TEST CASE DOCUMENTATION
AND TESTING RESULTS

TEST CASE ID ICFD-VER-1.2

The Poiseuille flow

Tested with LS-DYNA® v980 Revision Beta

Friday 28th December, 2012
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1 Introduction

1.1 Purpose of this Document

This document specifies the test case ICFD-VER-1.2. It provides general test case information like name and ID as well as information to the confidentiality, status, and classification of the test case.

A detailed description of the test case is given, the purpose of the test case is defined, and the tested features are named. Results and observations are stated and discussed. Testing results are provided in section 4.1 for the therein mentioned LS-DYNA® version and platforms.
2 Test Case Information

<table>
<thead>
<tr>
<th>Test Case Summary</th>
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<tr>
<td>Confidentiality</td>
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<tr>
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<tr>
<td>Test Case ID</td>
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<td>Metadata</td>
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Table 1: Test Case Summary
3 Test Case Specification

3.1 Test Case Purpose

The purpose of this test case is to study the laminar flow of the fluid between two fixed infinite planes and to compare the developed flow with the analytical solution.

3.2 Test Case Description

The Poiseuille flow is one of the few cases where, under some given hypotheses, the Navier-Stokes equations admit an analytical solution. For a stationary, incoming flow parallel to two infinite planes distant from $2h$ where the gravity forces are neglected, the Navier-Stokes equations reduce to ([1]):

$$0 = -\frac{\partial P}{\partial x} + \mu \frac{d^2U(y)}{dy^2} \quad (1)$$

$$0 = -\frac{\partial P}{\partial y} \quad (2)$$

which after integrating gives the following parabolic profile (see: Figure (1)):

$$\frac{U(y)}{U_{max}} = 1 - \frac{y^2}{h^2} \quad (3)$$

$$U_{max} = -\frac{dP}{dx} \frac{h^2}{2\mu} \quad (4)$$

For a cylindrical flow the Navier-Stokes equations simplify to:

$$0 = -\frac{\partial P}{\partial z} + \mu r \frac{\partial}{\partial r} \left( r \frac{\partial U(r)}{\partial r} \right) \quad (5)$$

$$0 = -\frac{\partial P}{\partial r} \quad (6)$$

$$0 = -\frac{1}{r} \frac{\partial P}{\partial \theta} \quad (7)$$

which after integrating gives a similar adimensional parabolic profile:
\[
\frac{U(r)}{U_{\text{max}}} = 1 - \frac{r^2}{a^2}
\]  \hspace{1cm} (8)

\[
U_{\text{max}} = -\frac{dP}{dx} \frac{a^2}{4\mu}
\]  \hspace{1cm} (9)

where \(a\) is the radius of the cylinder.

For this test case, both 2D and 3D models will be studied. Furthermore, the entry length i.e the distance needed for the flow to become fully developed will be examined. Reference data gives for a laminar flow ([2]) :

\[
L \approx 0.1aR_e
\]  \hspace{1cm} (10)
3.3 Model Description

Figure (2) offers a view of the geometry and mesh for the 2D model. The 3D model uses the same elements size and dimensions as the 2D model. Table (2) gives out some information about the mesh generated and Table (3) gives the physical parameters that will be used.

<table>
<thead>
<tr>
<th></th>
<th>2D model</th>
<th>3D model</th>
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</thead>
<tbody>
<tr>
<td>Surface Element size</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Volume Nodes</td>
<td>2972</td>
<td>60000</td>
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<tr>
<td>Volume Elements</td>
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<td>345000</td>
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<tr>
<td>Anisotropic Elements</td>
<td>2</td>
<td>2</td>
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Table 2: Test Case Mesh Information

![Test Case Mesh](image)

Figure 2: Test Case Mesh

<table>
<thead>
<tr>
<th></th>
<th>Fluid Density</th>
<th>Incoming average velocity</th>
<th>Viscosity</th>
<th>Thermal Coefficient</th>
</tr>
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<tr>
<td></td>
<td>1</td>
<td>0.66</td>
<td>0.02</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3: Test Case Parameters
4 Test Case Results

4.1 Test Case observations

4.1.1 2D model-Velocity profile

For this first series of testing, a parabolic inflow profile will be chosen in order to ensure that the flow is fully developed through the entire section (See Figure (3)). Figure (4) shows the good agreement for the velocity profile with the analytical solution. The 2D model can be considered as a 2D rectangle of infinite length, therefore, Equation (4) can be used for estimating the pressure gradient. For the chosen set of parameters, the analytical solution gives a pressure gradient $dP$ of 0.8 which is in good agreement with the numerical value of 0.808.

![Figure 3: 2D Model - Parabolic inflow: a) Velocity fringes, b) Pressure fringes](image)

![Figure 4: 2D Model - Parabolic inflow: Velocity profile](image)
4.1.2 2D model-Entry Length

The entrance velocity is now chosen as constant. As can be observed on Figure (5), as the fluid flows between the two planes, the velocity converges from an incoming constant velocity to a parabolic behavior along the Y axis. During this flow entry phase, a non null component of the Y Velocity can be observed. The literature (See Equation (10)) gives for the chosen set of parameters a entry length of \( \approx 1.7 \). Numerically, this corresponds to the point where \( U_{max} \) is within 1% of its expected value (See Figure (6)).

Figure 5: 2D Model - Constant inflow: a) X-Velocity fringes, b) Y-Velocity fringes

Figure 6: 2D Model - constant inflow: Maximum velocity \((y=0)\) along the X Axis
4.1.3 3D model-Velocity profile

Figure (7) offers a view of the pressure fringes and the velocity profile through the section. Consistently with Equation (8) and (3), the velocity profile does not change between the 2D and 3D model (See Figure (8). However, the pressure gradient is expected to be different with a analytical value of 1.6 in good agreement with the numerical value of 1.601.

![Figure 7: 3D Model - Parabolic inflow: a) Velocity fringes through the section, b) Pressure fringes](image)

**Figure 8: 3D Model - Parabolic inflow: Velocity Profile**

![3D Cylinder Poiseuille flow graph](image)
References
