TEST CASE DOCUMENTATION
AND TESTING RESULTS

TEST CASE ID ICFD-VER-6.1

Non Newtonian flow in a channel

Tested with LS-DYNA® v980 Revision Beta

Friday 28th December, 2012

Document Information	
Confidentiality	**external use**
Document Identifier	LSTC-QA-LS-DYNA-ICFD-VER-6.1-1
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Number of pages	8
Date created	Friday 28th December, 2012
Distribution	External

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Contents

1 Introduction

1.1 Purpose of this Document

This document specifies the test case ICFD-VER-6.1. It provides general test case information like name and ID as well as information to the confidentiality, status, and classification of the test case.

A detailed description of the test case is given, the purpose of the test case is defined, and the tested features are named. Results and observations are stated and discussed. Testing results are provided in section 4.1 for the therein mentioned LS-DYNA® version and platforms.

2 Test Case Information

Test Case Summary	
Confidentiality	external use
Test Case Name	Non Newtonian flow in a channel: The Power Law model
Test Case ID	ICFD-VER-6.1
Test Case Status	Under consideration
Test Case Classification	Verification
Metadata	INTERNAL FLOW

Table 1: Test Case Summary

3 Test Case Specification

3.1 Test Case Purpose

The purpose of this test case is to validate the Power Law model for Non-Newtonian flows.

3.2 Test Case Description

Classic fluid mechanics applications involve flows that use a constant viscosity independent of temperature and shear rate. Those fluids are said to follow Newton's law of viscosity and are therefore called Newtonian fluids (Water, most aqueous solutions, oil, air and other gases). However, for some fluids this assumption does not hold. The flow of Non-Newtonian fluids is also encountered in many branches of engineering (physical chemistry, blood mechanics, hair gel, corn syrup, oobleck).

A Power-law fluid is a type of generalized Newtonian fluid for which the shear stress τ is given by :

$$\tau = K(\frac{\partial u}{\partial y})^n \tag{1}$$

where K is the flow consistency index and n is the flow behavior index.

The quantity :

$$\mu_{eff} = K(\frac{\partial u}{\partial y})^{n-1} \tag{2}$$

represents an apparent or effective viscosity function of the shear rate.

For $n = 1$, the classic shear stress expression for Newtonian fluids is obtained and $K = \mu$. For $n < 1$, the power law predicts that the effective viscosity would decrease with increasing shear rate. Those kind of fluids are called pseudo-plastic. The opposite case $n > 1$ is far less common and those fluids are called dilatant.

Similarly to the Newtonian Poiseuille flow, it can be proved [1] that for a stationary, incoming flow parallel to two infinite planes distant from $2h$ where the gravity forces are neglected, the velocity profile can be expressed as :

$$U(y) = \frac{n}{n+1}(\frac{1}{K}\frac{dP}{dx})^{(\frac{1}{n})}h^{\frac{n+1}{n}}(1-(\frac{y}{h})^{\frac{n+1}{n}}) \tag{3}$$

$$\tag{4}$$

And similarly for a cylinder pipe of radius a :

$$U(r) = \frac{n}{n+1}(\frac{1}{2K}\frac{dP}{dx})^{(\frac{1}{n})}a^{\frac{n+1}{n}}(1-(\frac{r}{a})^{\frac{n+1}{n}}) \quad (5)$$

$$(6)$$

For this test case both 2D and 3D models will be studied.

3.3 Model Description

Similarly to the Poiseuille test case, Figure (1) offers a view of the geometry and mesh for the 2D model. The 3D model uses the same elements size and dimensions as the 2D model. Table (2) gives out some information about the mesh generated and Table (3) gives the physical parameters that will be used. It is too be noted that for the $n = 0.6$ case, a twice as fine mesh is needed and has been used in order to get accurate results.

	2D model	3D model
Surface Element size	0.05	0.05
Volume Nodes	2972	60000
Volume Elements	5690	345000
Anisotropic Elements added to the Boundary Layer	2	2

Table 2: Test Case Mesh Information

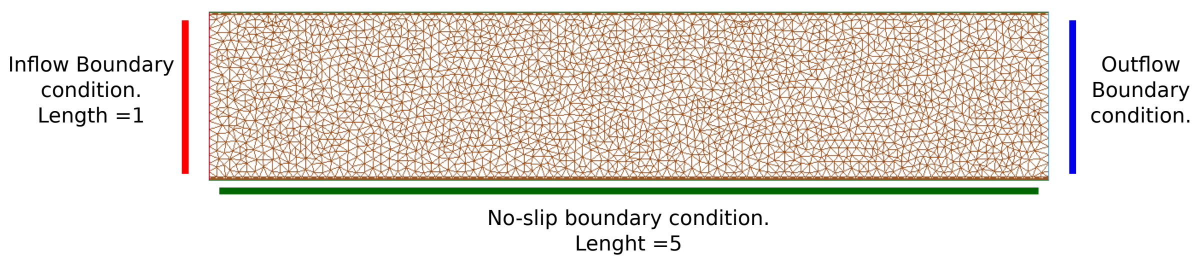

Figure 1: Test Case Mesh

Model physical parameters	
Fluid Density	1
Incoming average velocity	0.66
Viscosity,K	0.02
Thermal Coefficient	0

Table 3: Test Case Parameters

4 Test Case Results

4.1 Test Case observations

Figure (2) shows the adimentional velocity profiles obtained for different arbitrary chosen n values. The Non-Newtonian characteristics can clearly be identified. For $n < 1$, the velocity profile tends to be concave or blunt while for $n > 1$, the velocity profiles tend to have a sharper point. Figure (3) shows the absolute velocity profiles and offers a comparaison with the analytical solutions. It can be observed that the bigger n, the bigger U_{max} will be. The results are generally in good agreement with the analytical solutions. However, the further n differs from the Non-Newtonian fluid, the bigger the error. In order to diminish this error for such cases, the mesh can be progressively refined (A mesh using half the surface element size has already been used for the $n = 0.6$ case in order to obtain consistent results.

Figure (4) offers some velocity profiles for the 3D case. Again, the results are consistent with the analytical solutions and a finer mesh may be needed in cases where n differs too much from 1.

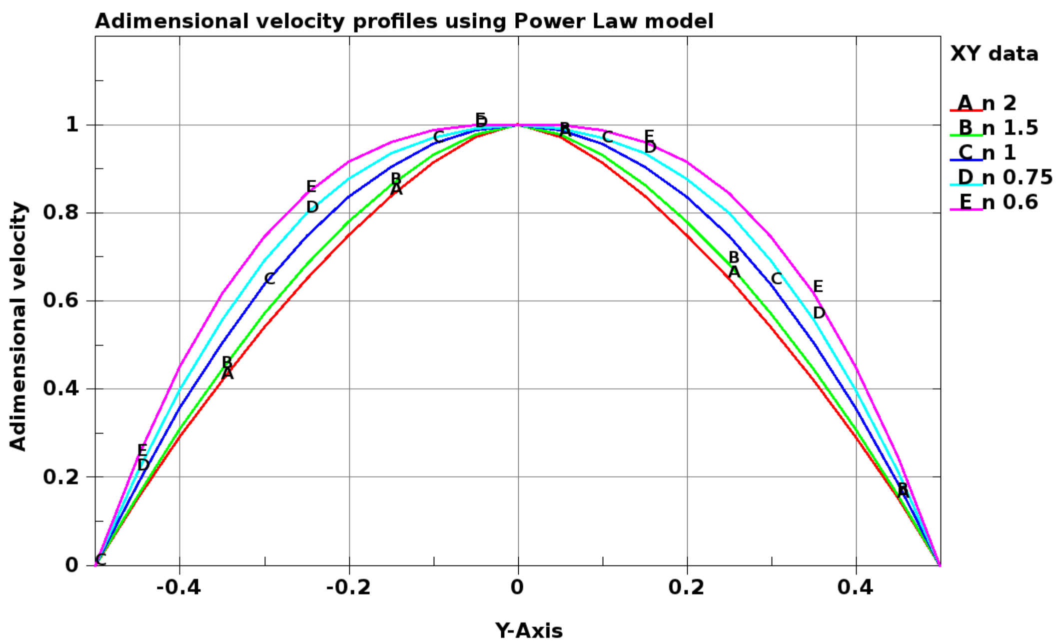

Figure 2: 2D Model - Adimensional velocity profiles for different choices of n ($\frac{U}{U_{max}}$).

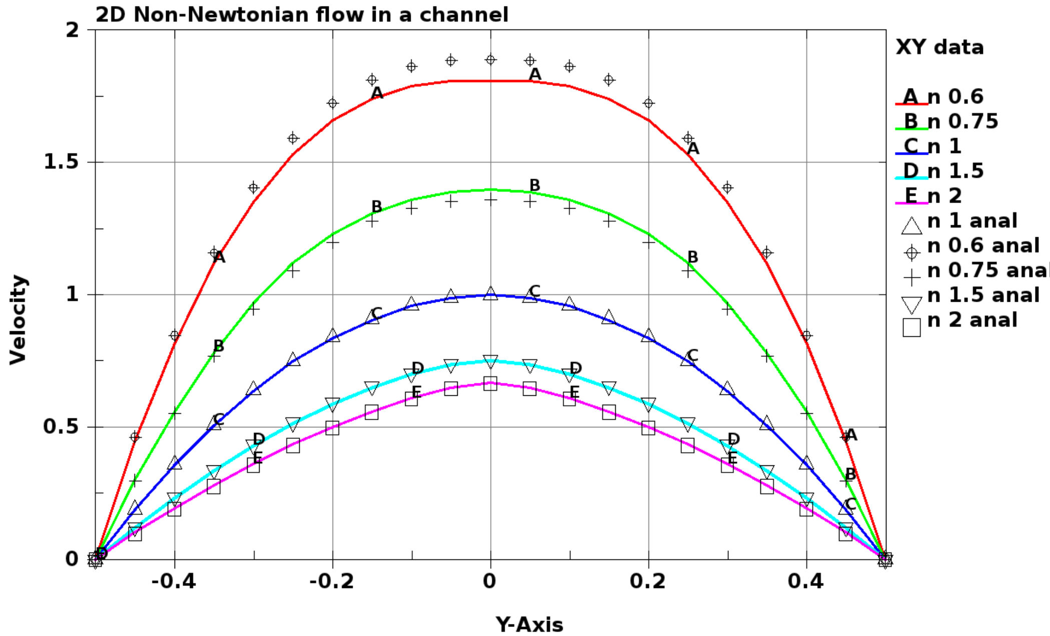

Figure 3: 2D Model - Absolute velocity profiles for different choices of n and comparison with analytical results.

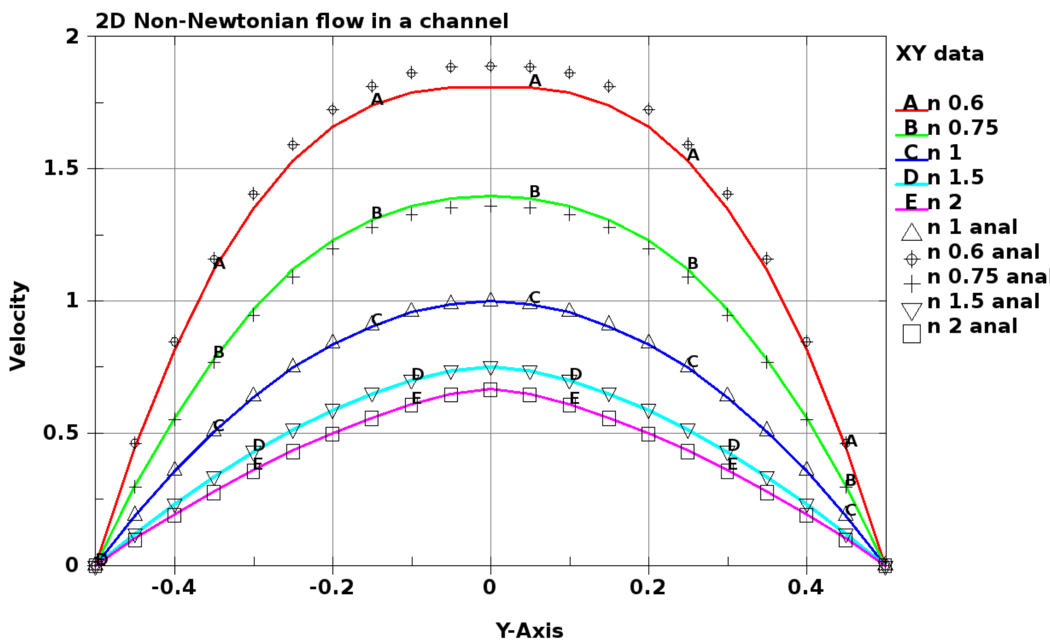

Figure 4: 3D Model - Absolute velocity profiles for different choices of n and comparison with analytical results.

References

[1] D. Pnueli and C. Gutfinger, *Fluid Mechanics*, Cambridge University Press, 1997.