



Discrete Element Method in LS-DYNA® (DEM)

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Content p. 1



Preface

These notes are a <u>basic</u> unofficial introduction to the new Discrete Element Method (DEM) recently implemented in LS-DYNA[®]. At current time, November 2012, the shown features are new and still under development so changes and fixes can occur in later versions of LS-DYNA[®]. In these notes, figures marked with 2 are embedded avi's in the pdf file.

It is the hope of the authors that the reader will be able to:

- Understand the idea behind the DEM method.
- Know the keywords in order to use this new feature.
- Use LS-PrePost[®] to build the DEM model.
- Start LS-DYNA[®] with the correct syntax to obtain additional output files.
- Know the files related to the DEM method.
- Post-process a DEM model applying LS-PrePost[®].



Contents

- 1. <u>Introduction</u>
- 2. <u>Main Keywords</u>
- 3. <u>Generating DEM Particles in LS-PrePost®</u>
- 4. <u>Examples</u>
- 5. <u>Post-Processing DEM Models using LS-PrePost</u>®
- 6. <u>General Comments</u>
- 7. Sphere Bond Models
- 8. Interaction Between DEM Particles and Structure
- 9. <u>Coupling to New Blast Particle Method</u>
- 10. <u>References</u>



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Content p. 3



Chapter 1

Introduction





 The Discrete Element Method (DEM) is implemented in LS-DYNA[®] from version ls971 R5.1.1.

 The method models parts with rigid spheres and handle interaction between these as well as interaction between particles and other structural parts. The interaction between the spheres are done in contact points using springs and dampers.



• First is searched for contact using bucket sorting as for regular contacts. Then forces are applied accordingly. A radius is given on the element card that is taken into account for sphere to sphere contact and also for sphere contacting other structural parts.



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 The default settings leads to a connection that only consider compression, the spheres will separate for tension. Further, the spheres has only 3 DOF by default so they don't rotate. Both settings can be overwritten in the keyword cards as shown in Chapter 2. Furthermore, both damping and friction can be applied.

 There are several new keyword cards related to this method as seen in Chapter 2 and later chapters. The part will use *PART and typically *MAT_020. Other materials can be given but all that is used from the material card is the bulk modulus and the density, the latter only if the _VOLUME option is used at *ELEMENT_DISCRETE_SPHERE. The bulk modulus used is the one that LS-DYNA[®] will calculate internally for all materials.

 The method has been used for modeling granular materials, e.g. sand and rocks. But there are many other applications for this approach as well, e.g. powder compaction.





 Several stages has been carried out in implementing this new method. First was implemented reaction between spheres and also contact to structural parts. Then Capillary forces was implemented in order to model wet sand. DEM was then extended to been coupled with a modified version of the *AIRBAG_PARTICLE feature for modeling Mine Blast. Recently, a packing algorithm was implemented in LS-PrePost[®] to pack the spheres and a new bound model is implemented in LS-DYNA[®] for modeling continuums.

• The base for the general implementation of the DEM method is [P. A. Cundall et al, 1979]. This is the interaction between the particles.

 To model wet sand, e.g. for mine blast Capillary forces needs to be taken into account. This is done based on the paper by [Y. I. Rabinovich et al, 2005].



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• LSTC's Bound Model was developed to handle "bonding" of the discrete spheres. This allows a continuum to be modeled and also make it possible for the spheres to obtain tension. The feature is described in more details in Chapter 7.

 Blast load is done by *PARTICLE_BLAST based on experience with *AIRBAG_PARTICLE and makes it possible to model the explosive with discrete spheres. Further, these can be coupled to discrete spheres that can model sand, which can be both dry or wet. This approach is especially for modeling burred land mines. Details is shown in Chapter 9 where keyword and examples are illustrated.

 In order to create parts using discrete spheres a new packing algorithm has been implemented in LS-PrePost [Z. Han et. al, 2012]. The algorithm fills closed volumes with spheres and is shown in Chapter 3.



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• Here the funnel is made transparent. Notice the nice packing of the particles.

 The main difference between these models are the setting for sand, which is done at *CONTROL_DISCRETE_ELEMENT. The related parameters are CAP, GAMMA, VOL and ANG.



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 Here the *DEFINE_DE_BOND is used to bond the DEM particles together. It is seen that the fries have elastic behaviour, as the bonding is elastic.

Courtesy of Kirk A. Fraser Roche, Canada



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Chapter 2

Main Keywords





• There are several relevant keyword cards for the DEM method:

- ***NODE:** Gives the location of the particle.
- *ELEMENT_DISCRETE_SPHERE_option: Element ID, mass, inertia and radius of the particle.
- ***CONTROL_DISCRETE_ELEMENT:** Setting damping coefficients, stiffness scaling and options for wet sand.
- *DEFINE_DE_TO_SURFACE_COUPLING: Define contact between DEM spheres and structural parts. In the moment only implemented for contact with shells. For solids one can use a _NODE_TO_SURFACE contact.
- ***DEFINE_DE_ACTIVE_REGION:** Gives a region of interest. If particles are outside this domain, they are deactivated in the particle to particle search and in the contact routines to structural parts.



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Introduction

- Keyword cards for the DEM method con't:
 - ***DEFINE_DE_BOND:** Define a bond model between the discrete spheres. Two method currently available, spring method or linear elastic for fracture analysis. The keyword is discussed in Chapter 7.

• *DEFINE_DE_INJECTION: The option makes it possible to generate a flow of discrete spheres without specifying them initially in the input deck. This saves CPU time as well as memory. The injection geometry is currently limited to a rectangular plane.

 *DATABASE_BINARY_DEMFOR: Setting the output interval for the binary DEM coupling file. It shows the forces in the interface defined by
*DEFINE_DE_TO_SURFACE_COUPLING. This is described in Chapter 5.

In the following these cards are described in more details.



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General Specification

 The DEM part is given by *PART and call's a *SECTION_SOLID or a SECTION_SHELL. The element formulation is ignored, since these are DEM elements and the radius for the particles is given at *ELEMENT_DISCRETE_SPHERE. The material often used is *MAT_RIGID. But others can be used, as mentioned earlier.

 This specification makes the input compatible with other parts specified in LS-DYNA[®].

*PAI	RT							
\$# t	title							
Glas	ss Beads							
\$#	pid	secid	mid	eosid	hgid	grav	adpopt	tmid
	1	1	1	0	0	0	0	0
*SE(CTION_SOLI	ED						
\$#	secid	elform	aet					
	1	0	0					
*MA	I_RIGID_TI	ITLE						
Glas	ss Beads							
\$#	mid	ro	е	pr	n	couple	m	alias
	1	2400.0	6.000E+09	0.240000	0.000	0.000	0.000	
\$#	CMO	conl	con2					
0	.000000	0	0					
\$#]	lco or al	a2	a3	vl	v2	v3		
	0.000	0.000	0.000	0.000	0.000	0.000		



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General Specification

 The density from the material card is only used is the _VOLUME option is applied at *ELEMENT_DISCRETE_SPHERE.

 Young's modulus and Poisson's ratio is used for contact between spheres and for contact with structural parts. In fact what is used is the Bulk modulus, internally calculated based on the material input. Notice that few material models in LS-DYNA[®] directly has the Bulk modulus as input.

• The stiffness of the material is used to calculate the time step for the spheres, see Chapter 6.

• The constraints on the material card (CMO, CON1, CON2) are ignored as are the rest of the flags. How to apply constraints is mentioned in Chapter 6.





Defining the Particle

 The location of the particle is done by specifying a node in that location using *NODE. This gives node number, X,Y and Z initial locations. The node number is then used in *ELEMENT_DISCRETE_SPHERE.

 The mass, inertia and the radius of the spheres is set using the *ELEMENT_DISCRETE_SPHERE_option card.

*ELEM	*ELEMENT_DISCRETE_SPHERE_{OPTION}											
Card 1	1	2	3	4	5	6	7	8				
Variable	NID	DID	MASS		DADIUS							
v ariable	NID	FID	MASS		KADIUS							
Туре	Ι	Ι	F	F	F							
Default	none	none	none	none	none							





Defining the Particle

NID:

Node ID.

PID:

Part ID for the part that the sphere belongs to.

MASS:

Mass of the particle. If the _VOLUME option is flagged, then the mass of the particle is MASS x RHO_{mat}, where the latter is the density given at the material card (*MAT_XXX). This means that MASS in this case is the volume.

INERTIA:

Gives the inertia of the sphere. If the _VOLUME option is flagged, then the inertia of the particle is calculated as:

$I = INERTIA \cdot RHO_{mat}$





Defining the Particle

INERTIA Con't:

Where MASS is the value of the given parameter MASS, R is the radius of the particle and RHO_{mat} is the density given at the material card (*MAT_XXX).

RADIUS:

The radius of the particle. This will not influence the mass of the part but is used for contact between particles and to structure.





 Defining global control options for the DEM elements is done at the *CONTROL_DISCRETE_ELEMENT card. This is for particle to particle contact.

*CONT	*CONTROL_DISCRETE_ELEMENT											
Card 1	1	2	3	4	5	6	7	8				
Variable	NDAMP	TDAMP	Fric	FricR	NormK	ShearK	САР	MXNSC				
Туре	F	F	F	F	F	F	Ι	I				
Default	0	0	0	0	0.01	2/7	0	0				

NDAMP:

Normal damping coefficient. This coefficient is multiplied to the relative velocity between two particles as NDAMP * $\rm V_{rel}.$ NDAMP is unitless.

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TDAMP:

Tangential damping coefficient. This coefficient is multiplied to the relative velocity between two particles as NDAMP * $\rm V_{rel}.$ TDAMP is unitless.

Fric:

0: 3 DOF

NE.0: 6 DOF

This is the translational friction between the particles. If not given, the particles will only have 3 DOF, the translational ones. However, if a translational friction is given, there will be created rotation due to that and the DOF will be set to both translational and rotational DOF. If Fric is set to 0 and and the particles only have translational DOF, the correct behaviour in many applications is not captured.

FricR:

Rolling friction coefficient. This is only valid for non-zero values of Fric. In fact, if FricR is specified, Fric is set so 6 DOF is activated.



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NormK:

Scale factor of normal spring coefficient. The spring stiffness is calculated as:

$$K_{Spring} = K_{Bulk} \cdot RADIUS \cdot NormK$$

where K_{Bulk} is the bulk modulus and RADIUS is the particle radius. If no value is given for NormK, it is default set to 0.01. If a negative value is given for NormK, then the absolute value is the spring stiffness, K_{Spring} .

ShearK:

Used to scale the shear spring stiffness. It is not given as a direct coefficient but as the ratio between scaling in shear and NormK. So the given value is ShearK_{given} = ShearK/NormK. The default value is 2/7.

MXNSC:

In the manual is stated maximum number of subcycling cycles, however it has nothing to do with traditional subcycling. The flag is reserved for an option to compute the DEM on GPU's. It is not yet implemented.



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<u>CAP:</u> 0:

0: Dry particles

NE.0: Wet particles. Requires an additional keyword card.

When CAP is given a value different than 0, capillary forces are considered. The implementation follows [Y. I. Rabinovich et. al, 2005]. This makes it possible to model e.g. both dry and wet sand using the DEM method. The forces are calculated as:

$$F = 4 \cdot \pi \cdot RADIUS^2 \cdot \alpha \cdot GAMMA \cdot COS(ANG \cdot \frac{d\alpha}{dH})$$





CAP not equal 0.											
Optional	1	2	3	4	5	6	7	8			
Variable	Gamma	Vol	Ang								
Туре	F	F	F								
Default	0	0	0								

GAMMA and ANG are described on the previous page.

VOL:

Volume fraction. This is used for calculating the liquid bridge volume. In LS-DYNA[®] is assumed that the liquid is equally distributed on each particle. When two wet spheres are in contact, part of the liquid forms a bridge and generates capillary forces.



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VOL, continued:

The liquid bridge volume is calculated as:

$$V = FAC \cdot VOL \cdot (\text{Volume of two spheres}) = \frac{FAC \cdot VOL \cdot 4}{3.0 \cdot \pi \cdot (R0^3 + R1^3)}$$

VOL is a user input and is given at the card. It is percentage volume that the water field is between the particles and is used in the above equation. The value is given in as values between 0 and 1. A value of 0.35 or less is often used. The FAC is to take into account that a sphere may be in contact with multiple spheres so for each bridge only FAC of its liquid contribute to the formation of a bridge. The FAC parameter is hardwired in LS-DYNA[®] to 0.1.





Defining Area of Interest

 In order to save CPU time, one can specify a region of interest. Particles inside this region are active but if the particle is outside, it is taken out of the calculation. In fact, the particle is deactivated for both particle interaction and contact between particle and structure. This is done with the *DEFINE_DE_ACTIVE_REGION option.

*DE	*DEFINE_DE_ACTIVE_REGION											
	Card 1	1	2	3	4	5	6	7	8			
	Variable	ID	TYPE	Xm	Ym	Zm						
	Туре	Ι	Ι	F	F	F						
	Default	None	0	0.	0.	0.						

The region can either be defined as a box or a set of parts. The parts gives the flexibility of regions of any shape. The region can be extended using scale factors for the margins, see next page.



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Defining Area of Interest

<u>ID:</u>

Part set ID or ID of *DEFINE_BOX_option, depending of the setting of TYPE.

TYPE:

0: The ID given above is for a *SET_PART

1: ID above represents a box ID, specified with *DEFINE_BOX_option.

<u>Xm, Ym, Zm:</u>

Factors to scale the regions margin. This means that the limits given by the specified region can be extended, creating a buffer zone. The equations used are here illustrated for the X-direction:





• If there is a flow of new particles that gets into the model, like e.g. sand flowing through a funnel, it can be beneficial to add the particles as they are needed. This can save CPU time and make the model less complex. The keyword *DEFINE_DE_INJECTION can do this.

• The current implementation limits the source geometry to be a finite rectangular plane. The maximum and minimum sizes of the particles are given together with a mass rate inflow.

 The new generated particles can be given a velocity, which can be specified using a vector.





*DEFINE_DE_INJECTION

Card 1	1	2	3	4	5	6	7	8
Variable	PID	SID	XC	YC	ZC	XL	YL	CID
Туре	Ι	I	F	F	F	F	F	I
Default	None	None	0.0	0.0	0.0	0.0	0.0	0
Card 2								
Card	1	2	3	4	5	6	7	8
Variable	RMASS	RMIN	RMAX	VX	VY	VZ	TBEG	TEND
Туре	F	F	F	F	F	F	F	F
Default	None	None	RMIN	0.0	0.0	0.0	0.0	1.0E20



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*DEFINE_DE_INJECTION

PID:

Part ID for new generated particles.

<u>SID:</u>

Node set ID for new generated particles. This option is optional but can be needed if the new generated particles are to be in contact with an other part using *DEFINE_DE_TO_SURFACE_COUPLING, since the slave side of that (DEM) can only be given as a node or node set.

<u>XC, YC, ZC:</u>

The coordinates for the center of the finite rectangular injection plane.

<u>XL, YL:</u>

The edge lengths of the rectangular injection plane. The lengths are along the global X and Y axis, unless a local coordinate system, CID, is specified.

CID:

Local coordinate system for defining the injection plane or for the velocity vector given for the new generated particles. See next page.



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RMASS:

The mass flow rate for the new generated particles. Mass pr Unit time.

RMIN:

Minimum particles radius – used for the automatic generation routine.

RMAX:

Maximum particles radius – used for the automatic generation routine.

The RMASS will specify how much mass pr time unit and this then has to be distributed to the new generated particles. To find how many is needed, the RMIN and RMAX radiuses are used in a packing routine.



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*DEFINE_DE_INJECTION

<u>VX, VY, VZ:</u>

Vector components for the initial velocity of the new generated particles. If the local coordinate system is define, i.e. if CID has a value, the components are in this local coordinate system.

TBEG:

Birth time for the generation of new particles.

TEND:

Death time for the generation of new particles.

If the CID is given it influences both the orientation of the plane AND the initial velocity vector. Both options will then be related to the same local coordinate system.



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Chapter 3

Generating DEM Particles in LS-PrePost[®]





Overview of Packing Routine

- DEM particles can be generated and packed using LS-PrePost[®]. The packing routine is implemented from version 4.0 [Z. Han et al., 2012].
- The DEM packing is done by a pure geometrical approach. Geometrical approach described in [Y. T. Feng et. al, 2003].
- Packing density is limited to a max value of 74%.
- A bounded volume is required. This volume must be made with shells. Both triangular and quads can be used.
- Reliability for arbitrary volumes.
- Efficiency for a large number of spheres (~100 million).

y^{corp.} Copyright © 2012 by LIVERMORE SOFTWARE TECHNOLOGY CORPORATION Generating Particles p. 3.1



Overview of Packing Routine

- Based on the advance-front approach, [Y. T. Feng et. al, 2003].
- Power diagram (3D weighted Delaunay triangulation) is used for contact searching, sphere inserting, and updating neighboring connection, [J. Jerier et al., 2010].
- Linear packing speed: ~10K spheres per second per CPU (single thread).
- One must verify the normal directions of the elements, especially for inner boundaries. This is required in order to find what side of the shell is the enclosed volume. This volume is used for the filling.
- The interface is fairly simple: Select the part and run the packing engine.



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Packing Routine in LS-PrePost[®]

This will remesh the shell geometry. But it is not really a remeshing but is used to show the user how the mesh would be based on Mass, if the specified radius is applied.

Options for viewing only. The style option makes it possible to see spheres instead of points. Divs is used to discretize each sphere. The higher the more accurate the shape. The Scale option will scale the radius of the spheres but this is only visual.

Create particles.









Dreifi F • The	acking Routine in LS-PrePost Dialog Box are different options for the DEM generation.	E®
discgendialog	 Normal Run Export Outer Shell Import Spheres Run & Write .k Normal Run: Start the packing on the local C implemented. Export Outer Shell: Export the boundary shel for a remote packing engine. A file named <i>lsp</i> is created. Import Spheres: Import the packing results of by a remote packing engine. Not implemented Run & Write .k: The packing results will be exan LS-DYNA® Keyword file. Will generate par "Normal Run" mode. Further is generated the <i>lspp.mesh</i> and <i>lspp.k</i>. The function is current 	PU. Fully Is to a file <i>p.points</i> generated d yet. cported as ticles as ticles as files ly not fully
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Start the Packing Routine



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Example 2: A Hollow Sphere A Doubly-connected Domain





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Reverse the Normals





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Start the Packing Routine





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Chapter 4

Examples













Examples

Benchmark Test – SMP and MPP

 A benchmark test provide by Dr. George Laird, Predictive Engineering and Kirk A. Fraser, Roche, Canada was used to test the performance in SMP and MPP.

 The model consists of 90k discrete particles that are bonded together 9 in a configuration using bond formulation 1 (see *DEFINE_DE_BOND in Chapter 7). This gives 10k individual "clusters" of particles.

There is 10k of these "clusters"

 The particles are impacting and being transported on a conveyor band that shakes.

 The set-up this way involves both interaction between the particles and several lagrangian parts.

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Examples p. 4.5







Examples

Benchmark Test – SMP and MPP

 The benchmark model was running on a Xeon64 machine using single precision of the latest Dev version which was Dev. 77498 (11/01/2012). The benchmark for MPP and SMP were on two different platforms where the one for MPP is slightly newer and hence faster.

MPP

# CPU	Elapsed time [Sec]
1	
4	59161
12	47636
24	40827
48	27682

SMP

# CPU	Elapsed time [Sec]
1	
2	68546
3	52378
4	41869
5	43001
6	51742
7	
8	



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Examples p. 4.8



Examples

Benchmark Test – SMP and MPP

 It is seen from this particular model that SMP shouldn't use more than 4 CPU's. This common to see this performance behaviour, see e.g. [M. R. Jensen, 2009].

 It can also be seen that work still has to be done to get better scaling in MPP but this is already in progress. It should be noticed that for MPP the hybrid version was not used which perhaps could improve the performance, also default decomposition was applied. The hybrid version is special version of the MPP executable and is described in [M. R. Jensen, 2009].



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Chapter 5

Post-Processing DEM Models using LS-PrePost[®]







DATABASES

- There are two special databases related to the DEM method. One is for binary output and one is for ASCII output. They both contains information about the force on structures impacted by the DEM particles. In the next slides these options are shown in more details.
 - *DATABASE_RCFORC. This will generate a file named *demrcf* that has the same format as the *rcforc file*. Thus this is an ASCII file and all that has to be given is the output frequency at the *DATABASE_RCFORC card.
 - *DATABASE_BINARY_DEMFOR. At this card the output frequency is given but no file will be written, unless the dem option is used at the command line:

ls971 I=inputfile.k dem=deminterface

The deminterface is an arbitrary name.

 There will be no stresses for the particles, since these are rigid spheres but nodal quantities such as velocity etc. can be plotted in LS-PrePost[®] in a usual manner.

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DATABASES

Generated Files

🖢 Organize 👻 🏢 Views 👻 📷	Share 🙆 Burn			
avorite Links	Name	Date modified	Туре	Size
Documents Pictures	2_falling_balls_on_plate	7/27/2012 6:01 PM	Video Clip	3,998 KB
	ball_wall	8/30/2012 3:27 PM	K File	47 KB
	binout	8/30/2012 3:30 PM	File	1,796 KB
Music ASCII	d3dump01	8/30/2012 3:30 PM	File	983 KB
Recently Changed	d3hsp	8/30/2012 3:30 PM	File	10,232 KB
Searches	d3plot	8/30/2012 3:27 PM	File	20 KB
	d3plot01	8/30/2012 3:30 PM	File	8,220 KB
	d3plot02	8/30/2012 3:30 PM	File	84 KB
olders	demrcf	8/30/2012 3:30 PM	File	16 KB
MODELS	interfacedem	8/30/2012 3:30 PM	<u>[File]</u>	2,696 KB
BALL	Ispost	8/29/2012 1:08 PM	CFILE File	1 KB
BLAST	Ispost	8/29/2012 1:07 PM	MSG File	0 KB
DEFINE DE INJECTION	messag	8/30/2012 3:30 PM	File	32 KB
	rcforc	8/30/2012 3:28 PM	File	1 KB
DRY	spooles.res	8/30/2012 3:30 PM	VisualStudio.res.8.0	1 KB
WET -	status.out	8/30/2012 3:28 PM	OUT File	2 KB

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DATABASES

demrcf ASCII File

 The file has the force components for the interface between particles and structure defined by the *DEFINE_DE_TO_SURFACE_COUPLING. Notice that only the master surface is available.





DATABASES dem Binary File

Open

LS-PrePost 4.0 (Beta) - 18Aug2012(09:20)-64bit

File Misc. View Geometry FEM Application Settings Help

LS-DYNA Binary Plot

Ctrl+B

.

- • ×

•

The binary dem file can be loaded to LS-PrePost using the option to open contact in CL 0 0 a

interface force files. There is currently no label for it so one select the All Files	Import LS-DYNA Keyword File Ctrl+K Recent Recent Time History Files % Save Command File Ctrl+C Save As Post.db File %	fGeo Urve
option. The file name is arbitrary.	Update Interface Force File Ctrl+J Run LS-DYNA IGES File St Print Ctrl+P STEP File St Movie Ctrl+M Nastran File Ge Exit Ctrl+X Lsplot File M	olid olid oTol
Favorite Links Name Date modified Type Favorite Links Name Date modified Type Time History(d?thd:r:thf) Command File(".cfile;".ses:*.cmd) Database File(".db) Project File(".cfile;".ses:*.cmd) Database File(".cfile;".ses:*.cmd) Database File(".db) Posktop No items match you Database File(".cfile;".ses:*.cmd) Database File(".cfile;".ses:*.cmd) Desktop No items match you Nastran File(".nas:*.dat".bdf) Nastran File(".nas:*.dat".bdf) Documents Computer Carck File(d3crck*.d3crack*.f.crck) FLD curves(".fid:*.file) Pictures Xydata Background JPEG(*.jpg) Ingrid File Music Background JPEG(*.jpg) Ingrid File IGES File(".ig*s) VDA File(".vda) Dynain Binary(dynain") STL Binary(*.stt) Abagus Input File (".inp) I/D ead to the set of	Save and Exit Others Mail VIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save and Exit Image: Save and Exit NIOU Image: Save	eTol
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DATABASES dem Binary File

The content of the file is similar to what is in the interface force file that is used for contact. There are interface pressure and forces. Only the master side is written to the database. It can both be a Fringe plot or a history plot of a segment.

: History plot for selected segment.



: Fringe plot.





Chapter 6

General Comments


Time Step:

 The used time step in the simulation is the smallest one calculated for all elements in the model. The time step for the DEM particles is calculated according to the springs following the work in [Cundall et al, 1979] but is slightly modified so the used equation in LS-DYNA[®] is:

$$\Delta t_{DES} = \text{TSSFAC} \cdot 0.2 \cdot \pi \cdot \text{DESTSSFAC} \cdot \sqrt{\frac{\text{m}}{\text{K}_{\text{Spring}}}}$$

Where:

m is the mass and K_{spring} is the spring stiffness. TSSFAC is the time step scale factor given at *CONTROL_TIMESTEP and DESTSSFAC is active for bond method 2 only (BDFORM=2 at *DEFINE_DE_BOND).

 In Chapter 2 is shown the calculation of the spring stiffness which leads to the following equation for the calculated time step:

$$\Delta t_{DES} = \text{TSSFAC} \cdot 0.2 \cdot \pi \cdot \text{DESTSSFAC} \cdot \sqrt{\frac{m}{\text{K}_{\text{Bulk}} \cdot \text{RADIUS} \cdot \text{NormK}}}$$

Where:

 K_{Bulk} is the bulk modulus of the material, RADIUS the particle radius and NormK is a scale factor of the normal spring (default 0.01).

 Next, consider an example that illustrates how the time step is calculated in a simple set-up.



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General Comments p. 6.2





<u>Time Step – Example Continued:</u>

• The mass of the particle is 4.084e-6 and the radius is 0.5. The NormK and TSSFAC are not changed from default so these are 0.01 and 0.9, respectively. This gives the time step as:





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General Comments p. 6.4



Time Step - Example Continued:

• In the *d3hsp* file the 100 smallest steps are not shown since this routine not yet has the information from the DEM routine. However, the used time step matches the calculated one.

No elements with calculated timesteps	
1 t 0.0000E+00 dt 3.96E-05 flu	sh i/o buffers 11/08/12 17:28:54

 Information is also written in the *d3hsp* file about time and energies:





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<u>Time Step:</u>

 The experience so far is that this approach is working very well and there is no general recommendation to lower the time step further based on the use of the DEM particles in a model.

Boundary Conditions and Loading:

• If used, the constraint setting in *MAT_RIGID is ignored for the DEM particles. This is true both for the global and local option. Constraints will have to be set using *BOUNDARY_SPC_option or the TC and RC flags at *NODE.

 Forces can be applied by using *LOAD_NODE_option or *LOAD_BODY_option for e.g. gravity loading.

 All the *INITIAL_VELOCITY_option cards can be used to apply an initial velocity for the DEM particles.

•*BOUNDARY_PRESCRIBED_MOTION_*option* can be used as well.



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Contact:

 The DEM particles can be in contact with structural parts. This can be done either by using a general _NODES_ contact. Some of the most used contacts that works are the following contacts:

> *CONTACT_NODES_TO_SURFACE *CONTACT_AUTOMATIC_NODES_TO_SURFACE *CONTACT_ERODING_NODES_TO_SURFACE

 Note that for all contacts the radius of the sphere is taking into account and there is no method to avoid this. Further, the contact thickness scaling and setting on the contact cards are not valid for the DEM particles.

Both the _MORTAR and the _SMOOTH option can be used where valid.



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Currently it is recommended to use the standard LS-DYNA[®] contacts, e.g. *CONTACT_AUTOMATIC_NODES_TO_SURFACE.



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Damping:

 The general damping cards used in LS-DYNA[®] for mass and stiffness damping (*DAMPING_option) are ignored for the DEM particles.

 Instead is used the NDAMP and TDAMP options at *CONTROL_DISCRETE_ELEMENT. These coefficients are <u>not</u> related to the critical damping coefficient but is given as a scale factor to the velocity of the DEM particle:

 $NDAMP * V_{Normal}$

 $TDAMP * V_{Tangential}$



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General Comments p. 6.9

<u>Misc.:</u>

• There is no internal energy generated for the DEM parts. There will however be kinetic energy.

 For the bonded DEM method, *DEFINE_DE_BOND, there is not yet implemented databases to visualize the forces.

There is Part section in the *d3hsp* file doesn't show that DEM particles are used.

	partid2sectionid2	
	material id 2	
	section title	
	material title	
	material type 20	
	equation-of-state type 0	
	hourglass type 2	
	bulk viscosity type 1	
	density = 7.80000E-06	
	hourglass coefficient = $1.00000E-01$	
	guadratic bulk viscosity = 1.50000E+00	
	lineer bulk viscosity = 1.50000E100	
	= 0.00000 F - 0.2	
	element type = 0	
	eq.0: 4, 6, 8, 10-node solid element or SPH element	
	eq.1: 2-node beam or truss or 2D shell element	
	eq.2: 3, 4-node membrane/shell or 2D continuum element	
I STC	eq.3: 8-node thick shell element	
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<u>Misc.:</u>

 However, the element information is shown for the DEM as it is for other elements:

> discete sphere elements node part mass inertia radius 9999 2 4.084E-06 4.084E-07 5.000E-01 99999 2 4.084E-06 4.084E-07 5.000E-01 109998 4 4.084E-06 4.084E-07 5.000E-01 199998 4 4.084E-06 4.084E-07 5.000E-01

• The timing spend for the DEM particles self contact is show in the timing table at the bottom of the *d3hsp* file.

Timing informatio CPU(second	n s) %CPU Clock(seconds) %	Clock
Rigid Bodies 2.2915E-0 DEM 1.3710E-0	L 3.28 8.8416E-02 L 1.96 1.8933E-02	3.26 0.70
DEM self contact 1.3672E-0 Other 1.3435E+0	L 1.96 1.8700E-02 D 19.25 5.0957E-01	0.69
Totals 6.9806E+0) 100.00 2.7099E+00 1	00.00



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General Comments p. 6.11



Chapter 7

Sphere Bond Models



Introduction

- So far DEM particles and their interaction has been considered. This has been done by a special interaction (contact) between the particles. They are all independent particles.
- Bond models to "bond" the particles have been developed. This makes modeling of a Continuum possible. It can also be seen as a system of particles.





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Draits

Introduction

- All particles are linked to their neighboring particles through bonds which are independent from the DEM model.
- There are currently two different bound methods implemented in LS-DYNA[®].
 - A method where material parameters are given at the specific DEM card. Failure of the bonds can also be given as stress values.
 - A more complex method where a linear bond is generated and the strength is taken from the material cards of the particles. Further, fracture release rates and an initial fracture plane can be given to model initialization and growth of fracture.
- The keyword related to specifying the bonds is *DEFINE_DE_BOND that will be described in more details later.





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Mechanical Behaviors of Bonds

- Every bond is subjected to:
 - Compression
 - Stretching
 - Shearing

· Drie

- Bending
- Twisting
- The bonds can break depending on the settings and method given at the *DEFINE_DE_BOND card.





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Card 1	1	2	3	4	5	6	7	8
Variable	SID	STYPE	BDFORM	IDIM				
Туре	Ι	Ι	Ι	Ι				
Default	None	0	1	3				

<u>SID:</u>

Node set or node for the discrete particles.

STYPE:

Type setting for SID.

EQ.0: Node set ID is given at SID.

EQ.1: A single node ID is given at SID.



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BDFORM:

Specification of the bond formulation between the particles. Currently, two options are available which both are under development and considered in the first beta phase.

EQ.1: Parallel-bond.

EQ.2: Linear elastic bond formulation.

The parallel-bond method is the simplest method and requires normal and shear stiffness to be given. A failure criteria given by a single parameter given for either shear or normal stress is implemented to fail the bond.

Linear elastic bond formulation is used to model fracture for brittle materials. This is done by applying a failure criteria that is based on fracture energy release rates.

IDIM:

Space dimension for the particles used for BDFORM=2.

EQ.2: 2D plane stress problems.

EQ.3: 3D problems.



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Card 2 if B	DFORM=1							
Card	1	2	3	4	5	6	7	8
Variable	PBN	PBS	PBN_S	PBS_S	SFA	ALPHA		
Туре	F	F	F	F	F	F		
Default	None	None	None	None	1.0	0.0		





Card 2 if BDFORM=2											
Card	1	2	3	4	5	6	7	8			
Variable	PBK_SF	PBS_SF	FENRGK	FENRGS	BONDR						
Туре	F	F	F	F	F						
Default	1.0	1.0	None	None	None						



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BDFORM=2: Define Bond Properties Based on Material Properties

- Bulk Modulus : K
- Shear Modulus: G
- Fracture Energy Release Rate: G_c
- Influence Distance: *R*
 - Smaller R for less bonds and smaller time steps.
 - Larger R for more bonds and larger time steps.
 - Choices of R are dependent on the bar wave speed and the time step.
 - Macro-behaviors are NOT dependent on R.









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Features Under Development

- The Bond Models are still under development but both formulations have successfully been applied in customers models.
- For the BDFORM=2 formulation work is done on:
 - Supporting Equation of State (EOS).
 - Non-linear material model. This is already in beta testing.
- For both formulations MPP implementation is currently under development and being beta tested.



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Chapter 8

Interaction Between DEM Particles and Structure





Introduction

- The DEM particles can get in contact with a structural part, e.g. particles on a conveyor belt.
- There are two ways to specify this interaction:
 - Using any of the "one way" contacts, which are the contacts with _NODES_ in their names. E.g. one can use the *CONTACT_AUTOMATIC_NODES_TO_SURFACE contact, where the slave side is the DEM particles. Notice that the SURFACE_TO_SURFACE contacts can not be used, since no segments can be generated based on the particles. Thus, SOFT=2 can't be applied either. The use of a regular contact is similar to the approach often used for SPH particles in contact with a structure.
 - Applying the new DEM specific keyword: *DEFINE_DE_TO_SURFACE_COUPLING. Here damping and friction can be given, together with velocity curves for the particles in contact. The keyword was introduced to handle certain shortcomings of the regular contact and it is recommended to use this option over the use of e.g. the *CONTACT_AUTOMATIC_NODES_TO_SURFACE contact.



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Using Standard Contact Specification

 If a standard contact is used, then the _NODES_TO_SURFACE option is to be used.

*COI	NTACT_AUT	OMATIC_NODE	ES_TO_SURF	ACE					
\$#	ssid	msid	sstyp	mstyp	sboxid	mboxid	spr	mpr	
	2	1	4	3	0	0	0	0	
\$#	fs	fd	dc	VC	vdc	penchk	bt	dt	
	0.000	0.000	0.000	0.000	0.000	0	0.000	0.000	
\$#	sfs	sfm	sst	mst	sfst	sfmt	fsf	vsf	
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	

- In the shown contact, the slave side (SSID) is given by node set number 2 (SSTYP=4). This is a set of DEM particles. The master side (MSID) is given by part number 1 since MSTYP is set to 3. No friction nor other options are used.
- The friction could be set with FS and FD as can contact damping be applied using the VDC flag. Scaling of the contact stiffness can also be done (SFS, SFM). However, the radius of the particles can't be changes with SST nor SFST.
- The forces on the master side can be found using the *rcforc* file, specified by applying the *DATABASE_RCFORC keyword.



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.STC

Using DEM Coupling Card *DEFINE_DE_TO_SURFACE_COUPLING

Card 1	1	2	3	4	5	6	7	8
Variable	SLAVE	MASTER	STYPE	MTYPE				
Туре	Ι	Ι	Ι	Ι				
Default	0	0	0	0				
Card 2	1	2	3	4	5	6	7	8
Variable	FricS	FricD	DAMP	BSORT	LCVx	LCVy	LCVz	
Туре	F	F	F	Ι	Ι	Ι	Ι	
Default	0	0	0	100	0	0	0	

The variables are described on the next slides.

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Using DEM Coupling Card *DEFINE_DE_TO_SURFACE_COUPLING

SLAVE:

Node ID or node set ID depending on the setting of STYPE. This is for the slave side and must be the DEM particles.

MASTER:

Part ID or part set ID depending on the setting of MTYPE. This must be the structure and specifies the master side of the coupling.

STYPE:

EQ.0: Specify a *SET_NODE ID for SLAVE.

EQ.1: Give a NODE ID (NID) for SLAVE.

MTYPE:

EQ.0: Specify a *SET_PART ID for MASTER.

EQ.1: Give a part ID for MASTER.

FricS:

Translational friction coefficient.



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Using DEM Coupling Card *DEFINE DE TO SURFACE COUPLING

FricD: Rolling friction coefficient.

DAMP:

Damping coefficient.

BSORT:

Number of cycles between bucket sorting. Default is 100 cycles. This is equivalent to the BSORT specified for contacts using *CONTACT_.

LCVx:

Loadcurve ID (LCID) gives the surface velocity in X-direction $(V_x(t))$.

<u>LCVy :</u>

Loadcurve ID (LCID) gives the surface velocity in Y-direction $(V_y(t))$.

LCVz :

Loadcurve ID (LCID) gives the surface velocity in Z-direction $(V_z(t))$.



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Using DEM Coupling Card

 The *DEFINE_DE_TO_SURFACE_COUPLING card has entity specification similar to the ones for regular contacts as seen on the previous slides.

*DEFINE_DE_TO_SURFACE_COUPLING								
\$#	slave 4	master 3	stype 0	mtype 1				
\$#	fricS	FricD	damp	bsort	lcvx	lcvy	lcvz	

- In the shown coupling cards, the slave side (SLAVE) is given by node set number 4 (STYPE=0). This is a set of DEM particles. The master side (MASTER) is given by part number 3 since MTYPE is set to 1. No friction nor other options are used.
- The friction could be set with FricS and FricD as can contact damping be applied using the DAMP flag.
- The coupling takes into account the radius of the particle and adds half the shell thickness on each side of the master segments as contact thickness. This is similar to the _AUTOMATIC_ standard contacts.
- The forces on the master side can be found using the *demrcf* file, specified by applying the *DATABASE_RCFORC keyword. This is described in more details in Chapter 5.



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Using DEM Coupling Card *DEFINE_DE_TO_SURFACE_COUPLING - LCVi

 The LCVx, LCVy and LCVz is used to apply a prescribed velocity to the DEM particles that would represent the velocity of the master side of the coupling. This means that in e.g. a conveyor simulation, the belt doesn't need to have a prescribed motion but the belt velocity can be given using one of the three load curves the coupling part. That is a significant simplification when building the model.







Example of Interaction

- As an example consider one particle placed on a plate made of shells. The plate is fully constrained around the boundary. A gravity load is applied to the ball and the force in the interface can be calculated and compared to the force in the interface force files.
- The force is slowly applied and the maximum value is reached with the first 20% of the simulation time.



The density of the particle is set to 7.8e-6ton/mm³ which is rather heavy. The radius is specified to be 0.5mm. The volume is given by : $V=4/3 * \P * r^3$ which gives a volume of 0.523mm³ and a mass of 4.084e-6ton. Since the force in the interface is F=M * A one gets that the force is F=(4.084e-6 * 9820)N = 0.04N



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Chapter 9

Coupling to New Blast Particle Method



Coupling to new Particle Blast Method

• The card to be used is *PARTICLE_BLAST. This option is under development and not released to the public yet. It is expected it should be ready within the next 6-12 months depending on the beta testing results.

Currently the option is only implemented in SMP, not in MPP.
However, this will hopefully be done by the end of the year.

 The method can be coupled with particles modeling sand and be used for simulations of buried land mines. This is very important since this type of simulation has been difficult to model correctly using *LOAD_BLAST since that is for developed for air blasts. And the ALE method is very CPU intensive to use and requires a good understanding of the ALE modeling technique.



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Chapter 10

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