COMPUTATIONAL MODEL OF DUCTILE DAMAGE AND FRACTURE IN SHELL ANALYSIS

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ABSTRACT

A continuum damage mechanics (CDM) model has been implemented in the explicit finite element code LS-DYNA for shell analysis using co-rotational shell elements. The constitutive model is coupled with an element-kill algorithm available in the code. A strain localization condition and a critical value of damage are adopted as optional indicators for the onset of material failure. A numerical simulation of a plate perforation problem is presented to demonstrate the capabilities of the CDM model.

INTRODUCTION

In numerical simulation of plastic forming, crashworthiness and structural impact it is of interest to predict the risk of fracture in the various structural components. For thin-walled structures, fracture is typically caused by dramatic localization of plastic deformation in a local band, which leads to rapid growth of damage in the material and finally rupture occurs.

In this paper, the theoretical basis and the numerical implementation of a continuum damage mechanics (CDM) model now available in LS-DYNA [1] are described. The CDM model includes elastoplasticity and ductile damage, as proposed by Lemaitre [2], and is coupled with criteria for onset of fracture based on a strain localization condition and a critical value of damage, as proposed by Billardon and Doghri [3]. The fracture criterion is used to trigger an element-kill algorithm available in LS-DYNA. The model was implemented using a fully vectorized backward-Euler algorithm, which has been shown to be efficient, accurate and robust for elastoplastic and viscoplastic materials (Berstad et al [4]).

The CDM model has applications to shell analysis of thin-walled metal structures subjected to quasi-static and dynamic loading, and has been implemented as material type 104 in LS-DYNA version 950. Material type 104 includes in addition Hill's yield condition and viscoplasticity, but anisotropy and rate effects are not discussed here. Also material type 105 includes the CDM model described in this paper. A numerical simulation of a plate perforation problem is presented to demonstrate the capabilities of the CDM model.

CONTINUUM DAMAGE MECHANICS MODEL AND CRITERIA FOR LOCALIZATION AND FRACTURE

In this section, the CDM model proposed by Lemaitre [2] for ductile materials is described in some detail. The strain tensor ε is decomposed as

$$\varepsilon = \varepsilon_e + \varepsilon_p \quad (1)$$

where ε_e is the elastic strain tensor and ε_p is the plastic strain tensor. The elastic stress-strain relation is defined as

$$\sigma = (1 - D)C:\varepsilon_e \quad (2)$$

where σ is the stress tensor, D is the damage variable, and **C** is the fourth order tensor of elastic constants, which is uniquely defined by Young's modulus E and the Poisson's ratio ν in the isotropic case. The effective stress, $\tilde{\sigma}$, is the stress calculated over the section that effectively resists the forces (Lemaitre [2]) and reads

$$\tilde{\sigma} = \frac{\sigma}{1 - D} \quad (3)$$

The elastic domain of the material is defined by the yield function $f = f(\sigma, R, D)$. The material is elastic for $f(\sigma, R, D) < 0$. If $f(\sigma, R, D) = 0$, the material is in the elastoplastic domain, while $f(\sigma, R, D) > 0$ is not admissible. The yield function is adopted in the form

$$f(\sigma, R, D) = \frac{\sigma_{eq}}{1 - D} - (\sigma_0 + R) \quad (4)$$

where σ_0 is the yield stress, σ_{eq} is the von Mises equivalent stress (i.e. the second invariant of the deviatoric stress tensor σ')

$$\sigma_{eq} = \sqrt{\frac{3}{2}\sigma':\sigma'} \quad (5)$$

and R is the isotropic strain-hardening variable defined as

$$R = \sum_k R_k; \quad R_k = Q_k[1 - \exp(-C_k r)] \quad (6)$$

in which r is the damage accumulated plastic strain. The loading/unloading conditions in elastoplasticity are given by

$$f \le 0, \quad \dot{\lambda} \ge 0, \quad \dot{\lambda} f = 0 \quad (7)$$

where $\dot{\lambda}$ is the plastic parameter; $\dot{\lambda}$ is equal to zero in the elastic domain and greater than zero in the elastoplastic domain.

The normality principle is adopted to define the flow rule of the damaged elastoplastic material (Lemaitre [2])

$$\dot{\varepsilon}_p = \dot{\lambda}\frac{\partial f}{\partial \sigma} = \frac{3}{2}\frac{\dot{r}}{1 - D}\frac{\sigma'}{\sigma_{eq}} \quad (8)$$

where it has been used that for the current choice of the yield function, the plastic parameter $\dot{\lambda}$ coincides with the damage-plastic strain rate $\dot{r}$ and thus the relation $\dot{r} = \dot{\lambda}$ holds (Lemaitre [2]). The evolution equation for the damage variable is defined as

$$\dot{D} = \begin{cases} 0 & \text{for} \quad r \le r_D \\ \dfrac{Y}{S(1-D)}\dot{r} & \text{for} \quad r > r_D \end{cases} \tag{9}$$

where r_D is the damage threshold, S is a positive material constant, and Y is the so-called strain energy density release rate

$$Y = \frac{1}{2}\boldsymbol{\varepsilon}_e{:}\mathbf{C}{:}\boldsymbol{\varepsilon}_e = \frac{\sigma_{eq}^2 R_v}{2E(1-D)^2} \tag{10}$$

The triaxiality function R_v is defined as (Lemaitre [2])

$$R_v = \frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2 \tag{11}$$

in which $\sigma_H = (\sigma_{11} + \sigma_{22} + \sigma_{33})/3$ is the hydrostatic stress (i.e. the first invariant of the stress tensor σ).

The accumulated plastic strain p is defined by

$$\dot{p} = \sqrt{\frac{2}{3}\dot{\varepsilon}_p{:}\dot{\varepsilon}_p} = \frac{\dot{r}}{1-D} \tag{12}$$

where Eqns. (5) and (8) have been used to obtain the last equality.

Using the consistency condition for elastic-plastic loading conditions, it is further possible to establish the relation between the stress-rate $\dot{\sigma}$ and the strain-rate $\dot{\varepsilon}$ in the form

$$\dot{\sigma} = \mathbf{H}{:}\dot{\varepsilon} \tag{13}$$

where $\mathbf{H}$ is a fourth order tangent stiffness tensor, which is important in material bifurcation analysis. According to Billardon and Doghri [3], an upper bound to the onset of bifurcation (or strain localization) in the structure is defined by the condition

$$\det(\mathbf{m} \cdot \mathbf{H} \cdot \mathbf{m}) = 0 \tag{14}$$

for some unit vector $\mathbf{m}$ somewhere in the structure. The localization criterion is local in character and may be applied anywhere within the structure regardless of the boundary conditions and material properties. The criterion for localized necking can be considered as an indicator for material failure, since at this point a dramatic localization of plastic strains and damage will occur inside a band (Billardon and Doghri [3]).

Another possible fracture criterion is obtained by assuming that a macrocrack occurs in the material as the damage variable reaches a critical value D_C (Lemaitre [2]). A fracture criterion is thus defined as

$$D = D_C \tag{15}$$

where D_C for simplicity is considered as a material constant. Note that a refined fracture criterion has been proposed by Lemaitre [2] in which the critical damage is a function of the stress state of the material, based on the assumption that the amount of energy dissipated in damage growth at fracture is constant.

NUMERICAL IMPLEMENTATION

The constitutive equations are implemented in LS-DYNA using a fully vectorized backward-Euler integration algorithm (Berstad et al [4]), which is efficient, accurate and robust and well suited for large-scale shell analysis.

The implementation of the damage model into LS-DYNA is described as follows:

1. The strain increment between time steps N and N+1 is $\Delta\varepsilon_{N+1/2}$.

2. Trial values of the effective stress and internal variables are calculated assuming that the step is elastic, i.e.

$$\tilde{\sigma}_{N+1}^{tr} = \frac{\sigma_N}{1-D_N} + \mathbf{C}:\Delta\varepsilon_{N+1/2}; \tag{16}$$
$$p_{N+1}^{tr} = p_N; \quad r_{N+1}^{tr} = r_N; \quad R_{k,N+1}^{tr} = R_{k,N}; \quad D_{N+1}^{tr} = D_N$$

3. The elastic assumption is checked by substitution into the yield criterion

$$f_{N+1}^{tr} = \sqrt{\frac{3}{2}\tilde{\sigma}_{N+1}'^{tr}:\tilde{\sigma}_{N+1}'^{tr}} - (\sigma_0 + \sum_k R_{k,N+1}^{tr}) \tag{17}$$

4. If $f_{N+1}^{tr} \le 0$ then the step is elastic, and the effective stress and internal variables at step N+1 are set equal to the trial values. If $f_{N+1}^{tr} > 0$ then the step is elastic-plastic, and the trial stress state must be returned to the yield surface:

$$p_{N+1} = p_{N+1}^{tr} + \Delta p_{N+1} \tag{18}$$

$$r_{N+1} = r_{N+1}^{tr} + (1 - D_{N+1})\Delta p_{N+1} \tag{19}$$

$$\tilde{\sigma}_{N+1} = \tilde{\sigma}_{N+1}^{tr} - \frac{3}{2}\Delta p_{N+1}\mathbf{C}:\frac{\tilde{\sigma}'_{N+1}}{\tilde{\sigma}_{eq,N+1}}; \quad \tilde{\sigma}_{eq,N+1} = \sqrt{\frac{3}{2}\tilde{\sigma}'_{N+1}:\tilde{\sigma}'_{N+1}} \tag{20}$$

$$R_{k,N+1} = R_{k,N+1}^{tr} + (1 - D_{N+1})\Delta p_{N+1} C_k\left(Q_k - R_{k,N+1}\right) \tag{21}$$

$$D_{N+1} = D_{N+1}^{tr} + \begin{cases} 0 & \text{for} \quad r_{N+1} \le r_D \\ \dfrac{\tilde{\sigma}_{eq,N+1}^2 R_{v,N+1}}{2ES} \cdot \Delta p_{N+1} & \text{for} \quad r_{N+1} > r_D \end{cases} \tag{22}$$

5. These equations are then used in the yield condition

$$f_{N+1} = \tilde{\sigma}_{eq,N+1} - (\sigma_0 + \sum_k R_{k,N+1}) = 0 \tag{23}$$

   to solve for Δp_{N+1} using an iterative solution method.

6. Finally, the stresses are obtained from the effective stresses as

$$\sigma_{N+1} = \left(1 - D_{N+1}\right)\tilde{\sigma}_{N+1} \tag{24}$$

The model is implemented for co-rotational shell elements in plane stress, i.e. the out-of-plane normal stress is assumed to be zero. The shell elements have one-point integration in the plane and several integration points or layers through the thickness where the constitutive equations are evaluated. In the implementation, the out-of-plane shear stresses are treated elastically; i.e. only the in-plane stress components enter into the coupled constitutive model of elastoplasticity and ductile damage. The thickness change of the shell can be accounted for in the simulations, and it is thus possible to represent (at least approximately) the thinning of the shell in a localized neck.

In the model, it is possible to adopt the localization condition, the critical damage criterion, or a combination of both as a fracture criterion, which is coupled with the element-kill algorithm available in LS-DYNA. As the fracture criterion is reached in one layer of a shell element this layer becomes inactive; i.e. the stress components in the layer are all taken equal to zero. If all the layers of a shell element become inactive, the element is removed from the finite element model. This means that it is, in principle, possible to follow the evolution of a "crack" through the structure. The fracture criterion is checked in all integration points in the structure for each time step throughout the loading process.

The criterion for localization is in general quite complicated, but for plane stress states (i.e. a two-dimensional case) a relatively simple algorithm is available (Ortiz et al [5]). In problems were extreme localization of deformation occurs, the numerical results tend to become mesh dependent, i.e. the results are influenced by shape functions, element orientation and element size. The criterion for localization may thus be considered as a limit beyond which the numerical solutions are regarded to be non-objective (Billardon and Doghri [3]).

NUMERICAL EXAMPLE

In order to show an example of the possible applications of the CDM model, the problem of a circular and fully clamped aluminium plate struck by a rigid hemispherically tipped cylindrical projectile is considered. The projectile is given an initial velocity in direction normal to the target and impacts at the centre point. The projectile velocity is chosen above the ballistic limit of the plate to study the perforation process.

The target plate is clamped on a 120 mm diameter, while the target thickness is 1.27 mm. The projectile has a 12.7 mm diameter and a mass of 37.7 g. The initial projectile velocity is 82 m/s. The target material is modelled using the following constants: $E = 70\ 000$ MPa, $\nu = 0.3$, $\sigma_0 = 115$ MPa, $Q_1 = 73$ MPa, $C_1 = 96$, $Q_2 = 176$ MPa, $C_2 = 4$, $S = 1.7$ MPa, $r_D = 0.03$ and $D_C = 0.23$. The projectile is assumed to be a rigid body.

The true stress vs plastic strain curve of the supposed aluminium material is shown in Figure 1. It is clearly shown that the material is assumed to strain harden up to about 35 % plastic strain, and at this point softening occurs due to damage evolution.

The conceptual model used in the numerical simulations of plate perforation is shown in Figure 2. The target is modelled using 30800 Belytschko-Wong-Chiang shell elements [1], while the projectile is discretized using rigid brick elements. Eroding surface-to-surface contact is prescribed between the projectile and target. Friction is not included. The critical damage criterion is used to trigger fracture, while the localization condition is used merely as an indicator of incipient material failure. In order to introduce imperfections into the model, the element thickness was defined by a random Gauss distribution with mean value 1.27 mm and standard deviation 0.0127 mm.

The results from the simulation are presented in Figure 3 showing the deformed geometry after perforation. It is seen that the model is able to predict a combined failure mode by plugging and petalling that has been found experimentally for the considered problem (Levy and Goldsmith [6]). The elements that failed in the simulation due to a critical level of damage are shown in Figure 4. It is seen that a spherical cap (or plug) and six petals are created during the perforation process.

CONCLUDING REMARKS

A continuum damage mechanics (CDM) model has been implemented in the explicit finite element code LS-DYNA. A fully vectorized backward Euler algorithm was adopted to integrate the rate constitutive equations. The constitutive model was coupled to an element-kill algorithm available in the code, using the localization condition and the critical damage criterion to predict the onset of fracture. The computational cost of introducing damage and fracture indicators into the elastoplastic constitutive relations is small, and the CDM model is thus applicable to large-scale impact and perforation simulations. Promising results are obtained in simulations of normal perforation of thin plates by hemispherically tipped projectiles.

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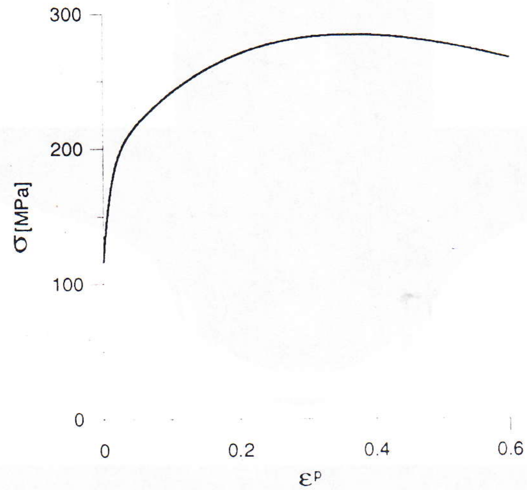

Figure 1. True stress vs plastic strain curve

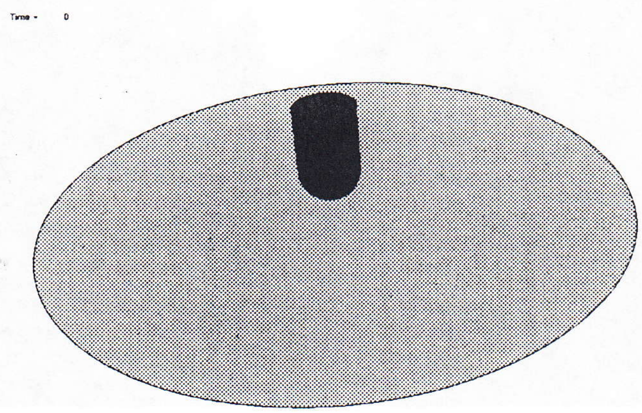

Figure 2. Conceptual model of target and projectile

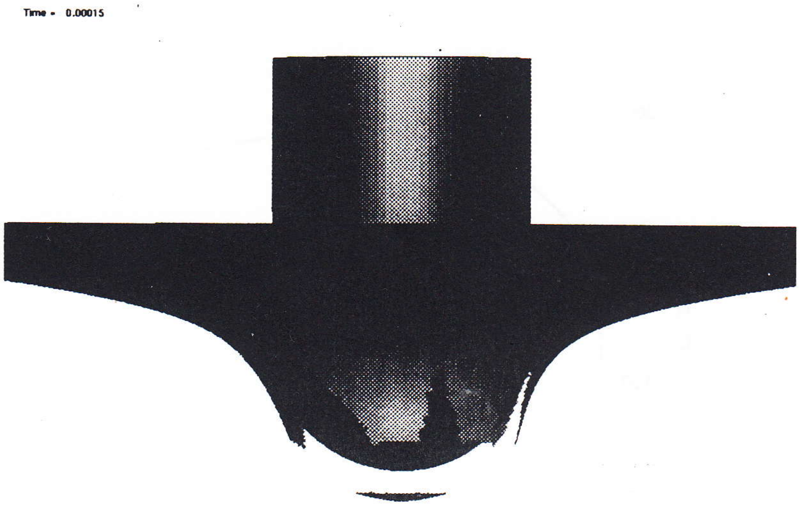

Figure 3. Plots of deformed geometry after perforation

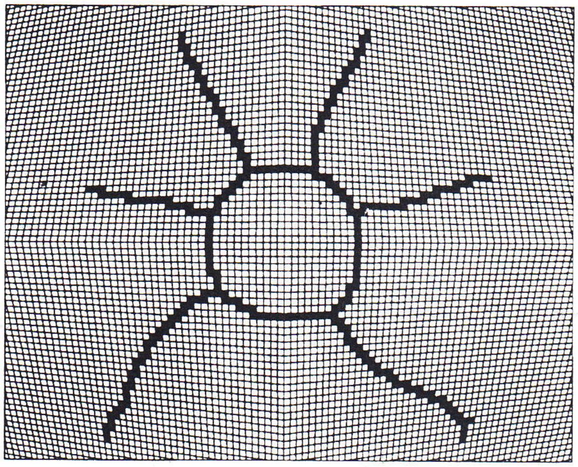

Figure 4. Element removed in the simulation by the element-kill algorithm