TEST CASE DOCUMENTATION AND TESTING RESULTS

TEST CASE ID ICFD-VER-1.2

The Poiseuille flow

Tested with LS-DYNA® v980 Revision Beta

Friday 28th December, 2012

Document Information	
Confidentiality	**external use**
Document Identifier	LSTC-QA-LS-DYNA-ICFD-VER-1.2-1
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Number of pages	9
Date created	Friday 28th December, 2012
Distribution	External

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Contents

1 Introduction

1.1 Purpose of this Document

This document specifies the test case ICFD-VER-1.2. It provides general test case information like name and ID as well as information to the confidentiality, status, and classification of the test case.

A detailed description of the test case is given, the purpose of the test case is defined, and the tested features are named. Results and observations are stated and discussed. Testing results are provided in section 4.1 for the therein mentioned LS-DYNA® version and platforms.

2 Test Case Information

Test Case Summary	
Confidentiality	external use
Test Case Name	The Poiseuille flow: Parabolic flow developing in a pipe
Test Case ID	ICFD-VER-1.2
Test Case Status	Under consideration
Test Case Classification	Verification
Metadata	INTERNAL FLOW

Table 1: Test Case Summary

3 Test Case Specification

3.1 Test Case Purpose

The purpose of this test case is to study the laminar flow of the fluid between two fixed infinite planes and to compare the developed flow with the analytical solution.

3.2 Test Case Description

The Poiseuille flow is one of the few cases where, under some given hypotheses, the Navier-Stokes equations admit an analytical solution. For a stationary, incoming flow parallel to two infinite planes distant from $2h$ where the gravity forces are neglected, the Navier-Stokes equations reduce to ([1]) :

$$0 = -\frac{\partial P}{\partial x} + \mu \frac{d^2 U(y)}{dy^2} \quad (1)$$

$$0 = -\frac{\partial P}{\partial y} \quad (2)$$

which after integrating gives the following parabolic profile (see : Figure (1)) :

$$\frac{U(y)}{U_{max}} = 1 - \frac{y^2}{h^2} \quad (3)$$

$$U_{max} = -\frac{dP}{dx}\frac{h^2}{2\mu} \quad (4)$$

For a cylindrical flow the Navier-Stokes equations simplify to :

$$0 = -\frac{\partial P}{\partial z} + \mu \frac{1}{r}\frac{\partial (r \frac{\partial U(r)}{\partial r})}{\partial r} \quad (5)$$

$$0 = -\frac{\partial P}{\partial r} \quad (6)$$

$$0 = -\frac{1}{r}\frac{\partial P}{\partial \theta} \quad (7)$$

which after integrating gives a similar adimensional parabolic profile :

$$\frac{U(r)}{U_{max}} = 1 - \frac{r^2}{a^2} \quad (8)$$

$$U_{max} = -\frac{dP}{dx}\frac{a^2}{4\mu} \quad (9)$$

where a is the radius of the cylinder.

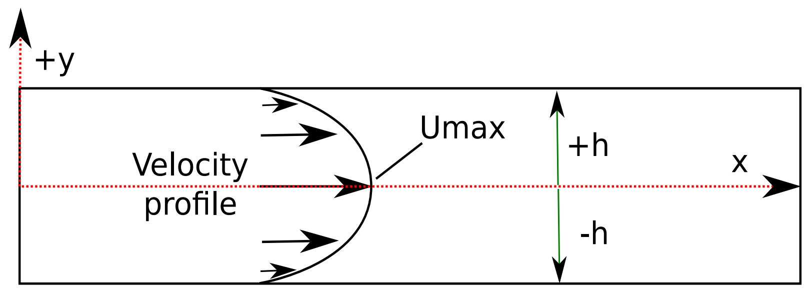

Figure 1: Test Case Sketch

For this test case, both 2D and 3D models will be studied. Furthermore, the entry length i.e the distance needed for the flow to become fully developed will be examined. Reference data gives for a laminar flow ([2]) :

$$L \approx 0.1aR_e \quad (10)$$

3.3 Model Description

Figure (2) offers a view of the geometry and mesh for the 2D model. The 3D model uses the same elements size and dimensions as the 2D model. Table (2) gives out some information about the mesh generated and Table (3) gives the physical parameters that will be used.

	2D model	3D model
Surface Element size	0.05	0.05
Volume Nodes	2972	60000
Volume Elements	5690	345000
Anisotropic Elements added to the Boundary Layer	2	2

Table 2: Test Case Mesh Information

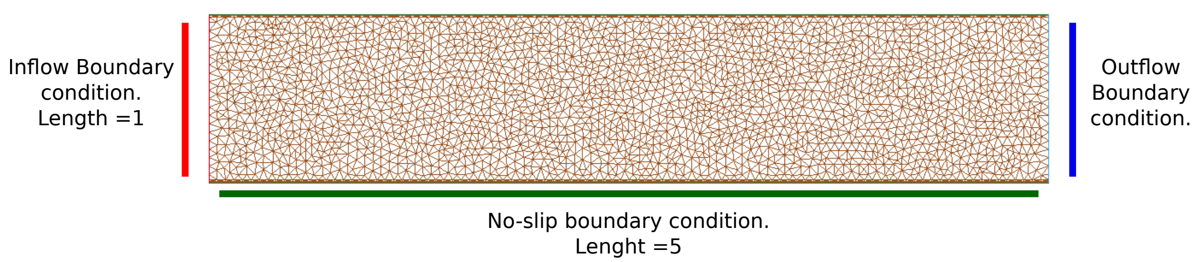

Figure 2: Test Case Mesh

Model physical parameters	
Fluid Density	1
Incoming average velocity	0.66
Viscosity	0.02
Thermal Coefficient	0

Table 3: Test Case Parameters

4 Test Case Results

4.1 Test Case observations

4.1.1 2D model-Velocity profile

For this first series of testing, a parabolic inflow profile will be chosen in order to ensure that the flow is fully developed through the entire section (See Figure (3)). Figure (4) shows the good agreement for the velocity profile with the analytical solution.
The 2D model can be considered as a 2D rectangle of infinite length, therefore, Equation (4) can be used for estimating the pressure gradient. For the chosen set of parameters, the analytical solution gives a pressure gradient dP of 0.8 which is in good agreement with the numerical value of 0.808.

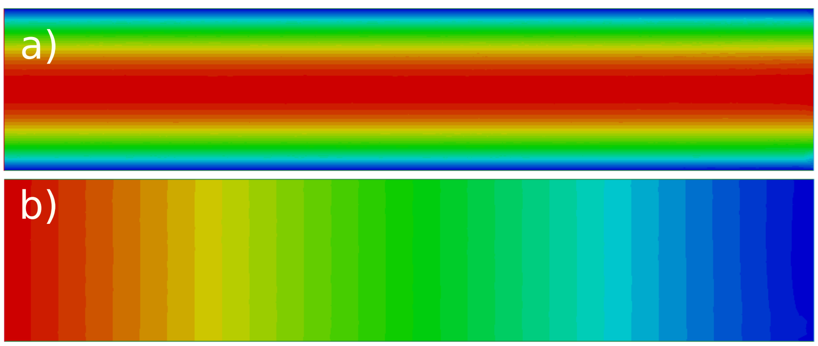

Figure 3: 2D Model - Parabolic inflow: a) Velocity fringes, b) Pressure fringes

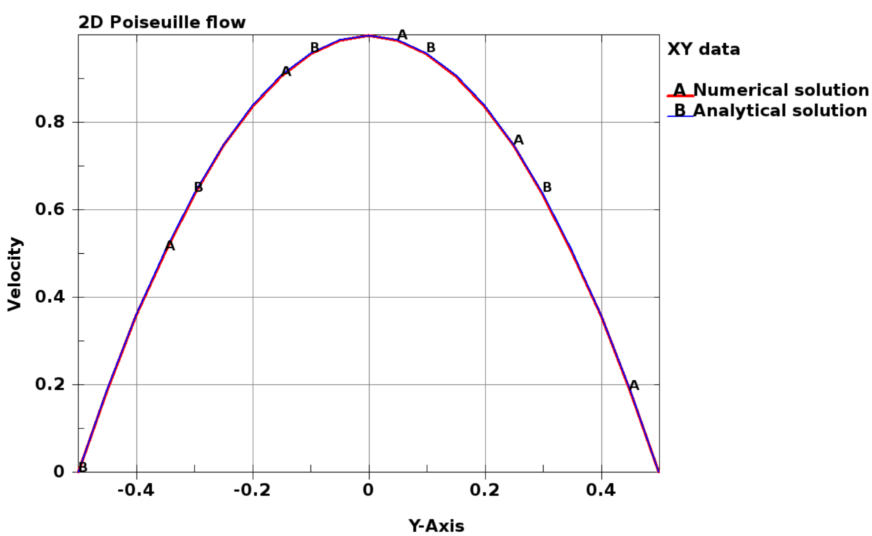

Figure 4: 2D Model - Parabolic inflow : Velocity profile

4.1.2 2D model-Entry Length

The entrance velocity is now chosen as constant. As can be observed on Figure (5), as the fluid flows between the two planes, the velocity converges from an incoming constant velocity to a parabolic behavior along the Y axis. During this flow entry phase, a non null component of the Y Velocity can be observed. The literature (See Equation (10) gives for the chosen set of parameters a entry length of ≈ 1.7. Numerically, this corresponds to the point where U_{max} is within 1% of its expected value (See Figure (6)).

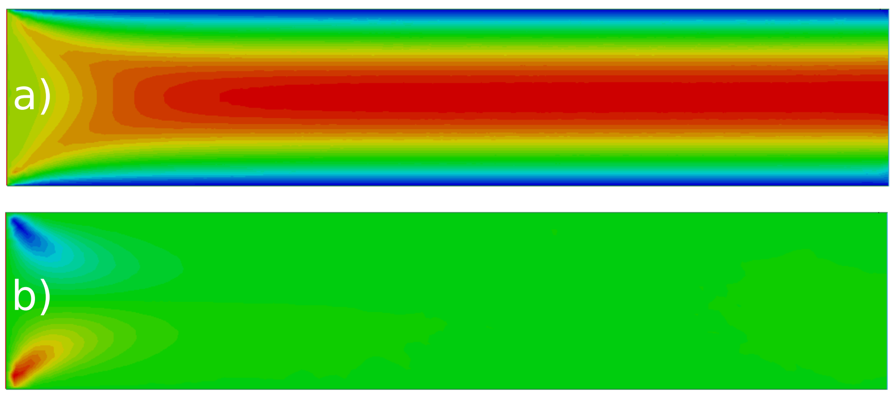

Figure 5: 2D Model - Constant inflow: a) X-Velocity fringes, b) Y-Velocity fringes

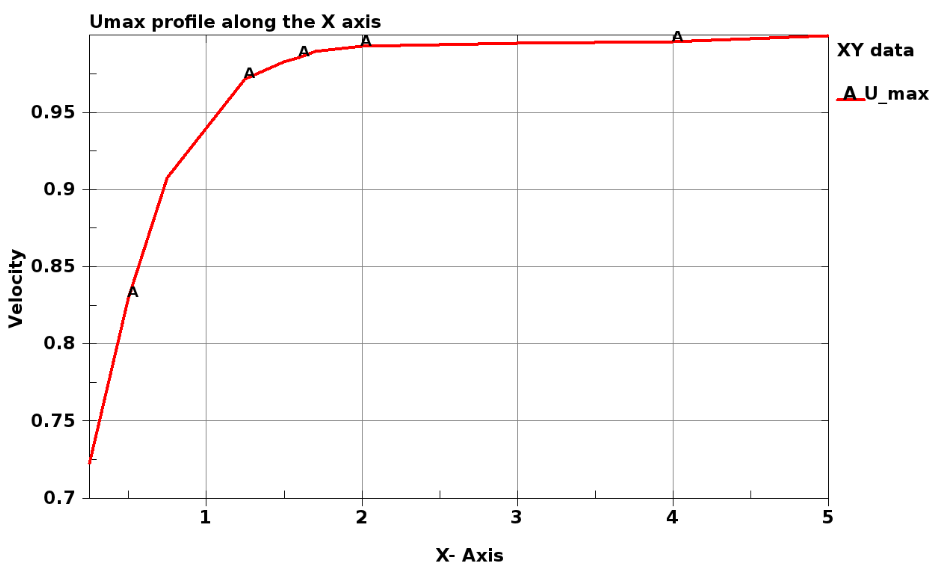

Figure 6: 2D Model - constant inflow : Maximum velocity (y=0) along the X Axis

4.1.3 3D model-Velocity profile

Figure (7) offers a view of the pressure fringes and the velocity profile through the section. Consistently with Equation (8) and (3), the velocity profile does not change between the 2D and 3D model (See Figure (8). However, the pressure gradient is expected to be different with a analytical value of 1.6 in good agreement with the numerical value of 1.601.

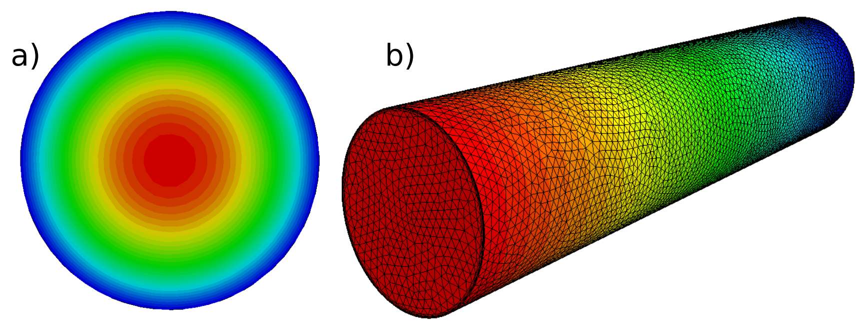

Figure 7: 3D Model - Parabolic inflow: a) Velocity fringes through the section, b) Pressure fringes

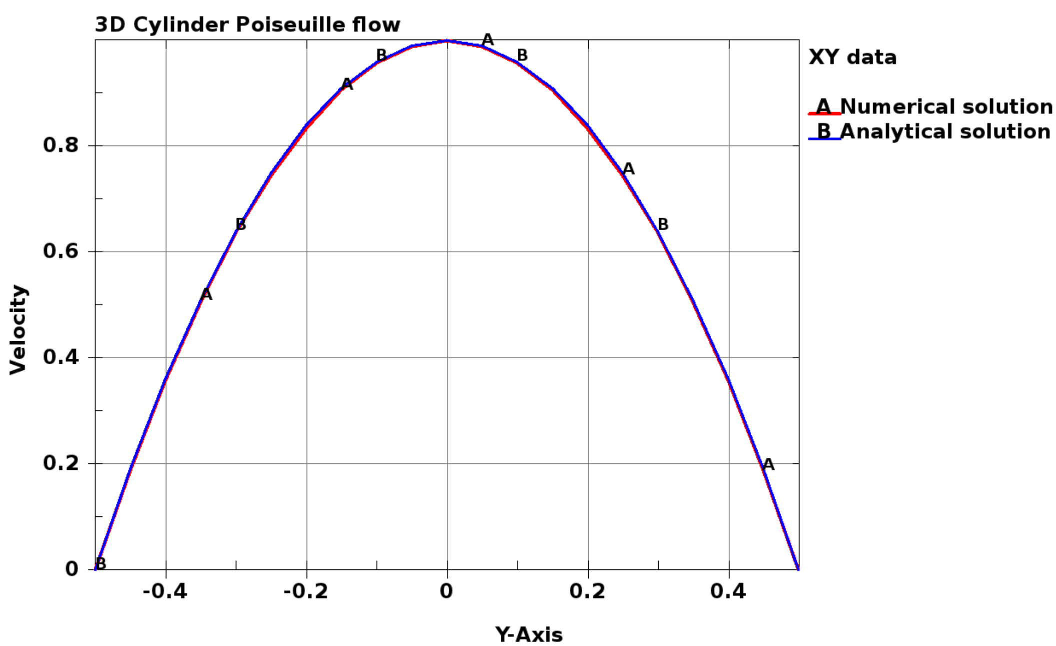

Figure 8: 3D Model - Parabolic inflow: Velocity Profile

References

[1] P. CHASSAING, *Mecanique des fluides. Elements d'un premier parcours*, CEPADUES Editions, 1982.

[2] W. Kays, *McGraw-Hill series in mechanical engineering*, McGRAW-HILL Book Company, 1966.