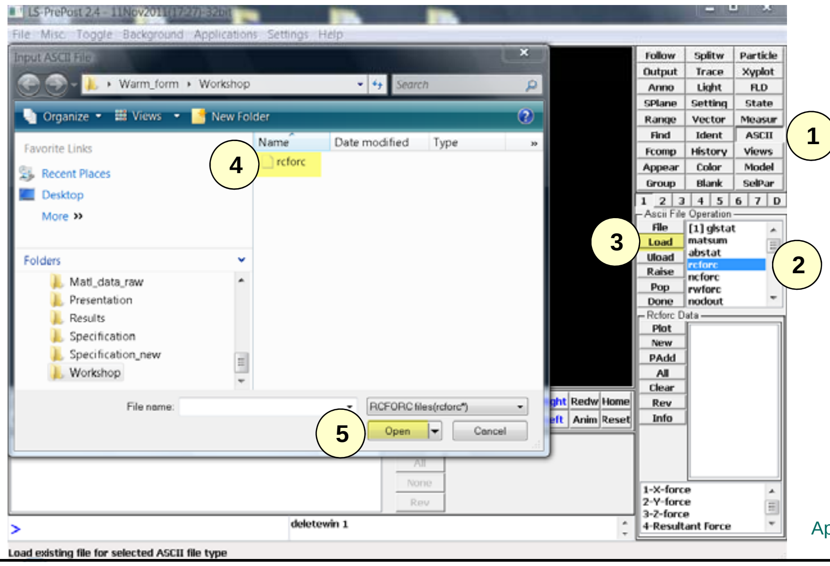
The solution is valid if for all times $E_{tot} = W_{ext}$

LSTC



The RCFORC file contains contact surface forces

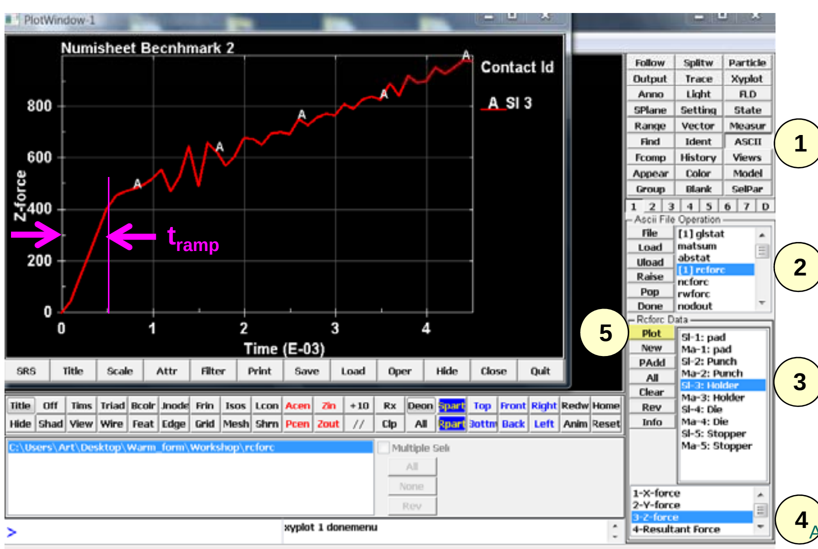
LSTC



Appendix E 27

Display the blank holder force. It should linearly increase from 450 – 990N (see slide 12)

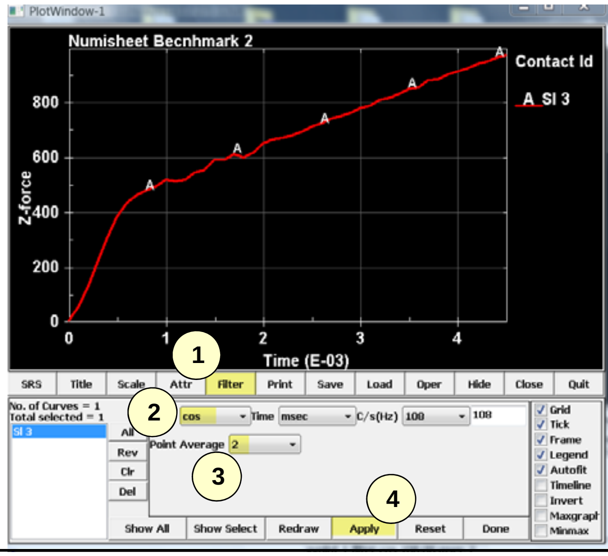
LSTC



Appendix E 28

The curve can be smoothed using filter

LSTC



Appendix E 29

LSTC

Appendix E 30

LSTC

Appendix E 31

LSTC

Appendix E 32

Appendix F – workshop problems

LSTC

WARNING

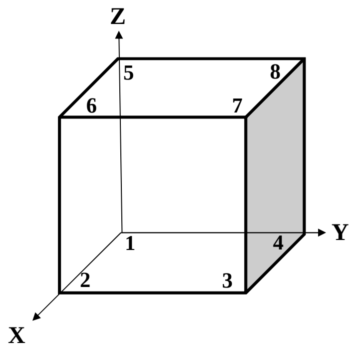
suggestions

LSTC

1. **Make a copy of the input file** so you can always go back to the original. You will be making changes to the input file to see the affect on problem solution. Many of the changes have already been made and all you have to do is comment and un-comment lines in the input file.
2. **Delete the d3plot file family** when solving a new problem. Otherwise, there may be a d3plot remnant fro a previous run and the results will be corrupted.

cp01.k – coupled thermal-stress

LSTC



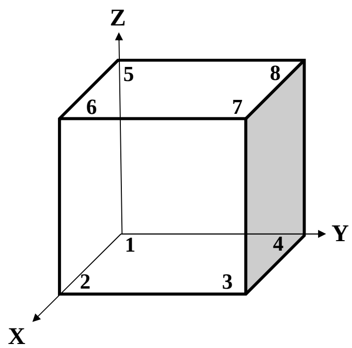
Calculate the expansion due to heating

1. analytical solution
2. numerical solution
3. why are there wiggles in a node displacement vs time plot
4. adjust input to attenuate wiggles
5. solve with implicit mechanics

cp01.k – coupled thermal-stress

Problem definition

LSTC



Aluminum 1100

Density	**2700 kg/m^3**
modulus of elasticity	**70.e+09 Pa**
Poisson Ratio	**0.3**
coeff. of expansion	**23.6e-06 m/m K**
heat capacity	**900 J/kg K**
thermal conductivity	**220 W/m K**
heat generation	**2.43e+07 W/m^3**

cp01.k – coupled thermal-stress

Analytical heat transfer solution

LSTC

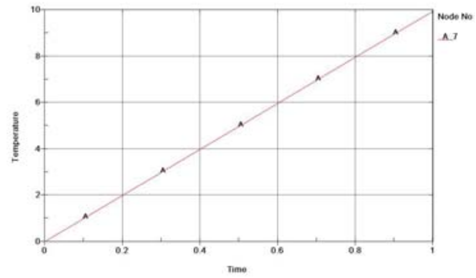
Analytical solution

Internal energy change = heat addition

$$\rho c \frac{\Delta T}{\Delta t} = \rho c \frac{(T_t - 0)}{(t-0)} = \dot{q}'''$$

$$T = \frac{\dot{q}''' t}{\rho c} = \frac{(2.43e+07)(t)}{(2700)(900)} = 10t$$

LS-DYNA results



cp01.k – coupled thermal-stress

Analytical mechanical solution

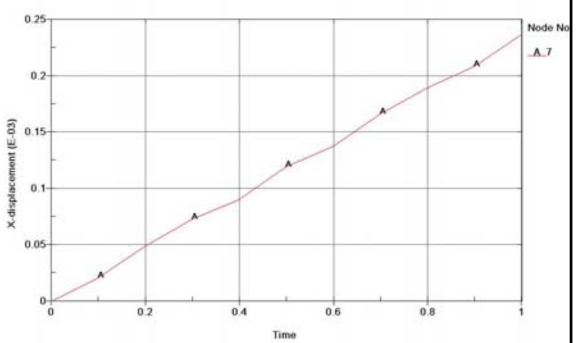
LSTC

Analytical solution

Linear expansion

$$\begin{aligned}\Delta x &= \alpha \Delta T \\ &= (23.6e-06)(10t) \\ &= 23.6e-05\end{aligned}$$

LS-DYNA results



cp01.k – coupled thermal-stress

Workshop exercise

LSTC

Workshop

1. Use LS-Dyna to solve problem cp01.k. Look at the end of the information printed to the screen. How many mechanical time steps (problem cycles) were taken? (ans. 6825)
2. How many thermal time steps were taken? (ans. 10)
3. Open the file tprint. Look at the output for time=0.

   What is the initial temperature? (ans. 0.)

   If not specified, the default is T=0. You may want to set the initial tempertaure to room temperature using the keyword INITIAL_TEMPERATURE.

cp01.k – coupled thermal-stress

Workshop exercise

LSTC

4. Using LS-PrePost
   a) Select node 7 and plot the temperature time history.
   b) Select node 7 and plot the X-displacement time history. Note the small oscillation in the curve.

5. Edit the input file cp01.k with a text editor or using LS-PrePost. Change the plot interval from 0.1 to 0.01 on the `*DATABASE_BINARY_D3PLOT` keyword. Re-run LS-Dyna, and look at the X-displacement versus time history for node 7. Note the oscillation. The oscillation is numerical and not physical.

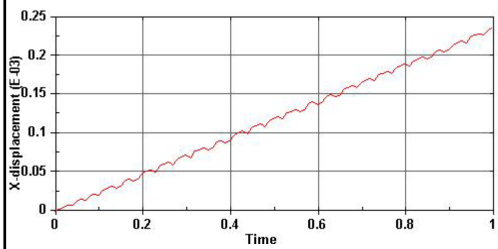
Text editor in LS-DYNA manger

1) Top menu item → Misc → edit last run input with wordpad
2) Click “SAVE FILE” icon in wordpad top menu bar

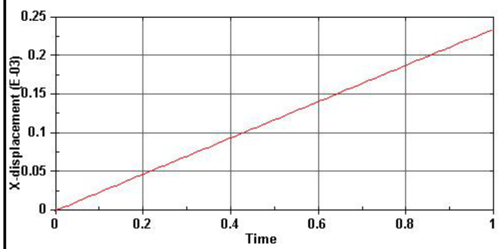
cp01.k – coupled thermal-stress

Workshop exercise

LSTC



X-displacement versus time history for node 7. Note the oscillation. The oscillation is numerical and not physical.



Oscillations can be attenuated by reducing the time step or using *CONTROL_IMPLICIT_GENERAL

cp01.k – coupled thermal-stress

Workshop exercise – using a smaller time step

LSTC

6. Change the default mechanical time step size by setting the scale factor TSSFAC=0.7

```
*CONTROL_TIMESTEP
    DTINIT      TSSFAC
         0          .7
```

Scale the time step by 0.7 (default is 0.9)

Calculate the default mechanical explicit time step size

Re-run using LS-Dyna and look at the X-displacement versus time history for node 7. There should be no oscillations. How many mechanical computational cycles were required? (ans. 8440)

How many thermal cycles? (ans. 10)

cp01.k – coupled thermal-stress

Workshop exercise – using *CONTROL_IMPLICIT_GENERAL

LSTC

7. Delete the keyword *CONTROL_TIMESTEP

   Add the keyword

```
*CONTROL_IMPLICIT_GENERAL
         1         .1
```

Implicit time step size

This keyword specifies an implicit mechanical analysis

Re-run using LS-Dyna and look at the X-displacement versus time history for node 7. There should be no oscillations. Implicit mechanical analysis is quasi-static and the high frequency numerical noise is not detected. How many mechanical computational cycles were required? (ans. 42)

How many thermal cycles? (ans. 10)

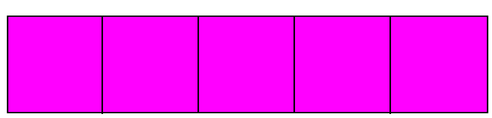
crank_oscillations.k - Crank-Nicolson oscillations

Problem description

LSTC

The region 0<x<*l* with zero initial temperature, with the surface x=0 insulated and the surface x = *l* kept at constant temperature T_l for t > 0.

insulated



T_l = 1

X=0 **X=*l***

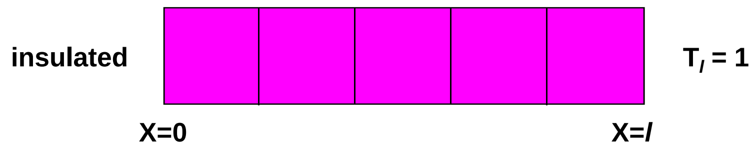
Carslaw & Jaeger, Conduction of Heat in Solids, Oxford Press, 2nd ed., eq. 4, p 100.

G.E. Myers, “The critical Time Step for Finite Element Solutions to Two-dimesnional Heat Conduction Transients”, Journal of Heat Transfer, pp120-127, V100, 1978.

crank_oscillations.k - Crank-Nicolson oscillations

Analytical solution

LSTC



$$T = T_l - \frac{4T_l}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\exp[-(2n+1)^2\pi^2\alpha t/4l^2]\cos\left[\frac{(2n+1)\pi x}{2l}\right]$$

The solution is described by decreasing exponentials with various rates of decrease (i.e., function of location x). The time step must be appropriate to capture both the slow and fast decay rates.

crank_oscillations.k - Crank-Nicolson oscillations

Numerical solution

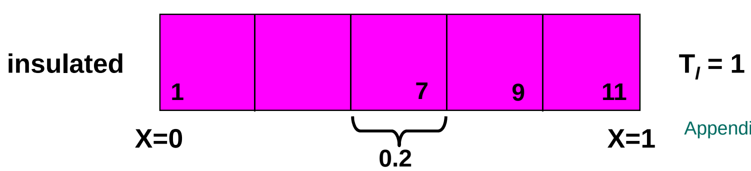
LSTC

Workshop

1. **Run crank_oscillations.k using a time step of Δt=0.1**
2. **Using LS-Post, plot the temperature time history for nodes 1, 7, 9, and 11. Note the temperature oscillations for node 9**
3. **Re-run using a time step calculated by** $\Delta t = (\Delta x)^2/\alpha$
4. **Using LS-Post, plot the temperature time history for nodes 1, 7, 9, and 11. Look for oscillations.**
5. **Re-run as implicit (TIP=1) with Δt=0.1. Look for oscillations.**

Although 2nd order accurate, the solution is oscillatory

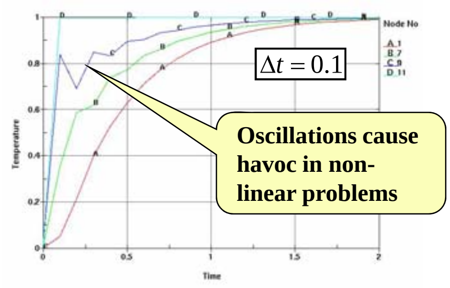
Δx = 0.2
α = 1.

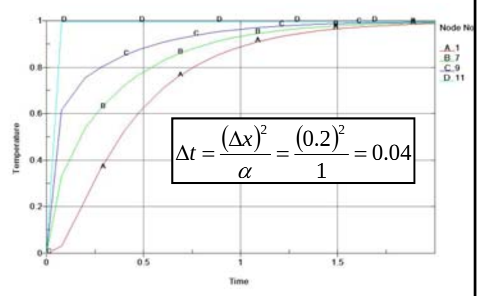


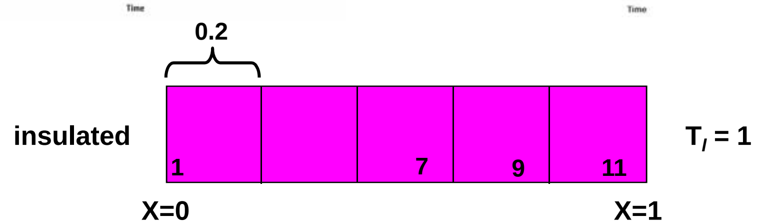
crank_oscillations.k - Crank-Nicolson oscillations

Numerical results

LSTC







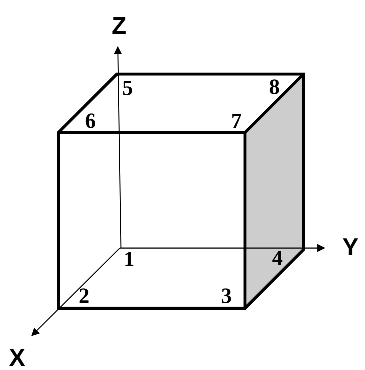
bcflux.k – investigate thermal time step parameters

Cube with flux boundary condition

LSTC

Problem description

A cube initially at 0C, is heated by a heat flux on all 6 faces.
What is the rate of temperature increase?



cube	edge = 1.0 m
density	ρ = 1 kg/m^3
heat capacity	c = 1 J/kg C
conductivity	k = 1 W/m K
flux	q = 1 W/m^2
initial temperature	T_0 = 0 C

bcflux.k – investigate thermal time step parameters

Analytical solution

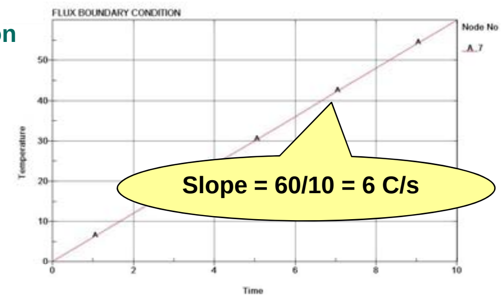
LSTC

Analytical solution

internal energy change = heat addition

$$\rho c V \frac{\Delta T}{\Delta t} = qA$$

$$\frac{\Delta T}{\Delta t} = \frac{qA}{\rho c V} = \frac{(1)(6*1*1)}{(1)(1)(1*1*1)} = 6$$



bcflux.k – investigate thermal time step parameters

Workshop exercise

LSTC

Workshop

1. **Run with a fixed time step**

```
*CONTROL_THERMAL_TIMESTEP
$         TS        TIP        ITS
           0        1.0        1.0
```

0 = fixed time step
1 = variable step

Use this keyword to dump ΔT/Δt to d3plot

***DATABASE_EXTENT_BINARY**

0
0
0
1

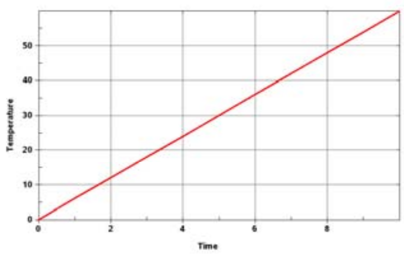
4th line, 1st parameter, DTDT

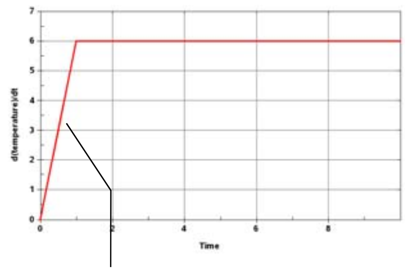
bcflux.k – investigate thermal time step parameters

Workshop exercise

LSTC

Using LS-PrePost, plot the temperature and ΔT/Δt time history of any node in the cube. Compare with the analytical answer.





Due to the discrete time step and the d3plot dump interval, we see a jump in the gradient. ΔT/Δt=0 at t=0 and ΔT/Δt=6 at t>0.

bcflux.k – investigate thermal time step parameters

Workshop exercise

LSTC

2. Edit the input file to run with a variable time step

```
*CONTROL_THERMAL_TIMESTEP
$     TS     TIP     ITS    TMIN    TMAX   DTEMP    TSCP
       1     1.0     1.0      1.      2.      1.     0.5
```

The problem will not run to completion and print the following error message.

```
thermal step        1 time    1.0000E+00 dt    1.0000E+00
*** Error - the minimum time step has been reached and
 the maximum temperature change is greater than specified
     solution time                      1.0000E+00
     time step size                     1.0000E+00
     node                                 1
     temperature change                 6.0000E+00
     max allowed temperature change     1.0000E+00
 suggestion - increase max allowed temperature change
              decrease minimum time step
```

bcflux.k – investigate thermal time step parameters

Workshop exercise

LSTC

3. Pick TMIN and/or DTEMP parameters so the problem will run.

```
*CONTROL_THERMAL_TIMESTEP
$     TS     TIP     ITS    TMIN    TMAX   DTEMP     TSCP
       1     1.0     1.0       ?      2.       ?      0.5
```

We know from the analytical solution that the heating rate is 6°/sec

If we keep TMIN=1 sec, then increase the maximum allowed temperature change → set DTEMP > 6°

Or

If we keep DTEMP=1, then decrease the minimum time step →

Set TMIN = $\frac{1^0}{6^0/\text{sec}}$ < 0.166

bcflux.k – investigate thermal time step parameters

Workshop exercise

LSTC

4. Use default values for the variable time step option.

```
*CONTROL_THERMAL_TIMESTEP
$     TS     TIP     ITS    TMIN    TMAX   DTEMP     TSCP
       1     1.0      0.      0.      0.      0.       0.
```

Open the d3hsp file with a text editor. Near the bottom of the file is a section titled "thermal variable time step data"

What is the minimum time step?

What is the maximum time step?

$$\Delta t_{resp} = l^2/\alpha = 1^2/1 = 1$$

$$TMIN = \Delta t_{resp}/100 = 0.01$$

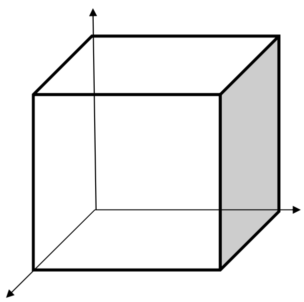
$$TMAX = 100\Delta t_{resp} = 100$$

phase1.k & phase2.k – nonlinear thermal material

LSTC

Problem description

A cube of water initially at 20C, is cooled by convection on all 6 faces. What is the length of time required to turn the water into ice?



Ice cube	edge = 0.1 m
density	ρ = 1000 kg/m^3
heat capacity	c = 2000 J/kg C
conductivity	k = 1 W/m K
latent heat	H_λ = 300,000 J/kg
phase temp.	T_λ = 0 C
Convection coef.	h = 100 W/m^2
environment T	T = -20 C
initial temperature	T_0 = 20 C

phase1.k & phase2.k – nonlinear thermal material

Analytical solution

LSTC

1. Calculate time to remove sensible heat from 20C to 0C

$$\frac{T_\lambda - T_\infty}{T_0 - T_\infty} = \exp\left(-\frac{hAt}{\rho c V}\right)$$

See page 6-29

$$\frac{0-(-20)}{20-(-20)} = \exp\left(-\frac{(100)(0.06)t}{(1000)(2000)(0.001)}\right)$$

$$t = 231\,\text{sec}$$

2. Calculate time to remove latent heat

$$hA(T_\lambda - T_\infty)t = H_\lambda \rho V$$

$$(100)(0.06)[0-(-20)]t = (300000)(1000)(0.001)$$

$$t = 2500\,\text{sec}$$

Phase change begins at 231 sec.

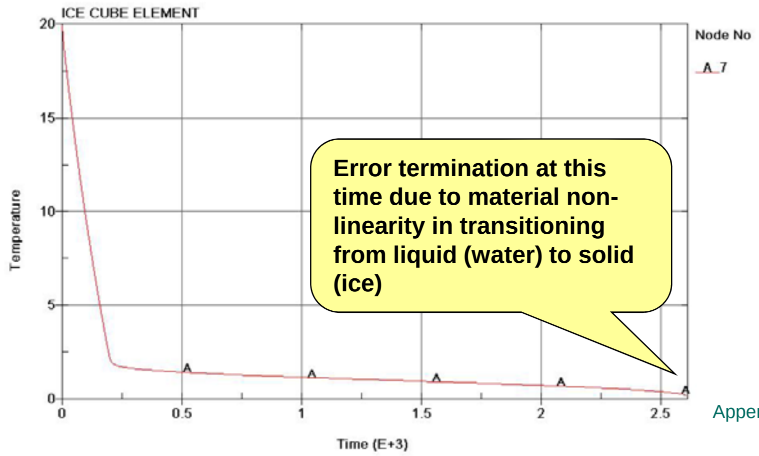
Phase change ends at 2500 + 231 = 2731 sec

phase1.k & phase2.k – nonlinear thermal material

phase1.k

LSTC

1. Run phase1.k. The problem will error terminate at about t=2620 sec due to the strong material nonlinearity as we exit phase change.



phase1.k & phase2.k – nonlinear thermal material

phase1.k → Error termination message printed on terminal

LSTC

```
130 t 2.6000E+03 dt 2.00E+01 thermal step
131 t 2.6200E+03 dt 2.00E+01 thermal step

*** Error - the solution is diverging with a fixed time step
specified
       solution time                2.6200E+03
       time step size               2.0000E+01
       convergence norm last step   9.3148E-01
       convergence norm this step   4.0693E+00
       convergence tolerance        1.0000E-04
       residual last step           1.6571E-01
       residual this step           1.4281E-01
suggestion -    increase convergence tolerance
                use line search
                decrease time step

 E r r o r   t e r m i n a t i o n
```

$$\frac{\|T_{new}\| - \|T_{old}\|}{\|T_{new}\|}$$

$$\left[K_T\left(T^i\right)\right]\left\{\Delta T^i\right\} = \left\{G\left(T^i\right)\right\}$$
$$T^{i+1} = T^i + \Delta T^i$$
$$\Delta T^i \to 0$$

phase1.k & phase2.k – nonlinear thermal material

phase1.k → Error termination message printed on terminal

LSTC

```
130 t 2.6000E+03 dt 2.00E+01 thermal step
131 t 2.6200E+03 dt 2.00E+01 thermal step

*** Error - the solution is diverging with a
fixed time step specified
       solution time                2.6200E+03
       time step size               2.0000E+01
       convergence norm last step   9.3148E-01
       convergence norm this step   4.0693E+00
       convergence tolerance        1.0000E-04
       residual last step           1.6571E-01
       residual this step           1.4281E-01
suggestion -   increase convergence tolerance
               use line search
               decrease time step

 E r r o r   t e r m i n a t i o n
```

Increase convergence tolerance is not a good option because the norm is >> 0. The convergence tolerance must be <0.

Appendix F - 27

phase1.k & phase2.k – nonlinear thermal material

phase1.k

LSTC

There are 2 things you can try to overcome the error termination.

Method 1: use nonlinear line search by setting THLSTL=0.9 on the CONTROL_THERMAL_NONLINEAR keyword.

Method 2: use a variable time step and setting DCP=0.1 on the CONTROL_THERMAL_NONLINEAR keyword.

Appendix F - 28

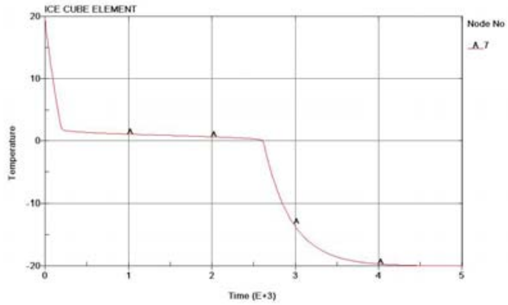
phase1.k & phase2.k – nonlinear thermal material

phase1.k, method 1: use nonlinear line search

LSTC

2. Set the line search tolerance to THLSTL=0.9 and re-run phase1.k

```
*CONTROL_THERMAL_NONLINEAR
$ REFMAX      TOL      DCP  LUMPBC    THLSTL
      50 1.0e-04      0.0      0.0        .9
```



Appendix F - 29

phase1.k & phase2.k – nonlinear thermal material

phase1.k, method 2: use a variable time step with DCP control

LSTC

3. Use a variable time step

- reset thlstl=0. on CONTROL_THERMAL_NONLINEAR
- set dcp=0.1 on CONTROL_THERMAL_NONLINEAR
- set variable time step parameters on CONTROL_THERMAL_TIMESTEP
- run problem

```
*CONTROL_THERMAL_TIMESTEP
$      TS      TIP      ITS      TMIN      TMAX     DTEMP      TSCP
        1      1.0     20.0        1.      20.0      5.0       0.5

*CONTROL_THERMAL_NONLINEAR
$ REFMAX      TOL      DCP  LUMPBC  THLSTL
      50  1.0e-04      .1              0.
```

Appendix F - 30

phase1.k & phase2.k – nonlinear thermal material

phase1.k, method 2: use a variable time step with DCP control

LSTC

D3HSP output

```
130 t 2.6000E+03 dt 2.00E+01 thermal step
131 t 2.6200E+03 dt 2.00E+01 thermal step
*** Warning - the solution is diverging,
time step is being reduced
    solution time                  2.6000E+03
    new time step size             2.0000E+00
131 t 2.6020E+03 dt 2.00E+00 thermal step
132 t 2.6060E+03 dt 4.00E+00 thermal step
133 t 2.6140E+03 dt 8.00E+00 thermal step
134 t 2.6300E+03 dt 1.60E+01 thermal step
135 t 2.6500E+03 dt 2.00E+01 thermal step
```

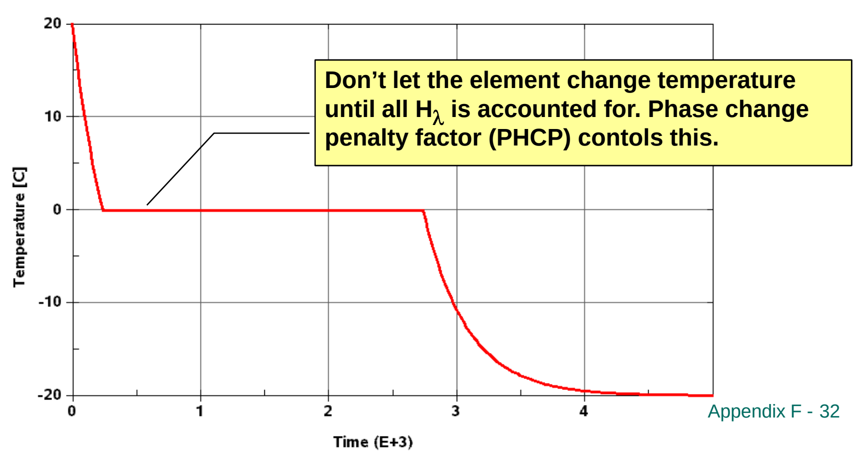
Time step is being reduced by DCP = 0.1

Note that the step counter is not increasing – the time step will be retaken until convergence is achieved

phase1.k & phase2.k – nonlinear thermal material

LSTC

4. Run phase2.k. Note that THLSTL=0.9 and PHCHPN=100 on the CONTROL_THERMAL_NONLINEAR keyword.

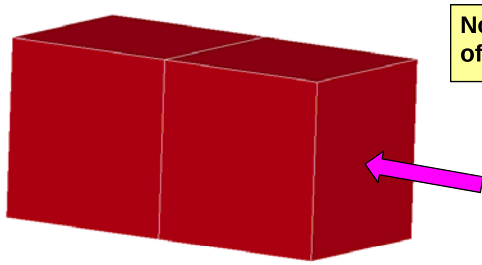


radiation.k – nonlinear boundary condition

LSTC

Calculate the steady state temperature distribution in the block.

T = 300K



Nonlinear because of 4th power

$$\dot{q}'' = \sigma\varepsilon\left(T_s^4 - T_\infty^4\right)$$

T_∞ = 600K
σ = 5.67e-08 W/m²K
ε = 1.

radiation.k – nonlinear boundary condition

LSTC

1) Run problem as is. It will error terminate with, "failed to converge in10 iterations".

2) Increase the number of allowed iterations by setting **REFMAX=100** on the CONTROL_THERMAL_NONLINEAR keyword.

```
*CONTROL_THERMAL_NONLINEAR
  REFMAX      TOL      DCP      LUMPBC      THLSTL
     100
```

Look at the bottom of the TPRINT file. How many iterations are required to achieve convergence? (answer 16)

radiation.k – nonlinear boundary condition

LSTC

3) Another way to help the calculation is to start off at a reasonable initial temperature. Since we are in degrees Kelvin, starting things off at T=0 is not good. Let's set T=298, room temperature. Add the keyword noting that NSID=0 means apply TIC to all nodes in the model.

```
*INITIAL_TEMPERATURE_SET
   NSID     TIC
      0    298.
```

Look at the bottom of the TPRINT file. Now only 8 iterations are needed.

4) Another useful switch when trying to solve a highly nonlinear problem is to turn on line search. Add THLSTL=0.9 to the CONTROL_THERMAL_NONLINEAR keyword.

```
*CONTROL_THERMAL_NONLINEAR
 REFMAX     TOL    DCP   LUMPBC   THLSTL
    100                             0.9
```

Look at the bottom of the TPRINT file. Now only 4 iterations are needed.

rod_gr_pr_nu.k - DEFINE_FUNCTION keyword

LSTC

How do you enter the convection coefficient

$$h = \left[0.14\left(Gr * \Pr\right)^{0.33}\right]\frac{k}{l}$$

1 Hand calculate h and enter it as a load curve using the BOUNDARY CONVECTION keyword

2 Use the DEFINE FUNCTION keyword and enter the equation

h can be a function of h(x, y, z, vx, vy, vz, temp, tinf, time)

Variable dropouts are permitted, eg., h(temp, tinf, time)

Variable names can not be changed (i.e., you must use vx and not change the name to something like vel_x.

rod_gr_pr_nu.k - DEFINE_FUNCTION keyword

LSTC

This problem has 5 rods. The right hand side has a temperature boundary condition. The left hand side at x=0 has a convection boundary condition. The convection boundary condition on the 5 rods are defined by:

1) load curve
2) single function entered inline FORTRAN syntax
3) nested functions
4) c-program
5) single function with a load curve argument

Run the problem. Use LS-PrePost to verify that the temperatures on all 5 rods are the same. Then, open rod_gr_pr_nu.k and look at how the 5 ways of defining h are entered.

Use this input file as a template for ***DEFINE_FUNCTION**.

rod_gr_pr_nu.k - DEFINE_FUNCTION keyword

1. h defined by a load curve

LSTC

```
*BOUNDARY_CONVECTION_SEGMENT
$        n1         n2         n3         n4
          1          2          4          3
$     LCIDH         HM      LCIDT         TM        LOC
         10   3.486784          0        10.          0
*DEFINE_CURVE
         10
                    0.                   1.
                 1000.                   1.
```

rod_gr_pr_nu.k - DEFINE_FUNCTION keyword

2. h defined by in-line FORTRAN syntax

LSTC

h defined using a single function. Note that the multiplication symbol '*' must not be in column 1. A '*' in column one is the flag for a keyword. The function name (e.g., h1 in this example) can be anything. This name is only used within the input file. Such as making nested functions(see example 3). Use *PARAMETER to define constants, such as grav, beta, rho, etc (llok at the input file to see how this is done).

```
$
$-----------------------------
*BOUNDARY_CONVECTION_SEGMENT
        45        46        48        47
$     LCIDH        HM     LCIDT        TM        LOC
         1        0.         0       10.          0
*DEFINE_FUNCTION
         1
h1(temp,tinf,y)= k/y *
 (0.55 * ( (grav*beta*rho**2*abs(temp - tinf)*leng**3 / mu**2) *
pr)**.25 )
```


rod_gr_pr_nu.k - DEFINE_FUNCTION keyword

3. h defined by nested functions

LSTC

h defined using nested functions. Note that FORTRAN syntax is used. The name xnu is used for the Nusselt number. In FORTRAN, xnu defaults to REAL. If, nu were used, then according to FORTRAN syntax this is an integer. INTEGERS are truncated and the calculated result will be wrong.

```
*BOUNDARY_CONVECTION_SEGMENT
        89        90        92        91
$     LCIDH        HM     LCIDT        TM        LOC
         4        0.         0       10.          0
*DEFINE_FUNCTION
         2 function for Grashof number
gr(temp,tinf) = grav*beta*rho**2*abs(temp - tinf)*leng**3
 / mu**2
*DEFINE_FUNCTION
         3 function for Nusselt number
xnu(temp,tinf)=0.55 * (gr(temp,tinf) * pr)**.25
*DEFINE_FUNCTION
         4 heat transfer coefficient
h4(y,temp,tinf)=xnu(temp,tinf)*k/y
```

rod_gr_pr_nu.k - DEFINE_FUNCTION keyword

4. h defined using a C program

LSTC

```
*BOUNDARY_CONVECTION_SEGMENT
$       n1         n2         n3         n4
       133        134        136        135
$    LCIDH         HM      LCIDT         TM         LOC
         5         0.          0        10.           0
*DEFINE_FUNCTION
         5 c-program
float h5(float y,float temp,float tinf)
{
 float gr, nu, h ;
 gr=grav*beta*rho**2*abs(temp - tinf)*leng**3 / mu**2 ;
 nu=0.55 * (gr * pr)**.25 ;
 h = nu*k/y ;
 printf("h= %e, temp=%e, tinf=%e\n",h,temp,tinf);
 return (h) ;
}
```

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rod_gr_pr_nu.k - DEFINE_FUNCTION keyword

5. h defined using a function with a load curve argument

LSTC

```
*BOUNDARY_CONVECTION_SEGMENT
      1213      1214      1216      1215
$    LCIDH        HM     LCIDT        TM        LOC
         7        0.         8        0.          0
*DEFINE_FUNCTION_TABULATED
         6 load curve function
tinfy
0.,20.
1.,30.
*DEFINE_FUNCTION
         7
h7(time,temp)= k/y *
 (0.14 * ( (grav*beta*rho**2*abs(temp - tinfy(time))*leng**3 / mu**2)
 * pr)**.33 )
*DEFINE_FUNCTION
         8
t8(time)=tinfy(time)
```

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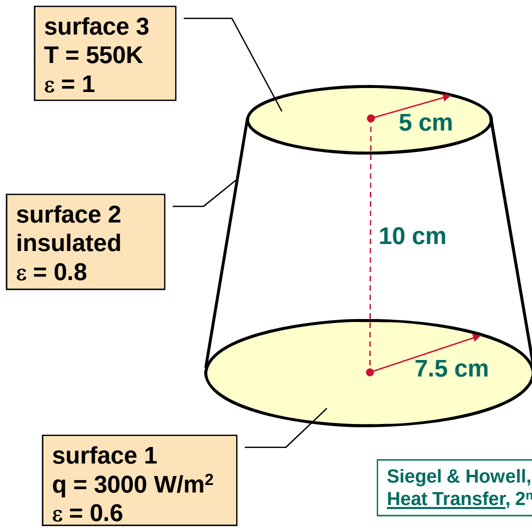
frustum.k - enclosure radiation problem

LSTC



A frustum of a cone has its base heated as shown. The top is held at 550K while the side is perfectly insulated. Surfaces 1 and 2 are gray and diffuse, while surface 3 is black. What is the temperature of surfaces 1 and 2. **(ans. T1=667, T2=720.)**

Siegel & Howell, Thermal Radiation Heat Transfer, 2nd ed., p. 245.

frustum.k - enclosure radiation problem

LSTC

Run frustum.k

Look at the file VIEWFL. The entries are A_iF_{ij}. Note the following:

The diagonal is the view factor of the surface to itself. Surfaces 1 & 3 do not see themselves (A_iF_{ii}=0), whereas surface 2 can see itself (A_iF_{ii}=0.02).

```
0.00000000E+00  1.51196584E-02  2.55180029E-03
1.51196584E-02  2.00566551E-02  5.30218134E-03
2.55180029E-03  5.30218134E-03  0.00000000E+00
```

The matrix is symmetric

frustum.k - enclosure radiation problem

LSTC

Look at the file VIEWFL and calculate the row sum. The row sum equals the surface area. This fact can be used as an accuracy criteria.

```
0.00000000E+00  1.51196584E-02  2.55180029E-03 → 0.01767
1.51196584E-02  2.00566551E-02  5.30218134E-03 → 0.04048
2.55180029E-03  5.30218134E-03  0.00000000E+00 → 0.007854
```

Surface	area
1	$A = p\ r^2 = p\ (0.075)^2 = 0.01766$
2	$A = p(r_1 + r_2)\mathrm{sqrt}[(r_1 - r_2)^2 + h^2] =$ $p(0.075+0.05)+\mathrm{sqrt}[(0.075-0.05)^2+0.1^2)]= 0.04048$
3	$A = p\ r^2 = p\ (0.05)^2 = 0.007854$

frustum.k - enclosure radiation problem

TPRINT file for frustum.k

LSTC

```
STEADY STATE RADIATION
                        ls-dyna ls971d.6614 beta         date 01/24/2006
view factor calculation error diagnostic
 row sum total should be        =  3.0000E+00
 row sum total is               =  3.0000E+00
********************************************************************
 steady state solution
     minimum temperature =     550.00000   at node       12
     maximum temperature =     721.58813   at node        1
  node     temperature       x-flux          y-flux          z-flux
       1    721.58813  0.0000E+00  0.0000E+00  0.0000E+00
       2    721.58812  0.0000E+00  0.0000E+00  0.0000E+00
       3    721.58812  0.0000E+00  0.0000E+00  0.0000E+00
       4    721.58813  0.0000E+00  0.0000E+00  0.0000E+00
       5    667.40170  0.0000E+00  0.0000E+00  0.0000E+00
number of solution iterations  =    22
 QA temperature norm            =   2.2527E+03
```

The view factor row sum is 1. Therefore, the sum of all rows is the number of surfaces. This can be used as an error criteria.

Fuel cask enclosure radiation problem

Reference: L.C. Sanchez, "Performance Testing of Thermal Analysis Codes for Nuclear Fuel Casks", Sandia National Laboratories, rpt. SAND84-1854, January 1987.

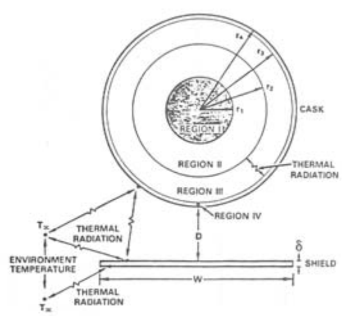
LSTC

Fuel cask enclosure radiation problem

Reference: L.C. Sanchez, "Performance Testing of Thermal Analysis Codes for Nuclear Fuel Casks", Sandia National Laboratories, rpt. SAND84-1854, January 1987.

LSTC

region	description	ρ kg/m³	Cp J/kg K	K W/m K
1	Fuel bundle R1=16.51 cm Q=38.32 kW/m3	2707	895	242
2	Gamma shield R2=38.74 cm	7833	473	45
3	Neutron shield Void region R3=53.98 cm	0	0	0
4	Outer shell R4=54.61 cm	7833	473	45
5	Truck bed W=109.2 cm d=2.54 cm D=30.48 cm	7833	473	45



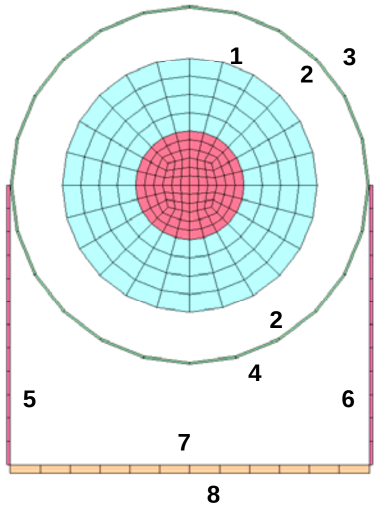
Model Problem 4
Cask with Annular Regions and Radiation Shield

Fire environment
T=54.4 C t=0
T=800. C 0<t<30 min
T=54.4 C 30<t<60 min

Fuel cask enclosure radiation problem

Model geometry

LSTC



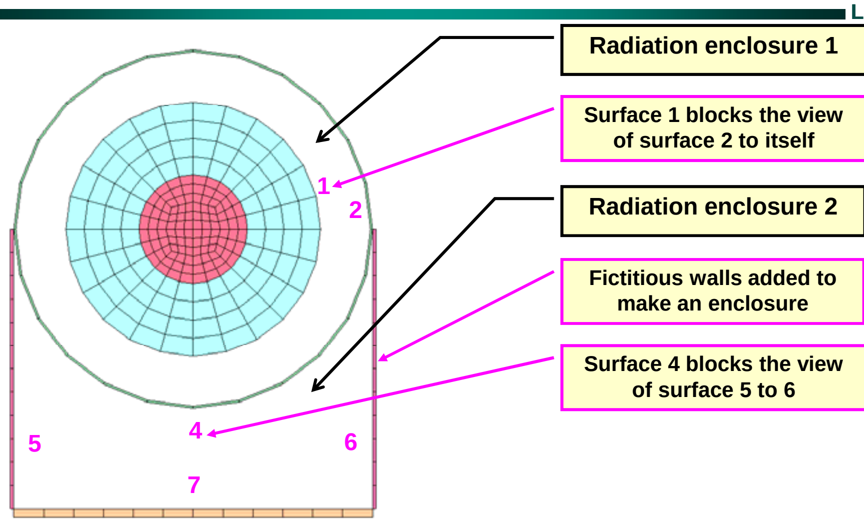
Segment sets

1. Gamma shield outer surface
2. Cask outer shell inner surface
3. Cask outer shell outer surface (top)
4. Cask outer shell outer surface (bottom)
5. Fictitious environment surface
6. Fictitious environment surface
7. Truck bed upper surface
8. Truck bed lower surface

Fuel cask enclosure radiation problem

Radiation enclosure model

LSTC



Fuel cask enclosure radiation problem

Keyword input

LSTC

```
$============ RADIATION ENCLOSURE 1 ==============
$
*BOUNDARY_RADIATION_SET_VF_CALCULATE
$       SID        TYPE    RAD_GRP    FILE_NO      BLOCK
          1           2          1          1          1
          0          1.
*BOUNDARY_RADIATION_SET_VF_CALCULATE
          2           2          1          1
          0          1.
$============ RADIATION ENCLOSURE 2 ==============
*BOUNDARY_RADIATION_SET_VF_CALCULATE
          4           2          2          2          1
          0          1.
*BOUNDARY_RADIATION_SET_VF_CALCULATE
          5           2          2          2
          0          1.
*BOUNDARY_RADIATION_SET_VF_CALCULATE
          6           2          2          2
          0          1.
*BOUNDARY_RADIATION_SET_VF_CALCULATE
          7           2          2          2
          0          1.
```

Define as radiation enclosure group 1

Segment set flagged as blocking

Write view factors to file viewfl_1

Segment set flagged as blocking

Write view factors to file viewfl_2

Define as radiation enclosure group 2

Segment sets 4, 5, 6, 7 belong to enclosure 2

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Fuel cask enclosure radiation problem

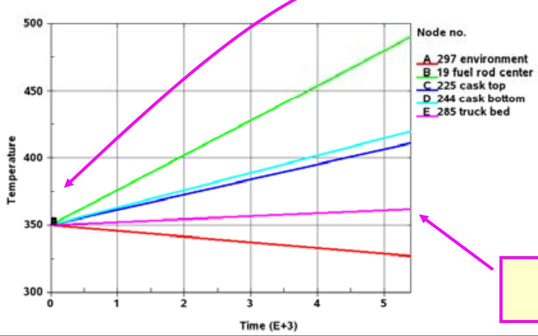
workshop problem cask_ss.k

LSTC

The problem is run in 2 steps

1. cask_ss.k → First, a steady state analysis is done to calculate an initial temperature profile resulting from the self heating of the nuclear fuel. The keyword *INTERFACE_SPRINGBACK_LSDYNA is used to create the file new_temp_ic.inc which is a keyword file containing the node temperatures at the last time state.

```
*INITIAL_TEMPERATURE_SET
   0   350.
*INTERFACE_SPRINGBACK_LSDYNA
$ PSID
     1
*SET_PART_LIST
$ PSID
     1
$  PID  PID  PID  PID  PID
     1    2    3    4    5
```



steady state temperatures

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Fuel cask enclosure radiation problem

workshop problem cask_tr.k

LSTC

2. cask_tr.k → Transient analysis. The *INCLUDE keyword is used to include the file new_temp_ic.k in the input file cask_tr.k.

```
*INCLUDE
new_temp_ic.inc
```

```
*KEYWORD
*INITIAL_TEMPERATURE_NODE
         1   .489E+03
         2   .489E+03
         -
         -
       342   .327E+03
       343   .327E+03
*END
```

The file new_temp_ic.inc is created using the keyword *INTERFACE_SPRINGBACK

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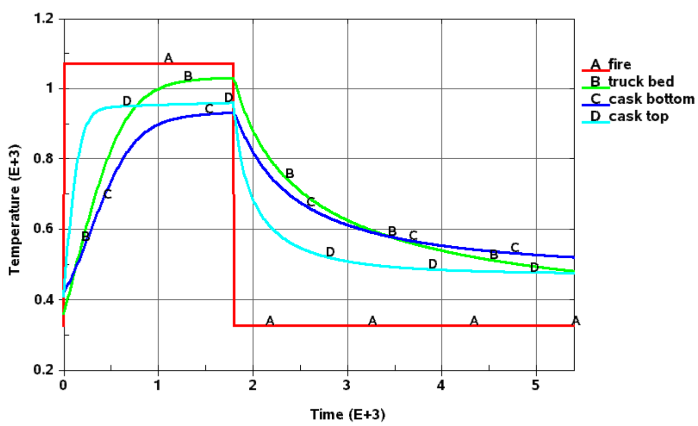
Fuel cask enclosure radiation problem

workshop problem cask_tr.k

LSTC

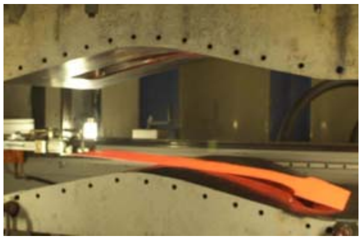


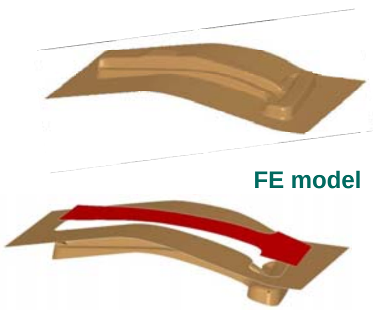
Appendix F - 54

Defining surfaces in contact

There are 2 choices for contact between parts defined by shells

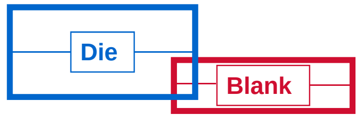
LSTC



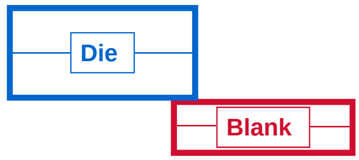


FE model

1. die mid-plane (CAD defined surface) contacts blank surface.



2. die surface contact blank surface



Defining surfaces in contact

Contact keyword parameters

LSTC

***CONTACT_(option)_FORMING_(option)**

do not account for master surface (die) shell thickness in calculating contact.

***CONTROL_CONTACT attribute SHLTHK (shell thickness)**

0 → shell thickness is not considered

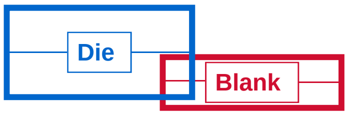
1 → shell thickness is considered but rigid bodies are excluded

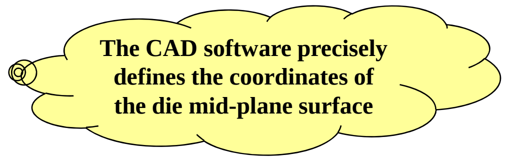
2 → shell thickness is considered including rigid bodies

Defining surfaces in contact

1. die mid-plane contacts blank surface

LSTC





Activated with

*CONTACT_FORMING_SURFACE_TO_SURFACE

Modeling technique

1. The die is a rigid material and does not deform.
2. The thermal analog is that the die does not change temperature.
3. The die is given a rigid body motion and a temperature boundary condition.
4. The mid-plane of the die is the reference surface for mechanical and thermal contact.

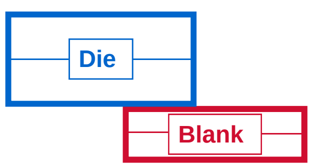
Workshop problem: **bouncing_shell_5.k**

Appendix F - 57

Defining surfaces in contact

2. Die surface contacts blank surface

LSTC



Activated with

*CONTACT_SURFACE_TO_SURFACE with SHLTHK=2

Modeling technique

1. The die is a rigid material or an elastic material.
2. The die is given an initial temperature.
3. Due to contact, a temperature gradient is calculated through the die thickness.

Workshop problem: **bouncing_shell_6.k**

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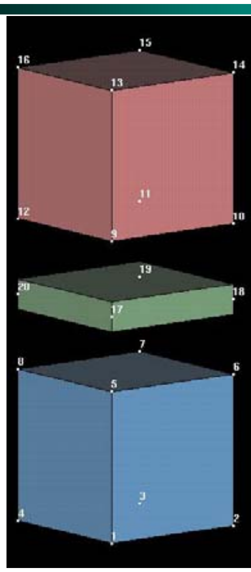
Workshop problem: bouncing_shell.k

LSTC



Note that shell has a top and bottom surface needing 2 contact definitions – get outward normal correct

LS-POST commands to display shell thickness
1. click ‘Appear’
2. click ‘thick’
3. click ‘All Vis’

Problem description

- Cold bottom block is stationary
- Top hot block is displaced downward
- Shell bounces between blocks and changes temperature when hit.

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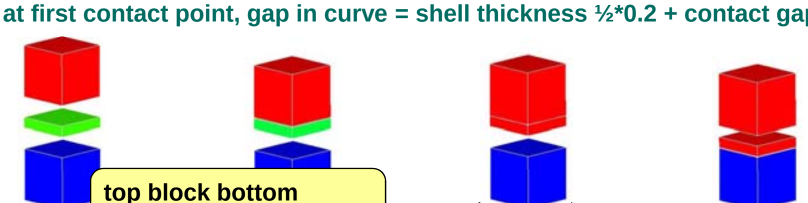
Workshop problem: bouncing_shell_1.k

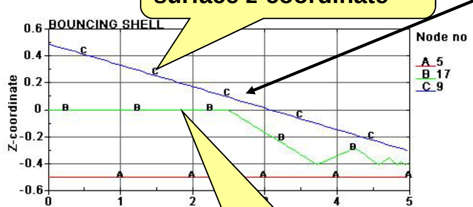
Problem 1: Investigate when contact occurs

LSTC

Run problem. Step through states and watch shell change temperature at contact
Look at first contact point, gap in curve = shell thickness ½*0.2 + contact gap 0.01



top block bottom surface z-coordinate



shell mid-plane z-coordinate

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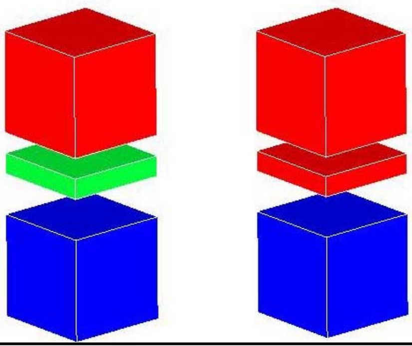
Workshop problem: bouncing_shell_2.k

Problem 2: change lmin & lmax

LSTC

Problem 2: Look at slide 7-17 for the definition of lmin & lmax. In problem 1 lmin=lmax=0.02 for contact #2. This means that thermal contact starts when gap<0.02. Change the thermal contact gap (lmin & lmax) from 0.02 to 0.2 for contact definition 2 and run the problem. Display temperature fringe and step through the time states. Note the response as shown below. Continue stepping through time states until the shell contacts the bottom block.



Note that thermal contact occurs before mechanical contact

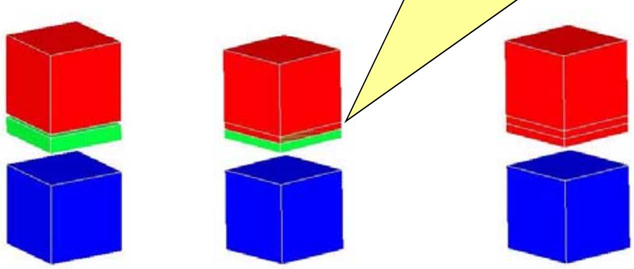
Workshop problem: bouncing_shell_3.k

Problem 3: set shlthk=0

LSTC

Problem 3: Set the parameter SHLTHK=0 on the *CONTROL_CONTACT keyword definition. Shell thickness will be ignored when calculating the gap. Set lmin=lmax=0.01. Step through the time states and note the response as shown below.

Note that the top block penetrates to the shell mid-plane before thermal or mechanical contact begins.



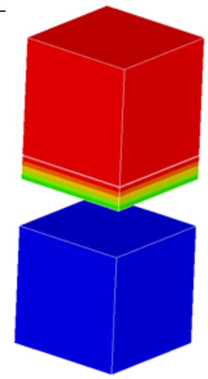
Workshop problem: bouncing_shell_4.k

Problem 4: use thick thermal shell

LSTC

Problem 4: Set the parameter TSHELL=1 on the *CONTROL_SHELL keyword to turn on the thermal thick shell option. Display temperature fringe and step through the time states. Note that a temperature gradient through the shell is now calculated.

```
time =  2.5600E+00     time step =  1.0000E-02     thermal step no.=   256
     minimum temperature =    0.0000E+00    at node      8
     maximum temperature =       1.00000    at node     16
    node temperature      t-bottom        t-top       x-flux       y-flux       z-flux
      14     1.00000                               0.0000E+00 -5.9605E-08  5.9605E-08
      15     1.00000                               0.0000E+00 -5.9605E-08  1.5930E-15
      16     1.00000                               0.0000E+00 -5.9605E-08  1.5930E-15
      17     0.99854     0.99797       1.00000     0.0000E+00  0.0000E+00  0.0000E+00
      18     0.99854     0.99797       1.00000     0.0000E+00  0.0000E+00  0.0000E+00
      19     0.99854     0.99797       1.00000     0.0000E+00  0.0000E+00  0.0000E+00
      20     0.99854     0.99797       1.00000     0.0000E+00  0.0000E+00  0.0000E+00
```



Note shell temperature gradient through the thickness (TPRINT).

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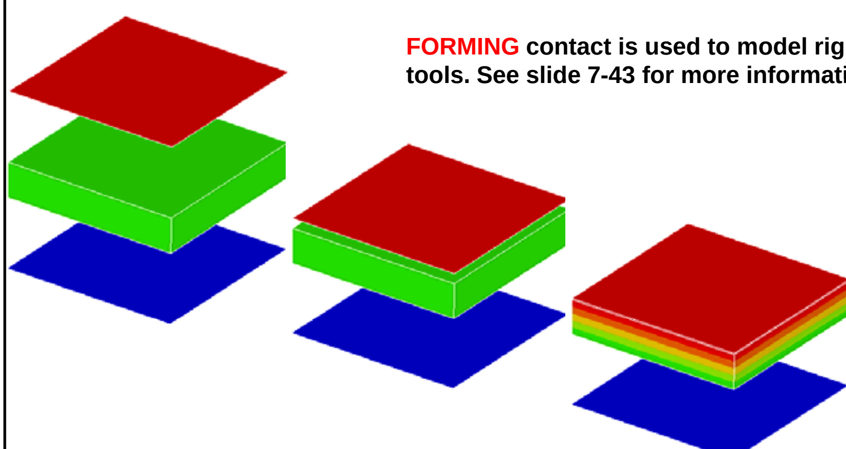
Workshop problem: bouncing_shell_5.k

Problem 5: model tools using shells, forming contact

LSTC

FORMING contact is used to model rigid tools. See slide 7-43 for more information



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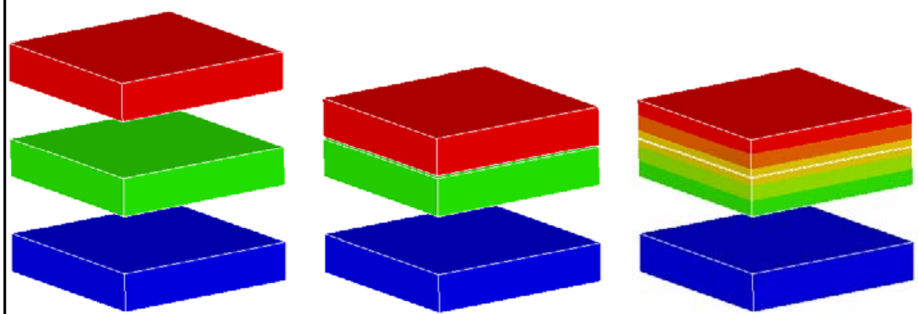
Workshop problem: bouncing_shell_6.k

Problem 6: deformable tools, automatic s_to_s, thick shells

LSTC

AUTOMATIC contact is used with deformable thick thermal shells to calculate a temperature gradient in the tools. Set SHLTHK=1 on the CONTROL_CONTACT keyword.

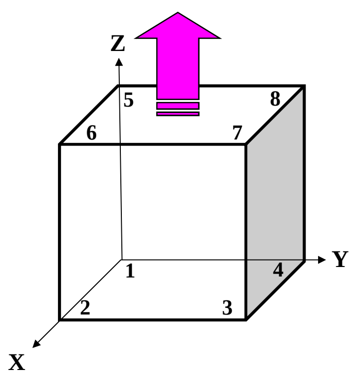


Conversion of mechanical work to heat

Workshop problem work_to_heat.k

LSTC

Z-displacement ramps up from Δz=0 to Δz=1 in 1 second.



Hypothetical mechanical properties

density	2000 kg/m^3
modulus of elasticity	70.e+09 Pa
Poisson Ratio	0.3
coeff. of expansion	0.0 m/m K
yield stress	2.0e+06 Pa
Tangent modulus	0.0

Hypothetical thermal properties

density	2000 kg/m^3
heat capacity	250 J/kg K
thermal conductivity	200 W/m K

Conversion of mechanical work to heat

LSTC

The mechanical work per volume, *w*, expended in deformation is equal to the area under the stress-strain curve.

$$\left(\frac{N}{m^2}\right)\left(\frac{m}{m}\right)=\left(\frac{J}{m^3}\right)$$

$$w=\int_0^{\varepsilon}\sigma d\varepsilon=\rho c\Delta T$$

$$\left(\frac{kg}{m^3}\right)\left(\frac{J}{kgK}\right)(K)=\left(\frac{J}{m^3}\right)$$

The above equation is dimensionally consistent when using SI units. However, a units conversion factor must be included when using other units.

Conversion of mechanical work to heat

Workshop problem work_to_heat.k

LSTC

1. **run work_to_heat.k**
2. **using LS-Post, click History. Report values at time=1 sec.**
   a) **what is the global internal energy? (ans. 1.39e+06)**
   b) **what is the element effective plastic strain? (ans. 0.693)**
   c) **what is the element effective stress? (ans. 2.e+06)**
   d) **what is the element temperature? (ans. 2.77)**
3. **(effective plastic strain) * (effective stress) = (ans. 1.39e+06)**
4. **Temperature = (above result) / (ρ cp) = (ans. 2.77)**

$$\frac{\sigma\varepsilon}{\rho c_p}=\frac{(0.693)(2.e+06)}{(2000)(250)}=2.77$$

Conversion of sliding friction to heat

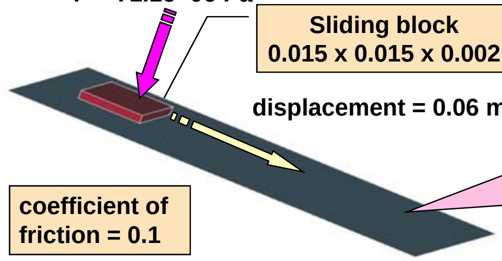
friction_to_heat.k

LSTC

P = 71.1e+06 Pa

Sliding block
0.015 x 0.015 x 0.002

displacement = 0.06 m



coefficient of friction = 0.1

The frictional heat is divided equally between the surfaces.
½ to the sliding block
½ to the stationary block

```
*CONTROL_CONTACT
  slsfac     rwpnal     islchk     shlthk
  usrstr     usrfrc      nsbcs .........
   sfric      dfric        edc .........
  ignore     frceng     skiprwg .........
```

Set shlthk = 1 to account for shell thickness

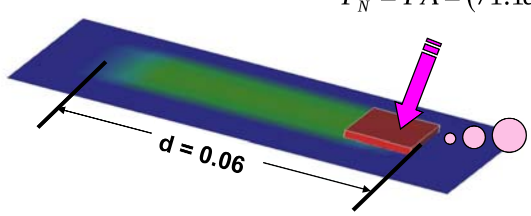
Set frceng = 1 to calculate contact frictional energy

Conversion of sliding friction to heat

friction_to_heat.k

LSTC

$$F_N = PA = (71.1e+06)\ (0.015 \times 0.015) = 16{,}000\ [N]$$



The small block is insulated. The frictional energy goes into increasing the block's internal energy

$$W_{friction} = (\mu F_N)\ d = (0.1)(16{,}000)(0.06) = 96\ J$$

$$Q = \frac{W}{2} = 48\ J$$

$$\Delta T = \frac{Q}{\rho c V} = \frac{48}{(7900)(460)(0.015 * 0.015 * 0.002)} = 29.4\ C$$

Conversion of sliding friction to heat

friction_to_heat.k

LSTC

Run friction_to_heat.k

The answers are reasonably close to the analytical solution for demonstration purposes. They would be better if a finer mesh where used.

Do the following with LS-PrePost and compare with the analytical solution.

Sliding interface energy

1. Click ASCI and select GLSTAT. Then click load.
2. Click “sliding energy” and then click plot
3. What is the sliding energy at time=1 sec. (ans. 94.7)

Temperature

1. Open d3plot
2. Click history – element (select the middle element in the sliding block)
3. Click temperature and then plot
4. What is the temperature at time=1 sec. (ans. 27.6)

Conversion of sliding friction to heat

friction_to_heat.k

LSTC

```
dt of cycle    1358 is controlled by shell      element
161

 time...........................    9.99990E-04
 time step......................    7.37916E-07
 kinetic energy.................    4.08254E+00
 internal energy................    9.94541E-03
 spring and damper energy.......    1.00000E-20
 system damping energy..........    0.00000E+00
 sliding interface energy.......    9.4696E+01
 external work..................    1.02005E+02
 eroded kinetic energy..........    0.00000E+00
 eroded internal energy.........    0.00000E+00
 total energy...................    9.91204E+01
 total energy / initial energy..    9.71725E-01
 energy ratio w/o eroded energy.    9.71725E-01
 global x velocity..............    5.21575E-01
 global y velocity..............   -3.95185E-03
 global z velocity..............    2.00433E-02
 time per zone cycle.(nanosec)..              0
```

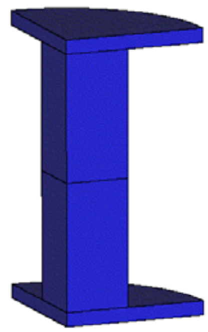
Sliding Interface Energy is printed to the GLSTAT file

Upset workshop problem

LSTC

The upsetting process is defined as the axial compression of an axisymmetric body between two perfectly rough, insulated plates.

- material is low carbon steel
- initial temperature: 20C
- no heat transfer to the environment – all plastic work goes into heating the part
- initial geometry: 9mm radius, 36 mm height
- imposed height reduction: $\Delta h/h = 0.44$
- loading time: 1.6 seconds



J. Van der Lugt, "Thermal Mechanically Coupled Finite Element Analysis in Metal Forming Processes", Computer Methods in Applied Mechanics and Engineering, 54 (1986) p. 145-160.

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Upset workshop problem

LSTC

Exercise 1

Run upset_mat003.k → after a few cycles

a) type ctrl C to interrupt execution

b) type sw2. to display diagnostics, then immediately type ctrl-C to freeze the screen. Look at the screen printout.

c) what is the time step size __________

d) what is the clock time to complete the run __________

d) type sw1. to terminate the run

Exercise 2

Run upset_mat003_dt2ms.k → repeat the above steps.

a) what is the time step size __________

b) what is the clock time to complete the run __________

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Upset workshop problem

LSTC

The only difference between upset_mat003.k and upset_mat003_dt2ms.k is the following keyword

```
*CONTROL_TIMESTEP
$    DTINIT      TSSFAC        ISDO     TSLIMT      DT2MS
         0.          0.           0         0.   1.0e-04
```

Upset_mat003.k is calculating a default explicit time step (dt = 9.79e-08) that satisfies the Courant condition for calculation stability.

upset_mat003_dt2ms.k is using a mass scaled time step specified by the *CONTROL_TIMESTEP keyword to be dt = 1.0e-04

Mass scaling OK for slow metal forming, not OK for crash dynamics

Upset workshop problem

Determination of the explicit time step size

LSTC

The time step of an explicit analysis is determined as the minimum stable time step in any deformable finite element in the mesh. In general this is determined by the so-called CFL-condition (Courant-Friedrichs-Lewy) that determines the stable time step in an element as a characteristic length divided by the acoustic wave speed. The CFL condition thus requires the numerical time step to be smaller than the time needed by the physical wave to cross the element. The physical stress wave propagates with the speed of sound c.

An element length of 5 mm will lead to a time step of 1 μs

$$\Delta t \leq \frac{l}{c}$$

Steel c = 5240 m/s
Aluminum c = 5328 m/s

$$c = \sqrt{\frac{E}{\rho\left(1-\upsilon^2\right)}} \quad \text{Shell element}$$

$$c = \sqrt{\frac{E\left(1-\upsilon\right)}{\rho\left(1+\upsilon\right)\left(1-2\upsilon\right)}} \quad \text{Brick element}$$

Upset workshop problem

Mass scaling used to reduce run time

LSTC

Mass-scaling refers to a technique whereby nonphysical mass is added to a structure in order to achieve a larger explicit time step and reduce run time.

Time step calculation for shell element

$$\Delta t = \frac{l}{c} \qquad c = \sqrt{\frac{E}{\rho(1-\upsilon^2)}}$$

$m\uparrow = \rho\uparrow = c\downarrow = \Delta t\uparrow$

Justifiable if KE < internal energy

Look at the end of the GLSTAT file for upset_mat003_dt2ms.k.

For a specified time step, $\Delta t_{specified}$, the density must be adjusted to satisfy the same equations. Solving the above equations for density:

Mass scaled density

$$\rho = \frac{(\Delta t_{specified})^2 E}{l^2(1-\upsilon^2)}$$

You can either enter a larger density on the material card or let Dyna calculate the density by entering the time step DT2MS

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Upset workshop problem

Mass scaling used to reduce run time

LSTC

Anytime you add nonphysical mass to increase the time step in a dynamic analysis, you affect the results (think of F = ma). Sometimes the effect is insignificant and in those cases adding nonphysical mass is justifiable where the velocity is low and the kinetic energy is very small relative to the internal energy. In the end, it's up to the judgment of the analyst to gage the affect of mass scaling. You may have to reduce or eliminate mass scaling in a second run to gage the sensitivity of the results to the amount of mass added.

Use the keyword `*DATABASE_GLSTAT` to create the text file glstat which contains system kinetic and internal energy

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Upset workshop problem

Mass scaling used to reduce run time

LSTC

Exercise 3

run upset_mat003_dt2ms.k to completion

a) open the GLSTAT file, what is the KE __________

b) open the GLSTAT file, what is the internal energy ________

c) is mass scaling justifiable __________

d) record the elapsed run time (at bottom of screen printout) ____

Run LS-POST

a) what is the maximum temperature __________

b) what is the minimum temperature __________

c) observe the deformed geometry & mesh at time=1.6.

We are going to observe the affect on the results by deleting hourglass control (exercise 4) and by changing the element formulation (exercise 5)

Upset workshop problem

Hourglass → D. Benson, "Zero Energy Modes in 1-Dimension: An Introduction to Hourglass Modes", FEA Information News, Feb, 2003.

LSTC

- Hourglass modes are caused by insufficient integration points (i.e., 1 point quadrature, elform=1)
- Fully integrated elements have no hourglass (elform=2)
- Hourglass often occurs in small displacement situations
- Hourglass modes are orthogonal to the real deformation
- General rule: hourglass energy < 10% of internal energy (see GLSTAT & MATSUM file)
- If hourglass occurs in an area where it does not influence the design area of concern, then it may be admissible.

Recommendation

Use reduced integration (i.e., elform=1) until you experience hourglassing, it is much faster

Upset workshop problem

hourglass control

LSTC

Exercise 4 → Run upset_mat003_dt2ms_viscous_hg.k to completion and observe the deformed geometry and mesh. Note: the problem may error terminate before reaching the end time of 1.6 seconds.

open upset_mat003_dt2ms_viscous_hg.k and look at the *HOURGLASS keyword

```
*HOURGLASS
$      HGID     HGTYPE
          1          1
```

HGTYPE=1 will activate the default (viscous form) hourglass.

Viscous hourglass (type 1) is a function of velocity. Stiffness hourglass (type 5) is a function of displacement.
This problem has large displacement but low velocity

Upset workshop problem

element formulation

LSTC

Exercise 5 → Run upset_mat003_dt2ms_elform_2.k to completion and observe the deformed geometry and mesh.

Open upset_mat003_dt2ms_elform_2.k and look at the *SECTION_SOLID keyword

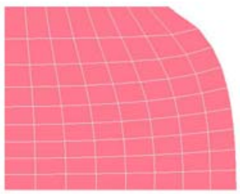
```
*SECTION_SOLID
$     SECID     ELFORM
          1          2
```

ELFORM = 2 for a fully integrated solid element . There are no hourglass modes for a fully integrated element, but the element is stiffer.

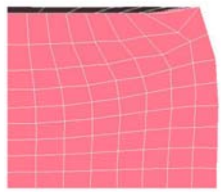
Upset workshop problem

Upset_mat003.k

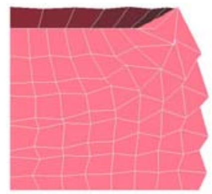
LSTC



baseline



Fully integrated solid element – a stiffer response



Default viscous hourglass

Upset workshop problem

Time scaling to reduce run time

LSTC

Exercise 6 → input file preparation, upset_mat003_dt2ms_time_scaled.k

1. Punch velocities are artificially increased but **not to exceed 2 to 5 m/s**. The punch velocity should be defined by a smooth curve (e.g., *DEFINE_CURVE_SMOOTH) and not a step function.

Baseline punch velocity

time	displacement
0.	0.
1.6	-0.00792

$$V = \frac{L}{t} = \frac{0.00792}{1.6} = 4.95 * 10^{-3}\ \frac{m}{\sec}$$

Time scaled – 1000x

time	displacement
0.	0.
0.0016	-0.00792

$$V = \frac{L}{t} = \frac{0.00792}{0.0016} = 4.95\ \frac{m}{\sec} = 4950\ \frac{mm}{\sec}$$

Upset workshop problem

Time scaling to reduce run time

LSTC

2. Define a mass scaled time step where cycles/mm is in the range of 100 to 1000.

$$dt2ms = \frac{1.}{(tool\ velocity)(cycles/mm)} = \frac{1.}{(4950)(200)} \approx 1.*10^{-6}\ \text{sec}$$

3. Use dt2ms to speed up the mechanical problem – do not artificially alter the density.

4. In the initial configuration, the punch should be in contact with the blank to avoid dynamic impact effects.

Upset workshop problem

Time scaling to reduce run time

LSTC

5. “Thermal velocity” terms (i.e., those with units of W = J/s) must be scaled by the same ratio as the punch velocity. Thermal velocity terms include
   - Thermal conductivity
   - Convection heat transfer coefficients
   - Contact heat transfer coefficients
   - Surface heat flux

$$k = (46)(1000) = 46000\ \frac{W}{m\ C}$$

Upset workshop problem

Mass and Time scaling to reduce run time

LSTC

Exercise 6 → results

baseline: upest_mat003.k
mass scaled: upset_mat003_dt2ms.k
mass & time scaled: upset_mat003_dt2ms_time_scaled.k

	Baseline	Mass scaled	Mass & time scaled
run time	**7 h**	**15 s**	**2 s**
Δt mech	1.e-07	1.e-04	1.e-06
cycles mech		16,000	1600
Δt thermal	1.e-06	1.e-03	1.e-05
cycles thermal		1,600	160

If you are doing a code shootout to see which is faster, make sure you understand the scaling that is going on.

Upset workshop problem

implicit analysis

LSTC

Exercise 7 → implicit

Open the file upset_mat003_implicit.k. The keyword *CONTROL_IMPLICIT_GENERAL has been added and both the thermal and mechanical times steps are set to 0.05 seconds

```
*CONTROL_IMPLICIT_GENERAL
         1        .05
*CONTROL_THERMAL_TIMESTEP
         0         1.        .05
```

a) Record the elapsed run time __________

b) Compare the run time with results in the previous table

Upset workshop problem

some things to remember about upset_mat003.k

LSTC

1

We are taking 10 mechanical time steps (dt=1.0e-04) for every thermal time step (dt=1.0e-03)

2

Material model 3, *MAT_PLASTIC_KINEMATIC is not a thermal material. However, plastic work is converted to heat.

3

1 point integration, elform=1

- reduces the computational cost
- softens the element so that the predicted stress is more accurate

4

Viscous hourglass (type 1) is a function of velocity. Stiffness hourglass (type 5) is a function of displacement. This problem has large displacement but low velocity. HG=5 gives better results.

Neutron heating

introduction

LSTC

R. E. Canaan, "Dynamic response of a pulse-heated, thick walled hollow sphere: validation of code numerics", UCRL-ID-137326, LLNL, January, 2000.

When fissionable metals are exposed to either an internal or external neutron source, fission heating and subsequent thermal expansion of the material can occur. If the heating pulse occurs rapidly enough, the temperature of the fissionable metal can rise faster than the material can respond by thermal expansion. In other words, there is a lag between the rise in temperature and thermal displacement of the material boundaries. Under such conditions, a portion of the thermal energy is converted to kinetic energy, producing vibrational displacements and potentially large dynamic stresses.

Early on in the pulse-heating transient, the mass-inertia effect mentioned above implies that while the material density remains essentially constant, there is an increase in material pressure as the temperature rises. Hence, the fissionable material is initially subject to a compressive stress. Later on, if the material remains elastic, the initial compression gives way to tension as the material elastically rebounds and also begins to finally expand in response to increasing temperature. After the heating pulse is complete and the total temperature rise is achieved, the expanded metal reaches a maximum displacement and peak tensile stress. Here, the dynamic expansion of the fissionable part exceeds the static expansion that would occur if the material were heated slowly. Furthermore, if the stresses remain below the tensile yield, another elastic rebound occurs, again sending the material into compression and contracting the material to a minimum expansion that is below the static value. The fission-heating-induced oscillations described above are of interest because the dynamic stresses involved may be large enough to result in material failure.

Neutron heating

Numerical model

LSTC

Numerical analysis of fission-heated mechanical systems typically begins with some sort of neutronics code that is capable of modeling neutron transport within the fissionable component of interest. If external neutron sources are present, the neutronics model may be additionally required to model neutron transport up to the boundaries of the fissionable part. Either way, the ultimate goal of such an analysis will be to determine the spatial and temporal history of the energy deposition in the fissionable component of interest. With this information in hand, the energy source term can then be directly coupled to the material response equations, i.e. the hydrodynamic problem, such that the resulting material displacements and thermal mechanical stresses can be determined subject to the appropriate initial and boundary conditions.

The geometry modeled is a 1/8 symmetry section of a thick walled sphere

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Neutron heating

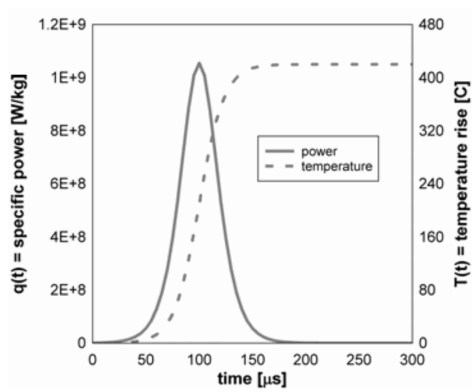
numerical model

LSTC

Neutron burst pulse

$$q(t) = C_p \frac{dT}{dt} = \frac{\dfrac{3.52 C_p T_{\max}}{b} \exp\left[-\dfrac{3.52}{b}\left(t - t_{pp}\right)\right]}{\left\{\exp\left[-\dfrac{3.52}{b}\left(t - t_{pp}\right)\right] + 1\right\}^2}$$

$$T(t) = \frac{T_{\max}}{\exp\left[-\dfrac{3.52}{b}\left(t - t_{pp}\right)\right] + 1}$$



T_{max} = maximum temperature rise [420 C]
T_{pp} = time to peak power [100e-06sec]
b = pulse duration [41 sec]
q = specific power [W/kg]

T = temperature [C]
t = time [sec]
Cp = heat capacity [177 W / kg C]

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Neutron heating

5 solutions with different options

LSTC

1. **neutron_heating_static.k** - implicit static solution. This is a mechanical only solution with thermal strains. The temperature change is defined using the LOAD_THERMAL keyword with the temperature rising from 0 to 420C in 1 load step.
2. **neutron_heating_load_thermal.k** – This is a mechanical only solution with thermal strains. The temperature change is defined using the LOAD_THERMAL keyword with the temperature defined by the burst temperature function entered as (T,t) data pairs.
3. **neutron_heating_coupled.k** – This is a coupled thermal mechanical solution. The energy deposition is defined using the LOAD_HEAT_GENERATION keyword with the deposited energy defined by the burst energy function entered as (q,t) data pairs.
4. **neutron_heating_coupled_function.k** – This is a coupled thermal mechanical solution. The energy deposition is defined using the burst energy function entered using the DEFINE_FUNCTION keyword.
5. **neutron_heating_coupled_function_decay.k** – this is the same as #4 but showing how to attenuate energy deposition with depth.

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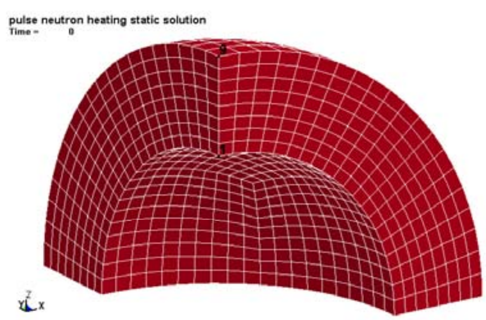
Neutron heating

1. run neutron_heating_static.k

LSTC

This is an implicit static solution. This is a mechanical only solution with thermal strains calculated . The temperature change is defined using the LOAD_THERMAL keyword with the temperature rising from 0 to 420C in 1 load step. Look at the input file and note how the following keywords are defined.

1. CONTROL_SOLUTION
2. CONTROL_IMPLICIT_GENERAL
3. CONTROL_TERMINATION
4. LOAD_THERMAL_LOAD_CURVE
5. DEFINE_CURVE



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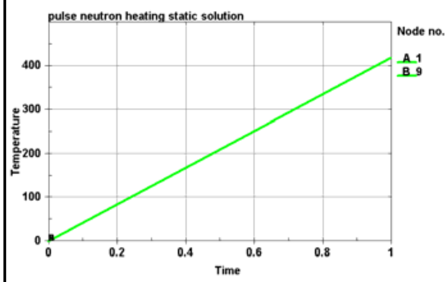
Neutron heating

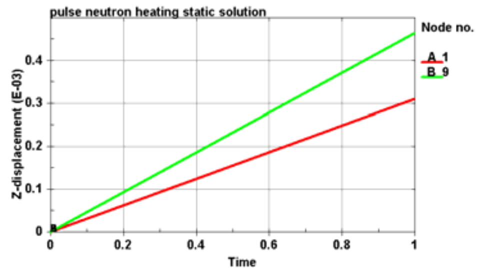
1. run neutron_heating_static.k

LSTC

Temperature response



Displacement response



Final radial displacement
- **inner 0.310 mm**
- **outer 0.465 mm**

The static solution provides baseline displacements to compare with the dynamic solution.

Neutron heating

Look at the following 3 problems

LSTC

Look at the input for the following 3 problems. They all calculate the same response but by 3 different modeling methodologies.

2. **neutron_heating_load_thermal.k**
3. **neutron_heating_coupled.k**
4. **neutron_heating_coupled_function.k**

Results are after the next 4 slides

The next 3 slides show the different keywords used for these modeling methods.

The final problem shows how to attenuate the deposition with depth. An exponential decay with depth is added to the function of problem 4.

5. **neutron_heating_coupled_function_decay.k**

Neutron heating

2. Run neutron_heating_load_thermal.k

LSTC

Mechanical only analysis using LOAD_THERMAL with a temperature CURVE defining the deposition response.

```
$
$ neutron heating defined by LOAD_THERMAL_LOAD_CURVE
$
*LOAD_THERMAL_LOAD_CURVE
         1
*DEFINE_CURVE
         1
          0.0000E+00          0.0000E+00
          1.0000E-05          1.8507E-01
          2.0000E-05          4.3645E-01
          3.0000E-05          1.0284E+00
          4.0000E-05          2.4188E+00
              *                   *
              *                   *
          2.4000E-04          4.2000E+02
          3.0000E-04          4.2000E+02
```


Neutron heating

3. Run neutron_heating_coupled.k

LSTC

Coupled analysis using LOAD_HEAT_GENERATION with an energy CURVE defining the deposition rate.

```
$
$ neutron heating defined by an energy deposition load curve
$
*LOAD_HEAT_GENERATION_SET
         1         1        1.
*DEFINE_CURVE
         1
          0.0000E+00          1.3551E+10
          1.0000E-05          3.1961E+10
          2.0000E-05          7.5328E+10
          3.0000E-05          1.7725E+11
          4.0000E-05          4.1549E+11
              *                   *
              *                   *
          2.4000E-04          4.3723E+08
          3.0000E-04          2.5326E+06
```

Neutron heating

4. Run neutron_heating_coupled_function.k

LSTC

Coupled analysis using LOAD_HEAT_GENERATION with an energy FUNCTION defining the deposition rate.

```
$
$ neutron heating defined by an energy deposition function
$
*LOAD_HEAT_GENERATION_SET
         1         1
*DEFINE_FUNCTION
         1
q(time)= 1.72e+04 * 3.52*117.*420./41.0e-06 *
 exp(-3.52*(time-100.e-06)/41.0e-06)
 / (exp(-3.52*(time-100.e-06)/41.0e-06) + 1.)**2
```

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Neutron heating

Run 5. neutron_heating_coupled_function_decay.k

LSTC

Coupled analysis using LOAD_HEAT_GENERATION with an energy FUNCTION defining the deposition rate attenuated exponentially with depth.

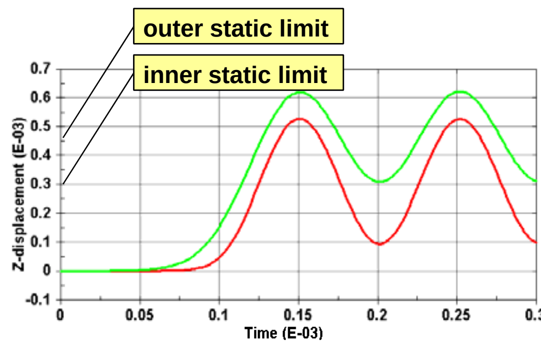
```
*LOAD_HEAT_GENERATION_SET
         1         1
*DEFINE_FUNCTION
         1
float heat_gen(float x, float y, float z, float time)
{
 float r, ri, ro, decay, q, hgen ;
 ri = 0.0508 ;
 ro = 0.0762 ;
 r = sqrt(x**2 + y**2 +z**2) ;
 decay = exp(-(ro-r)/(ro-ri)) ;
 q = 1.72e+04 * 3.52*117.*420./41.0e-06 *
 exp(-3.52*(time-100.e-06)/41.0e-06) /
 (exp(-3.52*(time-100.e-06)/41.0e-06) + 1.)**2 ;
 hgen = q * decay ;
 return (hgen) ;
}
```

Appendix F -100

Neutron heating

Radial displacement vs. time

LSTC

Inner surface node 1

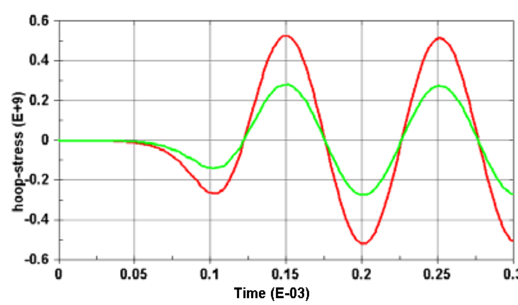
Outer surface node 9

Appendix F -101

Neutron heating

Hoop stress vs. time

LSTC

Inner surface element 1

Outer surface element 701

Appendix F -102

LSTC

Appendix F -103

LSTC

Appendix F -104