

In theory (e.g. papers from Camanho et al.), stiffness is reduced by $(1-d)$:

$$1-d = 1 - \frac{\sigma^F(\lambda - \sigma^0)}{\lambda(\sigma^F - \sigma^0)} \quad d \dots \text{damage variable}$$
$$= \dots = \frac{\sigma^0(\sigma^F - \lambda)}{\lambda(\sigma^F - \sigma^0)}$$

In *MAT-138 of LS-DYNA, we have damage variable defined as:

$$D = \frac{\lambda - \sigma^0}{\sigma^F - \sigma^0} \quad (\text{"plastic strain"})$$

but stiffness is reduced by this factor

$$r = \frac{1 - D}{1 + D(\frac{\sigma^F}{\sigma^0} - 1)}$$
$$= \dots = \frac{\sigma^0(\sigma^F - \lambda)}{\lambda(\sigma^F - \sigma^0)}$$

which is exactly the same as in "theory"

⇒ Stiffness reduction factor is the same in *MAT-138 and theory, only difference is the definition of "damage".