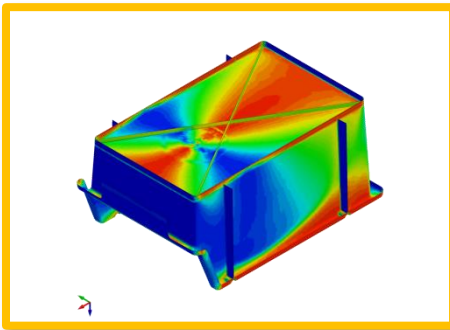
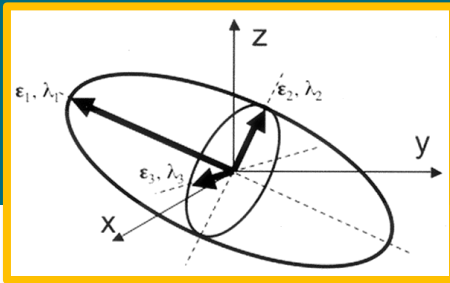


# Recent Enhancements on Short-Fiber Reinforced Plastics (SFRP) Modeling in LS-DYNA



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DYNAmore GmbH, Stuttgart

10<sup>th</sup> European LS-DYNA Conference  
Würzburg, Ger, 14-Jun-2015

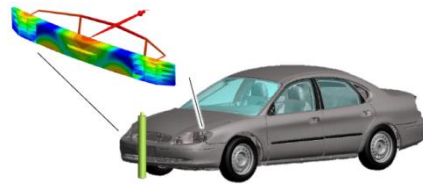
**DYNA**  
MORE



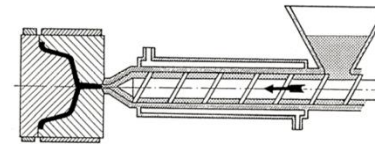
# Agenda

- **Introduction**
- **Homogenization procedures for SFR materials**
  - Eff. properties of UD-fiber reinforced composites
  - Orientation averaging
    - Closure approximation
- **Data Mapping (DYNAmap)**
- **Numerical application (proof of concept)**

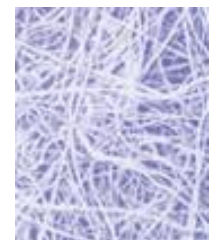
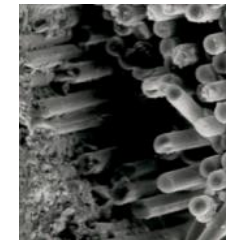
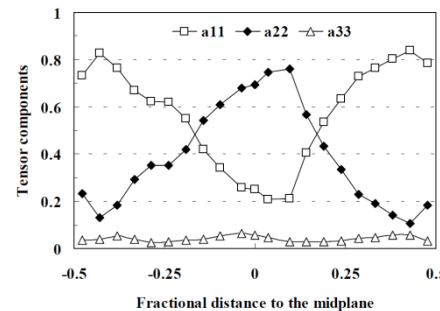
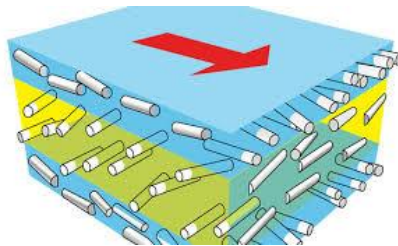
- Short fiber reinforced plastics (SFRP) are widely used within different industries



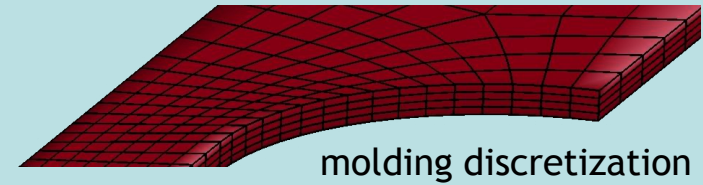
- Infiltration processes are used to manufacture all different kinds of SFRP components



- SFR materials show a strong anisotropic material behavior which caused by rather randomly orientated fibers



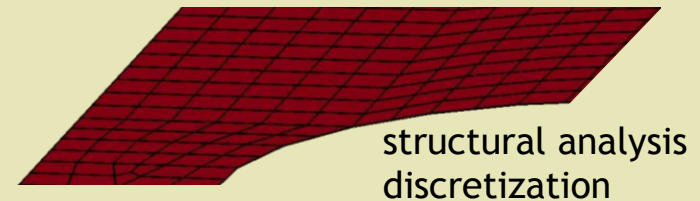
- Injection molding simulation:  
Fiber orientation / fiber content



- Mapping of fiber orientation tensor / fiber content

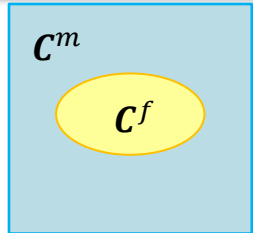
mapping

- Computation of homogenized  
elastic material properties

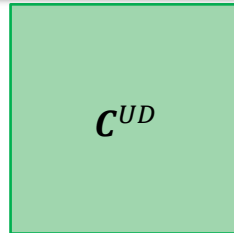


- Structural analysis using homogenized anisotropic material properties

## 1<sup>st</sup> step: Effective properties of unidirectional (UD) composite



fiber in matrix



equivalent homogeneous medium

Analytical homogenization:

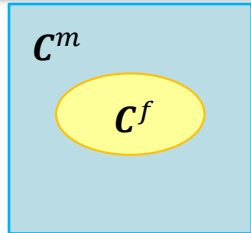
e.g. Eshelby + Mori-Tanaka,  
self-consistent model, bounds,...

## Eshelby's tensor (in isotropic medium)

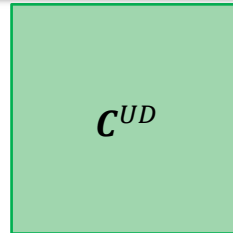
### ■ Analytical solutions exist for:

- Spherical inclusions ( $a = b = c$ )
- Ellipsoidal inclusions ( $a > b > c$ )
- Elliptic cylinders ( $c \rightarrow \infty$ )
- Flat ellipsoids ( $a > b \gg c$ )
- Penny-shaped inclusions ( $a = b \gg c$ )
- and some more...

## 1<sup>st</sup> step: Effective properties of unidirectional (UD) composite



fiber in matrix



equivalent homogeneous medium

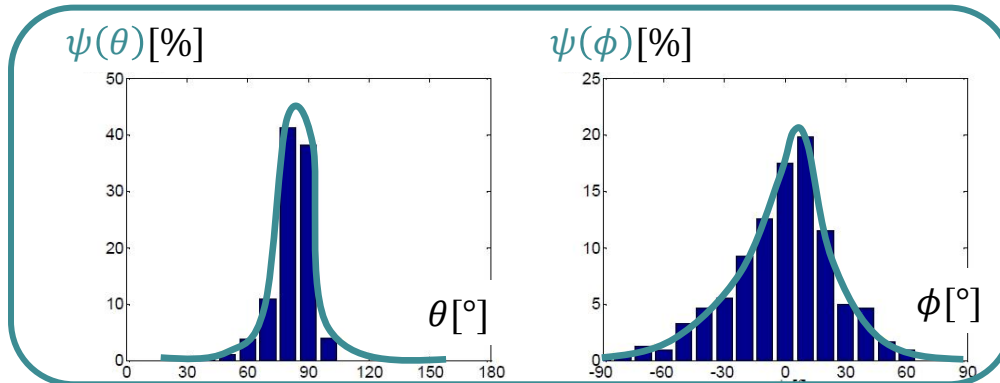
Analytical homogenization:

e.g. Eshelby + Mori-Tanaka,  
self-consistent model, bounds,...

## 2<sup>nd</sup> step: Effective properties of unaligned composite: Orientation averaging

- as function of fiber orientation distribution function  $\psi$ :

$$\mathbf{C} = \int \mathbf{C}^{UD}(\theta, \phi) \psi(\theta, \phi) d\Omega$$



- as function of fiber orientation tensors  $\mathbf{a}(p), \mathbf{A}(p)$ :

$$\mathbf{C} = f(\mathbf{C}^{UD}, \mathbf{a}(p), \mathbf{A}(p))$$

# 1<sup>st</sup> step: UD-homogenization

e.g. using Mori & Tanaka scheme:

1. Evaluate geometry dependent Eshelby tensor  $E$
2. Evaluate strain concentration tensor  $A$  using Eshelby tensor  $E$ :  
(With fiber volume fraction  $v_f$ , matrix compliance  $S^m$ )

$$A^{Eshelby} = \left( I + E : S^m : (C^f - C^m) \right)^{-1}$$

$$A^{MT} = A^{Eshelby} : \left( (1 - v_f)I + v_f A^{Eshelby} \right)^{-1}$$

3. Evaluate Unidirectional Stiffness Matrix:

$$C^{UD} = C^m + v_f (C^f - C^m) : A^{MT}$$

# 1<sup>st</sup> step: UD-homogenization

e.g. using Halpin-Tsai-Equations:

## ■ Derived from self-consistent model

$$\frac{M}{M_M} = \frac{1 + \xi \eta V_f}{1 - \eta V_f} \quad \text{with} \quad \eta = \frac{M_r - 1}{M_r + \xi}, \quad M_r = \frac{M_f}{M_m}$$

$M$	$M_f$	$M_m$	$\xi$
$E_{11}$	$E_f$	$E_m$	$2a$
$E_{22}$	$E_f$	$E_m$	$2$
$G_{12}$	$G_f$	$G_m$	$1$
$G_{13}$	$G_f$	$G_m$	$\frac{K_m}{G_m} / \left( \frac{K_m}{G_m} + 2 \right)$

$$a = \frac{L}{d} \quad \text{- Fiber aspect ration, L = fiber length, d = fiber diameter}$$

$$\vartheta_{21} = V_f \vartheta_f + V_m \vartheta_m$$

e.g. using Tandon-Weng Equations:

- Based on Eshelby's solution of an ellipsoidal inclusion in an infinite matrix and Mori-Tanaka's average stress

$$\frac{E_{11}}{E_m} = \frac{1}{1 + V_f \frac{A_1 + 2 \vartheta_m A_2}{A_6}}$$

$$\frac{E_{22}}{E_m} = \frac{1}{1 + V_f \frac{-2 \vartheta_m A_3 + (1 - \vartheta_m) A_4 + (1 + \vartheta_m) A_5 A_6}{2 A_6}}$$

$$\frac{G_{12}}{G_m} = 1 + \frac{V_f}{\frac{\mu_m}{\mu_f - \mu_m} + 2 V_m S_{1212}}$$

$$\frac{G_{23}}{G_m} = 1 + \frac{V_f}{\frac{\mu_m}{\mu_f - \mu_m} + 2 V_m S_{2323}}$$

$$\vartheta_{21} = \frac{\vartheta_m A_6 - V_f (A_3 - \vartheta_m A_4)}{A_6 + V_f (A_1 - 2 \vartheta_m A_2)}$$

$$\frac{K_{23}}{K_m} = \frac{(1 + \vartheta_m)(1 - 2 \vartheta_m)}{1 - \vartheta_m(1 + 2 \vartheta_{21}) + V_f \frac{2(\vartheta_{21} - \vartheta_m) A_3 + [1 - \vartheta_m(1 + 2 \vartheta_{21})] A_4}{A_6}}$$



■ with:

$$A_1 = D_1(B_4 + B_5) - 2B_2$$

$$A_2 = (1 + D_1)B_2 - (B_4 + B_5)$$

$$A_3 = B_1 - D_1B_3$$

$$A_4 = (1 + D_1)B_1 - 2B_3$$

$$A_5 = \frac{1 - D_1}{B_4 - B_5}$$

$$A_6 = 2B_2 B_3 - B_1(B_4 + B_5)$$

$$B_1 = V_f D_1 + D_2 + V_m(D_1 S_{1111} + 2S_{2211})$$

$$B_2 = V_f + D_3 + V_m(D_1 S_{1122} + S_{2222} + S_{2233})$$

$$B_3 = V_f + D_3 + V_m(S_{1111} + (1 + D_1)S_{2211})$$

$$B_4 = V_f D_1 + D_2 + V_m(S_{1122} + D_1 S_{2222} + S_{2233})$$

$$B_5 = V_f + D_3 + V_m(S_{1122} + S_{2222} + D_1 S_{2233})$$

$$D_1 = 1 + 2 \frac{\mu_f - \mu_m}{\lambda_f - \lambda_m}$$

$$D_2 = \frac{\lambda_m - 2\mu_m}{\lambda_f - \lambda_m}$$

$$D_3 = \frac{\lambda_m}{\lambda_f - \lambda_m}$$

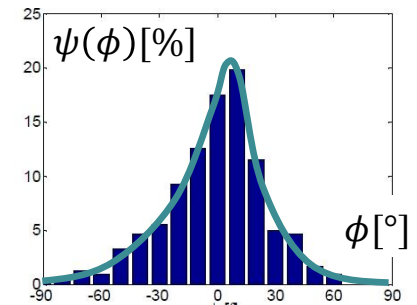
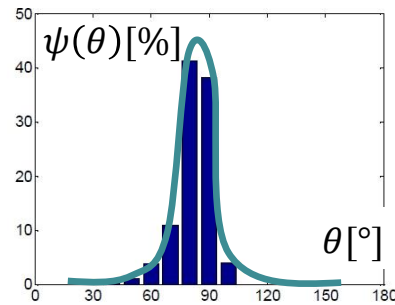
$\mu_f, \mu_m, \lambda_f, \lambda_m$  - Lamé constants

$S_{ijkl}$  - non-vanishing components of the Eshelby's tensor

## 2<sup>nd</sup> step: Orientation averaging

### ➤ Fiber orientation distribution function $\psi$

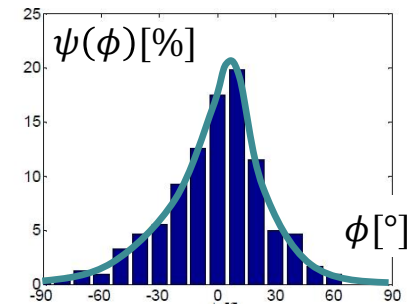
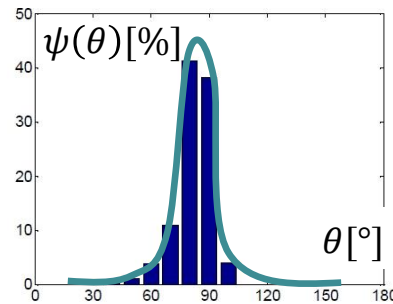
- probability to have fibers in a certain direction



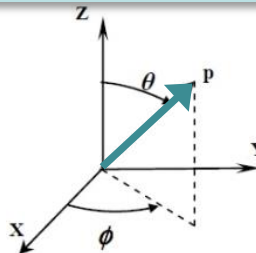
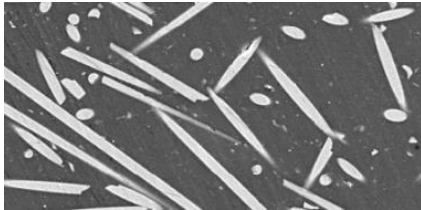
## 2<sup>nd</sup> step: Orientation averaging

### ➤ Fiber orientation distribution function $\psi$

- probability to have fibers in a certain direction



### ➤ Fiber orientation vector $p(\theta, \phi)$ : unit vector in fiber direction

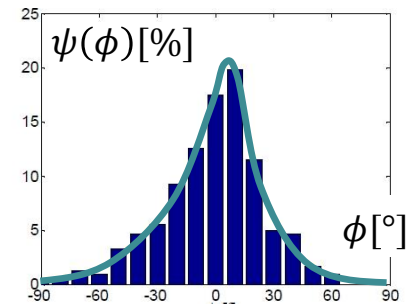
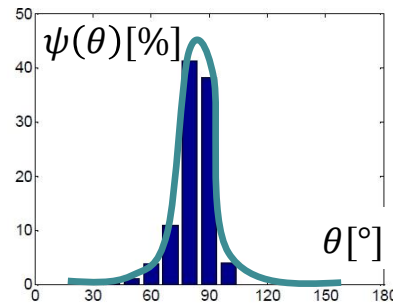


$$p = \begin{bmatrix} \sin\theta \cos\phi \\ \sin\theta \sin\phi \\ \cos\theta \end{bmatrix}$$

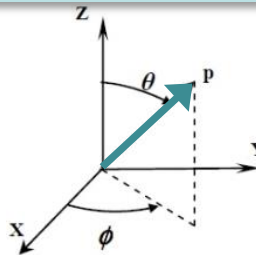
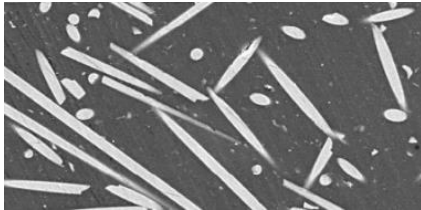
## 2<sup>nd</sup> step: Orientation averaging

### ➤ Fiber orientation distribution function $\psi$

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### ➤ Fiber orientation vector $p(\theta, \phi)$ : unit vector in fiber direction



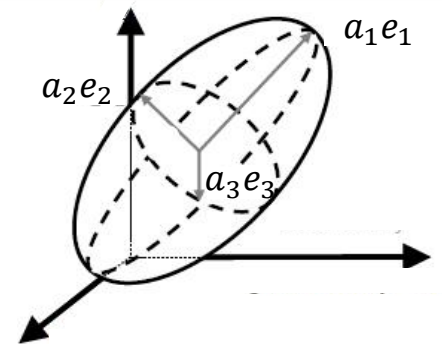
$$p = \begin{bmatrix} \sin\theta \cos\phi \\ \sin\theta \sin\phi \\ \cos\theta \end{bmatrix}$$

### ➤ Fiber orientation tensor (2<sup>nd</sup> and 4<sup>th</sup> order)

$$a_{ij} = \int p_i p_j \psi(\theta, \phi) d\Omega \quad \text{and} \quad a_{ijkl} = \int p_i p_j p_k p_l \psi(\theta, \phi) d\Omega$$

Eigenvectors → principal directions of the material

Eigenvalues → Orientation Distribution Probability



### Orientation averaging ... Advani & Tucker [1987]

- Starting point: 
$$C_{ijkl} = \int C^{UD}_{ijkl} \psi(\theta, \phi) d\Omega$$

- Advani & Tucker:

$$C_{ijkl} = B_1 A_{ijkl} + B_2 (a_{ij} \delta_{kl} + a_{kl} \delta_{ij}) + B_3 (a_{ik} \delta_{jl} + a_{il} \delta_{jk} + a_{jl} \delta_{ik} + a_{jk} \delta_{il}) + B_4 (\delta_{ij} \delta_{kl}) + B_5 (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})$$

with:

For  $x_1$  being the direction of fibers of  $C^{UD}$

$$B_1 = C^{UD}_{1111} + C^{UD}_{2222} - 2C^{UD}_{1122} - 4C^{UD}_{1212}$$

$$B_2 = C^{UD}_{1122} + C^{UD}_{2233}$$

$$B_3 = C^{UD}_{1212} + 1/2 (C^{UD}_{2233} - C^{UD}_{2222})$$

$$B_4 = C^{UD}_{2233}$$

$$B_5 = 1/2 (C^{UD}_{2222} - C^{UD}_{2233})$$

## 2<sup>nd</sup> step: closure approximation



- 2<sup>nd</sup> order orientation tensor available: Measured or output of molding simulation
- 4<sup>th</sup> order orientation tensor needed: Closure approximation

- Linear closure approximation:

$$A^L_{ijkl} = -\frac{1}{35}(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \frac{1}{7}(a_{ij}\delta_{kl} + a_{ik}\delta_{jl} + a_{il}\delta_{jk} + a_{kl}\delta_{ij} + a_{jl}\delta_{ik} + a_{jk}\delta_{il})$$

- quadratic closure approximation

$$A^Q_{ijkl} = a_{ij}a_{kl}$$

- hybrid closure approximation  
(mixture between linear and quadratic)

$$A^H_{ijkl} = (1 - f)A^L_{ijkl} + fA^Q_{ijkl} \qquad \text{e.g. } f = \frac{3}{2}a_{ij}a_{ij} - \frac{1}{2}$$

- Variety of fitted closure approximations

1<sup>st</sup> step:

- Eshelby tensor  $E$  (for unidirectional 1 fiber in matrix)
- Strain concentration tensor  $A$  (for unidirectional 1 fiber in matrix) if Mori-Tanaka
- Uni-directional elasticity tensor (Mori-Tanaka or directly from Literature):  $C^{UD}$

2<sup>nd</sup> step:

- Closure approximation orientation tensor 4<sup>th</sup> order  $a_{ij} \rightarrow A_{ijkl}$

- Orientation averaging of elasticity tensor:

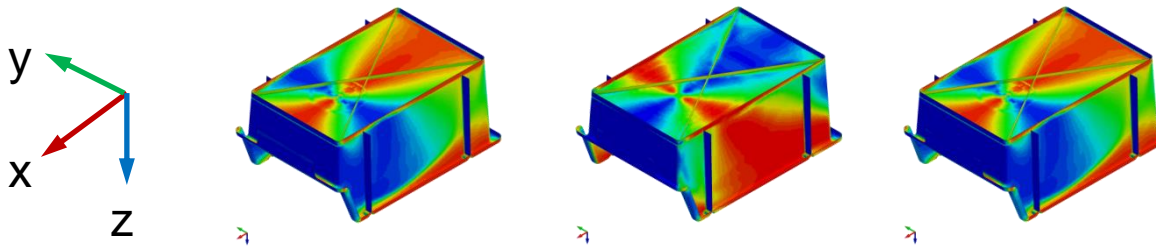
$$C_{ijkl} = B_1 A_{ijkl} + B_2 (a_{ij} \delta_{kl} + a_{kl} \delta_{ij}) + B_3 (a_{ik} \delta_{jl} + a_{il} \delta_{jk} + a_{jl} \delta_{ik} + a_{jk} \delta_{il}) + B_4 (\delta_{ij} \delta_{kl}) + B_5 (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})$$

Load step loop

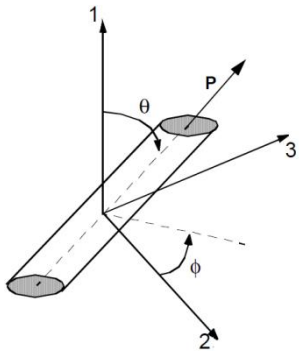
Composite (overall) strain increment  $\Delta \epsilon^c$

Composite stress increment  $\Delta \sigma^c = C \Delta \epsilon^c$

- In order properly consider the fiber orientation distribution within FE analysis, a proper data mapping has to be established.



- Output data from moldflow simulations usually contain fiber orientation tensors or the first two eigenvectors and corresponding eigenvalues

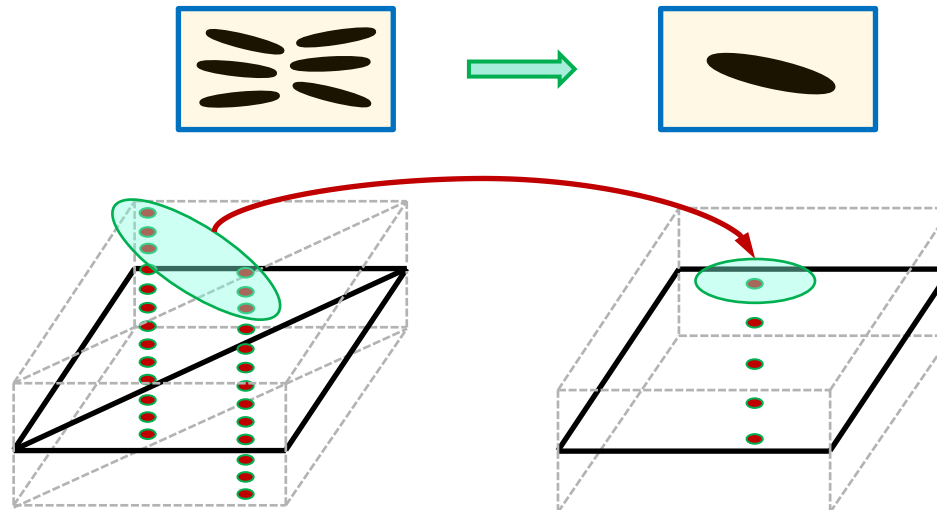


$$p = \begin{pmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{pmatrix} \quad a_{ij} = \frac{1}{n} \sum_{k=1}^n a_{ij}^k = \frac{1}{n} \left( \sum_{k=1}^n p_i^k p_j^k \right) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

- Orientation data will be initialized using \*INITIAL\_STRESS\_SOLID - card

## ■ Challenges for a proper data mapping:

- Process simulations are often performed in different unit systems than servicability analysis
- Different meshes are used during infiltration simulation and crushing analysis
- Number of integration points differs between infiltration simulation (e.g. 13) and crushing analysis (e.g. 5)
- Proper averaging techniques have to be established in order to properly consider tensor orientation and tensor shape during the data mapping



# Data Mapping (DYNAmap)

```
$#-----  
$# Main mapping definition  
$#-----  
DYNAMAP=MOLDFLOW-SOLID  
$#-----  
$# Activate transformation  
$#-----  
TRANSFORMATION=NO  
SourceUnitSystem=kg-m-s  
TargetUnitSystem=kg-mm-ms  
$#-----  
$# In- and output meshes  
$#-----  
SourceFile=MF_MESH.key  
TargetFile=Stapelbox.key  
MappingResult=Stapelbox mapped.key  
OrientationFile=DYNA.xml  
ViewMoldFlowData=Stapelbox_res.key  
$#-----  
$# Target - Options  
$#-----  
ETYP=1  
MapStress=YES  
$#-----  
$# Mapping-Options  
$#-----  
ALGORITHM=ClosestPoint  
$#-----  
$# END-OF-FILE  
$#-----
```

Main mapping command

Usually, unit-systems in moldflow-  
and FE-mesh are different!

Load Moldflow \*.xml-File

For visualization purposes in LS-  
PrePost.

Flag to choose element type

Necessary to generate  
\*INITIAL\_STRESS\_SOLID - cards

So far, tensor averaging is not  
implemented, results are mapped on  
a simple „closest point“ procedure

- First test simulations are preformed on a stackable box (fiber volume fraction 16.5%, aspect ratio = 25)

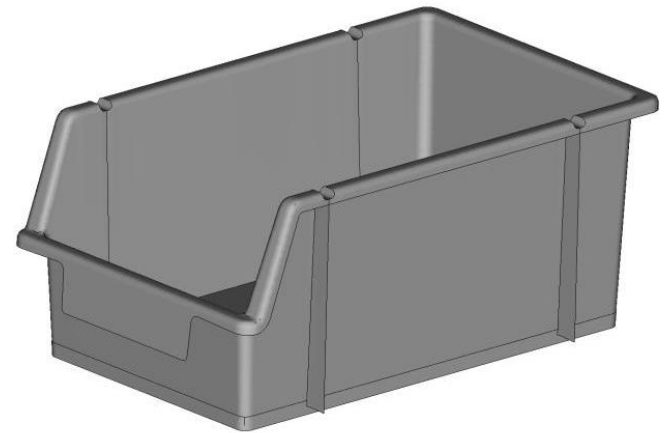
- Assumed material parameters:

$E_f$	$\nu_f$	$\rho_f$	$E_m$	$\nu_m$	$\rho_m$
74000.0 MPa	0.25	2.6 E-09 t/mm <sup>3</sup>	5100.0 MPa	0.35	1.2E-09 t/mm <sup>3</sup>

- Spherical impactor used ( $\varnothing$  19.6 mm,  $v = 1.0$  mm/ms, 5.2 kg )

- Homogenization using Halpin-Tsai equations

- Linear closure approximation

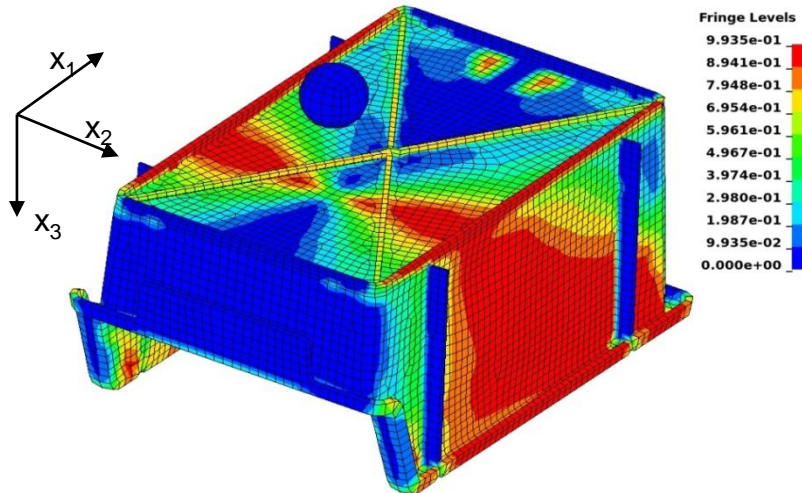


- For a first trial, Halpin-Tsai homogenization equations together with linear closure approximation show an acceptable accuracy compared to experiments

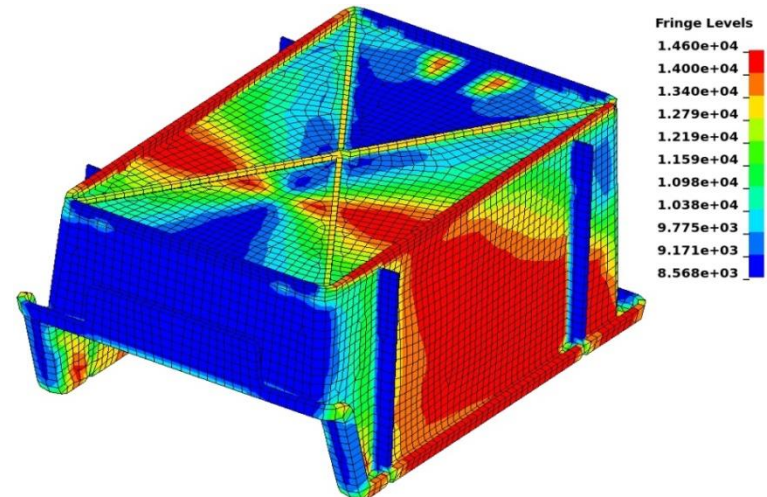
	$E_1$	$E_2 = E_3$
LS-DYNA, Halpin-Tsai, linear closure approximation	11600 MPa	7900 MPa
Dray et al. [2], experimental results	11800 MPa	7220 MPa

- Homogenization shows reasonable results for the components of the elasticity tensor compared to the fiber orientation tensor data

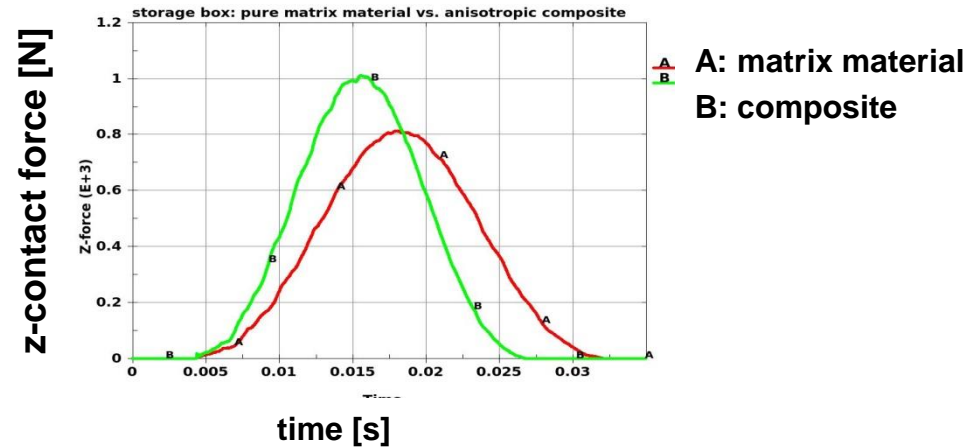
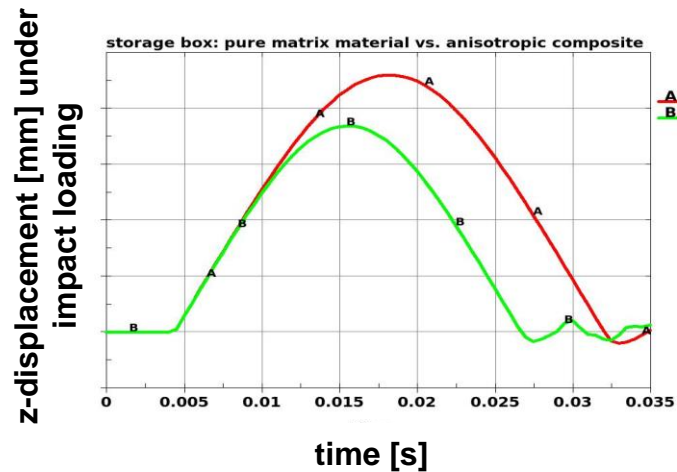
Fiber orientation tensor component  $a_{11}$ :



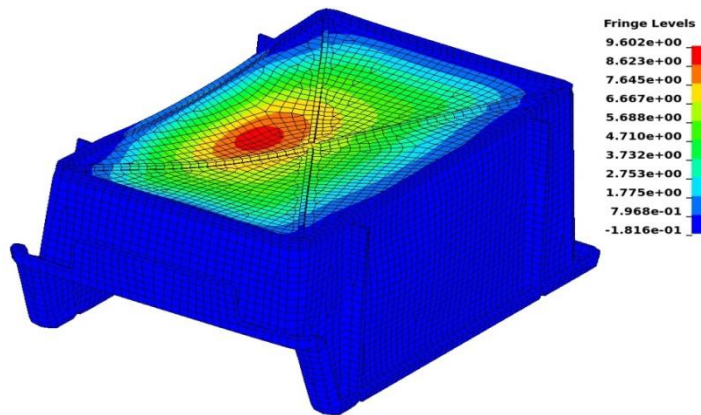
Elasticity tensor component  $C_{1111}$  of homogenized equivalent medium:



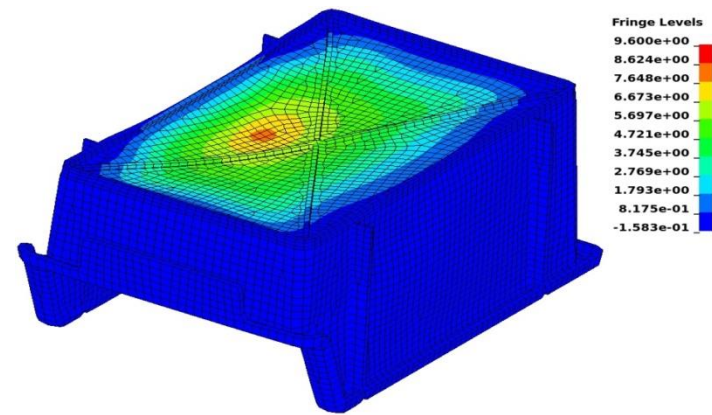
- Simulations using a pure matrix material compared to a consideration of the fiber inclusions show reasonable results as well.



pure matrix material:  $u_z$  [mm], time=0.016 s :



Composite:  $u_z$  [mm], time=0.016 s :





- A new material model for short fiber reinforced plastic materials is being introduced into LS-DYNA, considering various homogenization techniques and closure approximations
- Besides that, proper data mapping routines are being established in a new tool called DYNAmap (beta-status)
- So far, only a linear-elastic response is being implemented for solid elements only
- Material model will be extended for elasto-plastic behavior as well as thick shell and shell elements.
- Data mapping will be improved for a proper consideration of tensor direction and shape during averaging

- FIN -

