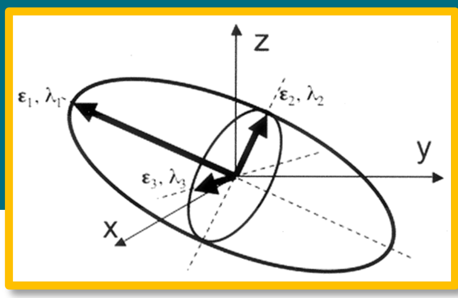

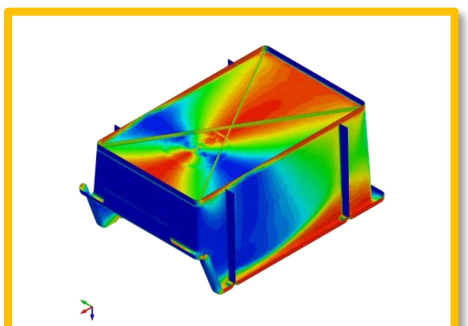

Recent Enhancements on Short-Fiber Reinforced Plastics (SFRP) Modeling in LS-DYNA

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10th European LS-DYNA Conference

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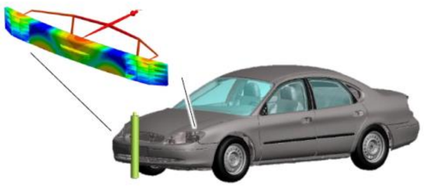
Agenda

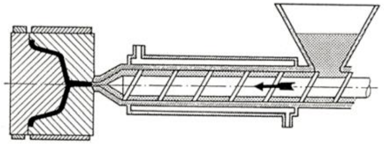
- **Introduction**
- **Homogenization procedures for SFR materials**
  - Eff. properties of UD-fiber reinforced composites
  - Orientation averaging
    - Closure approximation
- **Data Mapping (DYNAmap)**
- **Numerical application (proof of concept)**

Introduction

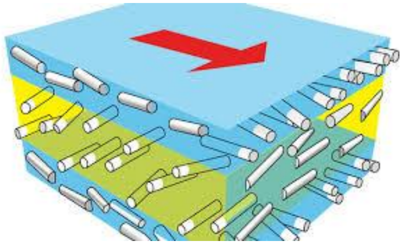
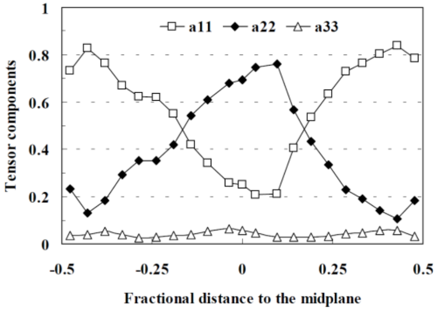
- **Short fiber reinforced plastics (SFRP) are widely used within different industries**

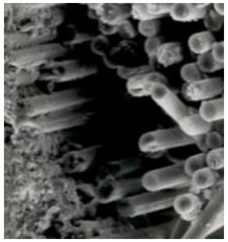



- **Infiltration processes are used to manufacture all different kinds of SFRP components**



- **SFR materials show a strong anisotropic material behavior which caused by rather randomly orientated fibers**




Process chain for SFRP

- **Injection molding simulation:**

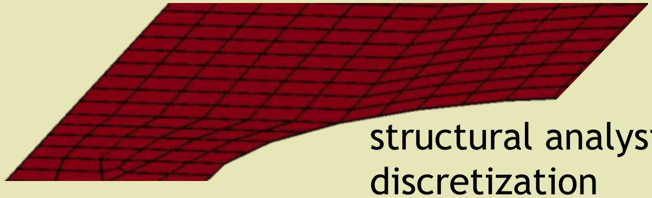
  Fiber orientation / fiber content

molding discretization

- **Mapping of fiber orientation tensor / fiber content**

mapping

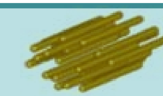
- **Computation of homogenized elastic material properties**

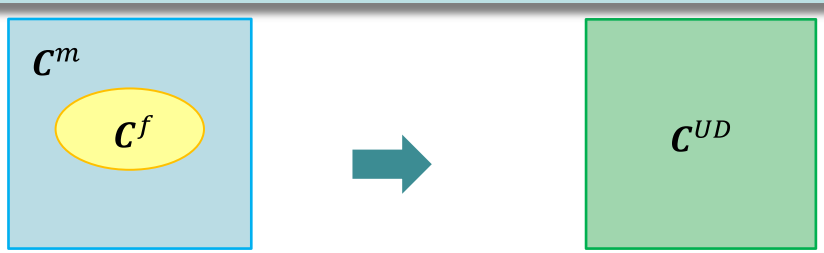


structural analysis discretization

- **Structural analysis using homogenized anisotropic material properties**

1^{st} step: Effective properties of unidirectional (UD) composite





fiber in matrix — equivalent homogeneous medium

Analytical homogenization:

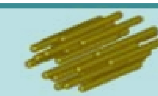
e.g. Eshelby + Mori-Tanaka, self-consistent model, bounds,…

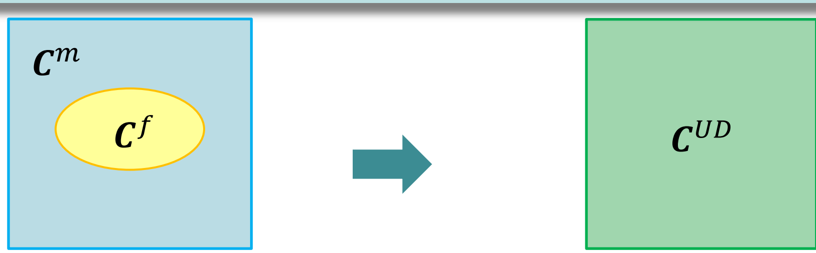
Eshelby's tensor (in isotropic medium)

- **Analytical solutions exist for:**
  - Spherical inclusions (a = b = c)
  - Ellipsoidal inclusions (a > b > c)
  - Elliptic cylinders (c -> ∞)
  - Flat ellipsoids (a > b >> c)
  - Penny-shaped inclusions (a = b >> c)
  - and some more…

Homogenization

1st step: Effective properties of unidirectional (UD) composite





Analytical homogenization:

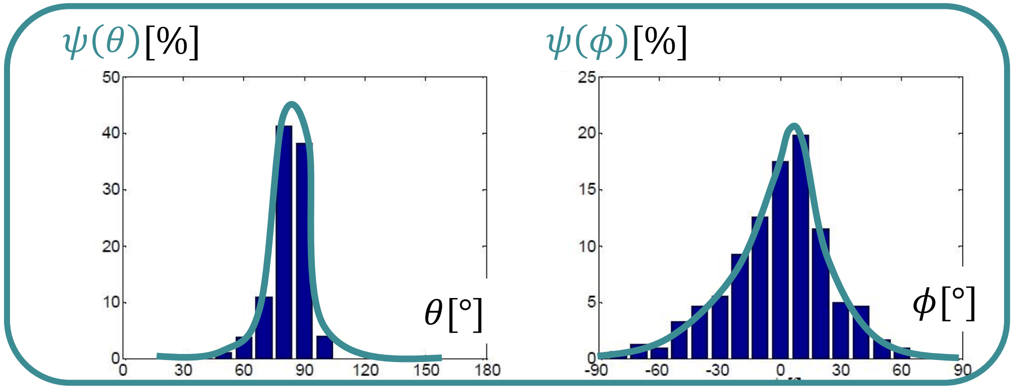
e.g. Eshelby + Mori-Tanaka,
self-consistent model, bounds,…

fiber in matrix — equivalent homogeneous medium

2nd step: Effective properties of unaligned composite: Orientation averaging

- as function of fiber orientation distribution function ψ:

$$C = \int C^{UD}(\theta, \phi)\ \psi(\theta, \phi)\ d\Omega$$



- **as function of fiber orientation tensors $a(p), A(p)$:**

$$C = f(C^{UD}, a(p), A(p))$$

1st step: UD-homogenization

e.g. using Mori & Tanaka scheme:

1. Evaluate geometry dependent Eshelby tensor $\boldsymbol{E}$

2. Evaluate strain concentration tensor $\boldsymbol{A}$ using Eshelby tensor $\boldsymbol{E}$:
(With fiber volume fraction ν_f, matrix compliance $\boldsymbol{S}^m$)

$$\boldsymbol{A}^{Eshelby} = \left(\boldsymbol{I} + \boldsymbol{E} : \boldsymbol{S}^m : \left(\boldsymbol{C}^f - \boldsymbol{C}^m\right)\right)^{-1}$$

$$\boldsymbol{A}^{MT} = \boldsymbol{A}^{Eshelby} : \left((\mathbf{1} - \boldsymbol{\nu}_f)\boldsymbol{I} + \boldsymbol{\nu}_f \boldsymbol{A}^{Eshelby}\right)^{-1}$$

3. Evaluate Unidirectional Stiffness Matrix:

$$\boldsymbol{C}^{UD} = \boldsymbol{C}^m + \boldsymbol{\nu}_f\left(\boldsymbol{C}^f - \boldsymbol{C}^m\right) : \boldsymbol{A}^{MT}$$

1st step: UD-homogenization

e.g. using Halpin-Tsai-Equations:

- **Derived from self-consistent model**

$$\frac{M}{M_M} = \frac{1+\xi\,\eta\,V_f}{1-\eta V_f} \quad \textbf{with } \eta = \frac{M_r - 1}{M_r + \xi} \;\; , \;\; M_r = \frac{M_f}{M_m}$$

M	M_f	M_m	ξ
E_{11}	E_f	E_m	$2a$
E_{22}	E_f	E_m	2
G_{12}	G_f	G_m	1
G_{13}	G_f	G_m	$\frac{K_m}{G_m} \Big/ \left(\frac{K_m}{G_m} + 2\right)$

$a = \dfrac{L}{d}$ **- Fiber aspect ration, L = fiber length, d = fiber diameter**

$$\vartheta_{21} = V_f\,\vartheta_f + V_m\,\vartheta_m$$

1st step: UD-homogenization

e.g. using Tandon-Weng Equations:

- **Based on Eshelby's solution of an ellipsoidal inclusion in an infinite matrix and Mori-Tanaka's average stress**

$$\frac{E_{11}}{E_m} = \frac{1}{1 + V_f \dfrac{A_1 + 2\,\vartheta_m A_2}{A_6}}$$

$$\frac{E_{22}}{E_m} = \frac{1}{1 + V_f \dfrac{-2\vartheta_m A_3 + (1-\vartheta_m)A_4 + (1+\vartheta_m)A_5 A_6}{2A_6}}$$

$$\frac{G_{12}}{G_m} = 1 + \frac{V_f}{\dfrac{\mu_m}{\mu_f - \mu_m} + 2\,V_m S_{1212}}$$

$$\frac{G_{23}}{G_m} = 1 + \frac{V_f}{\dfrac{\mu_m}{\mu_f - \mu_m} + 2\,V_m S_{2323}}$$

$$\vartheta_{21} = \frac{\vartheta_m A_6 - V_f(A_3 - \vartheta_m A_4)}{A_6 + V_f(A_1 - 2\vartheta_m A_2)}$$

$$\frac{K_{23}}{K_m} = \frac{(1+\vartheta_m)(1-2\,\vartheta_m)}{1 - \vartheta_m(1 + 2\,\vartheta_{21}) + V_f \dfrac{2(\vartheta_{21} - \vartheta_m)A_3 + [1 - \vartheta_m(1 + 2\vartheta_{21})]A_4}{A_6}}$$

1st step: UD-homogenization

- **with:**

$$
\begin{aligned}
A_1 &= D_1(B_4 + B_5) - 2B_2 \\
A_2 &= (1 + D_1)B_2 - (B_4 + B_5) \\
A_3 &= B_1 - D_1 B_3 \\
A_4 &= (1 + D_1)B_1 - 2B_3 \\
A_5 &= \frac{1 - D_1}{B_4 - B_5} \\
A_6 &= 2B_2\, B_3 - B_1(B_4 + B_5)
\end{aligned}
$$

$$
\begin{aligned}
B_1 &= V_f D_1 + D_2 + V_m(D_1 S_{1111} + 2S_{2211}) \\
B_2 &= V_f + D_3 + V_m(D_1 S_{1122} + S_{2222} + S_{2233}) \\
B_3 &= V_f + D_3 + V_m(S_{1111} + (1 + D_1)S_{2211}) \\
B_4 &= V_f D_1 + D_2 + V_m(S_{1122} + D_1 S_{2222} + S_{2233}) \\
B_5 &= V_f + D_3 + V_m(S_{1122} + S_{2222} + D_1 S_{2233})
\end{aligned}
$$

$$
\begin{aligned}
D_1 &= 1 + 2\frac{\mu_f - \mu_m}{\lambda_f - \lambda_m} \\
D_2 &= \frac{\lambda_m - 2\mu_m}{\lambda_f - \lambda_m} \\
D_3 &= \frac{\lambda_m}{\lambda_f - \lambda_m}
\end{aligned}
$$

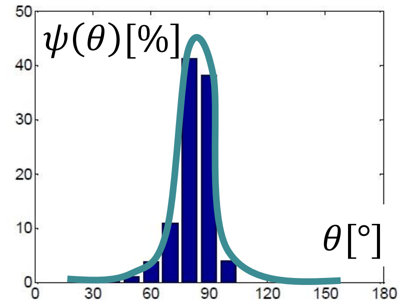
$\mu_f, \mu_m, \lambda_f, \lambda_m$ - Lamé constants

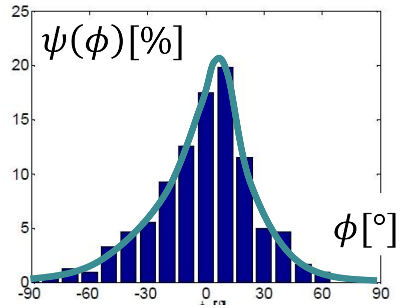
S_{ijkl} - non-vanishing components of the Eshelby‘s tensor

2nd step: Orientation averaging

➢ Fiber orientation distribution function ψ

➢ probability to have fibers in a certain direction





2nd step: Orientation averaging

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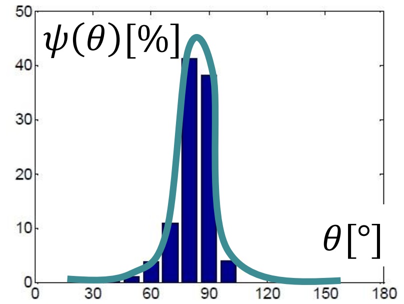
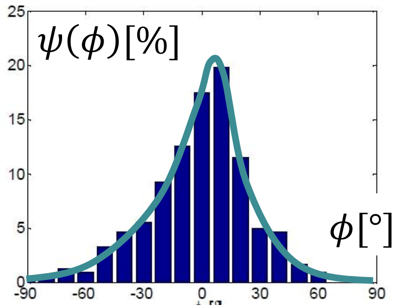
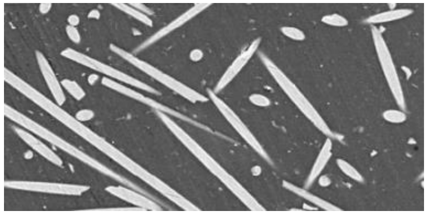
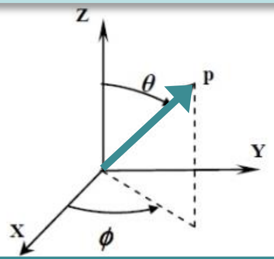




➢ Fiber orientation vector $\boldsymbol{p}(\theta, \phi)$: unit vector in fiber direction



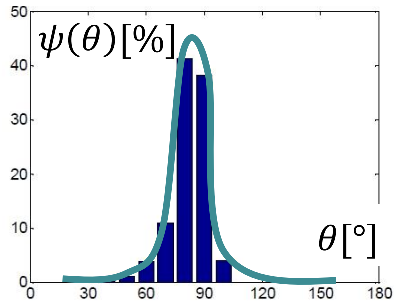


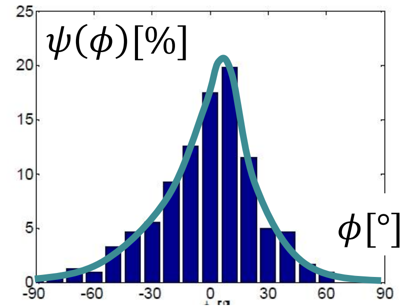
$$p = \begin{bmatrix} sin\theta\ cos\phi \\ sin\theta\ sin\phi \\ cos\theta \end{bmatrix}$$

2nd step: Orientation averaging

➢ Fiber orientation distribution function ψ

- ➢ probability to have fibers in a certain direction





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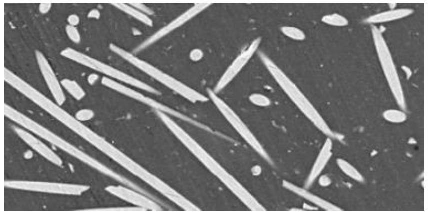
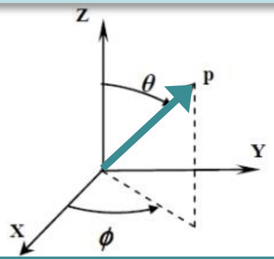
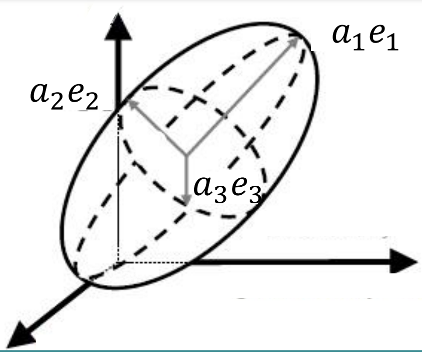




$$p = \begin{bmatrix} sin\theta \; cos\phi \\ sin\theta \; sin\phi \\ cos\theta \end{bmatrix}$$

➢ Fiber orientation tensor (2nd and 4th order)

$a_{ij} = \int p_i p_j \; \psi(\theta, \phi) \; d\Omega$ and $a_{ijkl} = \int p_i p_j p_k p_l \; \psi(\theta, \phi) \; d\Omega$



Eigenvectors → principal directions of the material

Eigenvalues → Orientation Distribution Probability

2^{nd} step: Orientation averaging

Orientation averaging ... Advani & Tucker [1987]

- Starting point: $$C_{ijkl} = \int C^{UD}{}_{ijkl}\, \psi(\theta,\phi)\ d\Omega$$

- Advani & Tucker:

$$C_{ijkl} = B_1 A_{ijkl} + B_2\left(a_{ij}\delta_{kl} + a_{kl}\delta_{ij}\right) + B_3\left(a_{ik}\delta_{jl} + a_{il}\delta_{jk} + a_{jl}\delta_{ik} + a_{jk}\delta_{il}\right) + B_4\left(\delta_{ij}\delta_{kl}\right) + B_5\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$$

with:

$$B_1 = C^{UD}{}_{1111} + C^{UD}{}_{2222} - 2C^{UD}{}_{1122} - 4C^{UD}{}_{1212}$$

$$B_2 = C^{UD}{}_{1122} + C^{UD}{}_{2233}$$

$$B_3 = C^{UD}{}_{1212} + {}^{1}\!/_{2}\,(C^{UD}{}_{2233} - C^{UD}{}_{2222})$$

$$B_4 = C^{UD}{}_{2233}$$

$$B_5 = {}^{1}\!/_{2}\,(C^{UD}{}_{2222} - C^{UD}{}_{2233})$$

For x_1 being the direction of fibers of $\boldsymbol{C}^{UD}$

2nd step: closure approximation

- 2nd order orientation tensor available: Measured or output of molding simulation
- **4th order orientation tensor needed: Closure approximation**

- **Linear closure approximation:**

$$A^{L}{}_{ijkl} = -\frac{1}{35}\left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right) + \frac{1}{7}\left(a_{ij}\delta_{kl} + a_{ik}\delta_{jl} + a_{il}\delta_{jk} + a_{kl}\delta_{ij} + a_{jl}\delta_{ik} + a_{jk}\delta_{il}\right)$$

- **quadratic closure approximation**

$$A^{Q}{}_{ijkl} = a_{ij}a_{kl}$$

- **hybrid closure approximation**
  (mixture between linear and quadratic)

$$A^{H}{}_{ijkl} = (1-f)A^{L}{}_{ijkl} + fA^{Q}{}_{ijkl}$$

e.g. $f = \frac{3}{2}a_{ij}a_{ij} - \frac{1}{2}$

- **Variety of fitted closure approximations**

Overview elastic algorithm

1st step:

- Eshelby tensor $\boldsymbol{E}$ (for unidirectional 1 fiber in matrix)
- Strain concentration tensor $\mathbf{A}$ (for unidirectional 1 fiber in matrix) if Mori-Tanaka
- Uni-direcetional elasticity tensor (Mori-Tanaka or directly from Literature): $\boldsymbol{C}^{UD}$

2nd step:

- Closure approximation orientation tensor 4th order $a_{ij} \rightarrow A_{ijkl}$
- Orientation averaging of elasticity tensor:

$$C_{ijkl} = B_1 A_{ijkl} + B_2\left(a_{ij}\delta_{kl} + a_{kl}\delta_{ij}\right) + B_3\left(a_{ik}\delta_{jl} + a_{il}\delta_{jk} + a_{jl}\delta_{ik} + a_{jk}\delta_{il}\right) + B_4\left(\delta_{ij}\delta_{kl}\right) + B_5\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$$

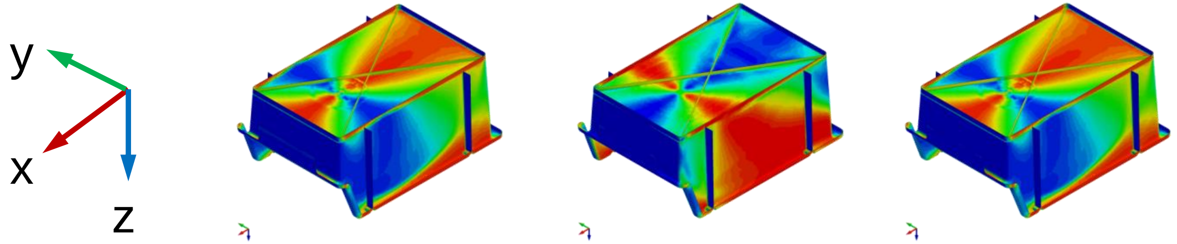
Load step loop

Composite (overall) strain increment $\Delta\boldsymbol{\varepsilon}^c$

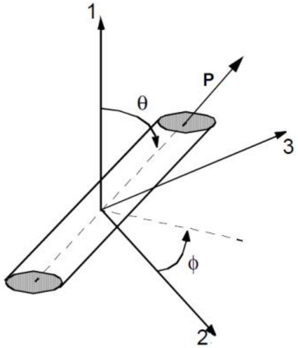
Composite stress increment $\Delta\boldsymbol{\sigma}^c = \mathbf{C}\,\Delta\boldsymbol{\varepsilon}^c$

Data Mapping (DYNAmap)

- In order properly consider the fiber orientation distribution within FE analysis, a proper data mapping has to be established.



- Output data from moldflow simulations usually contain fiber orientation tensors or the first two eigenvectors and corresponding eigenvalues

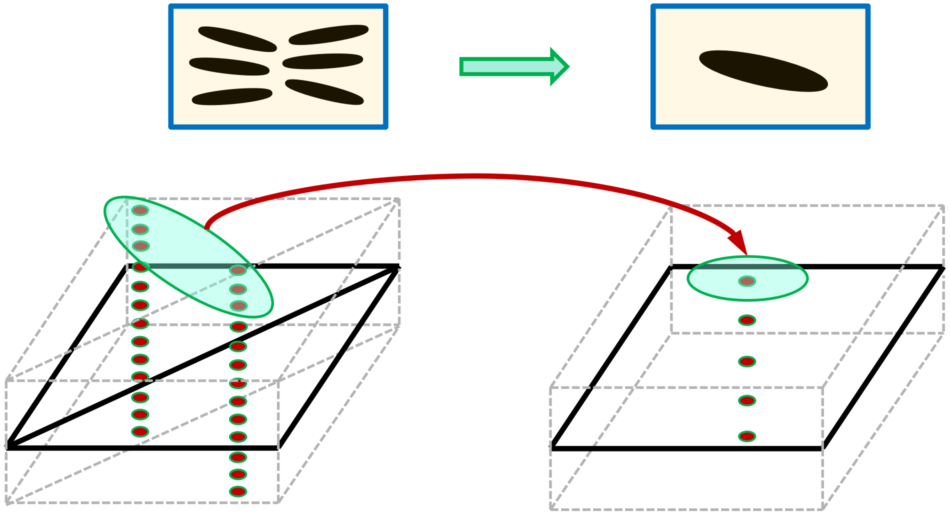


$$p = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix} \qquad a_{ij} = \frac{1}{n}\sum_{k=1}^{n} a_{ij}^{k} = \frac{1}{n}\left(\sum_{k=1}^{n} p_i^k p_j^k\right) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

- Orientation data will be initialized using `*INITIAL_STRESS_SOLID` - card

Data Mapping (DYNAmap)

- **Challenges for a proper data mapping:**
  - Process simulations are often performed in different unit systems than servicability analysis
  - Different meshes are used during infiltration simulation and crushing analysis
  - Number of integration points differs between infiltration simulation (e.g. 13) and crushing analysis (e.g. 5)
  - Proper averaging techniques have to be established in order to properly consider tensor orientation and tensor shape during the data mapping



Data Mapping (DYNAmap)

```
$#----------------------------------------
$# Main mapping definition
$#----------------------------------------
DYNAMAP=MOLDFLOW-SOLID
$#----------------------------------------
$# Activate transformation
$#----------------------------------------
TRANSFORMATION=NO
SourceUnitSystem=kg-m-s
TargetUnitSystem=kg-mm-ms
$#----------------------------------------
$# In- and output meshes
$#----------------------------------------
SourceFile=MF_MESH.key
TargetFile=Stapelbox.key
MappingResult=Stapelbox_mapped.key
OrientationFile=DYNA.xml
ViewMoldFlowData=Stapelbox_res.key
$#---------------------------------------
$# Target - Options
$#---------------------------------------
ETYP=1
MapStress=YES
$#---------------------------------------
$# Mapping-Options
$#---------------------------------------
ALGORITHM=ClosestPoint
$#---------------------------------------
$# END-OF-FILE
$#---------------------------------------
```

Main mapping command

Usually, unit-systems in moldflow- and FE-mesh are different!

Load Moldflow *.xml-File

For visualization purposes in LS-PrePost.

Flag to choose element type

Necessary to generate `*INITIAL_STRESS_SOLID` - cards

So far, tensor averaging is not implemented, results are mapped on a simple „closest point“ procedure

Numerical experiment (proof of concept)

- First test simulations are preformed on a stackable box (fiber volume fraction 16.5%, aspect ratio = 25)

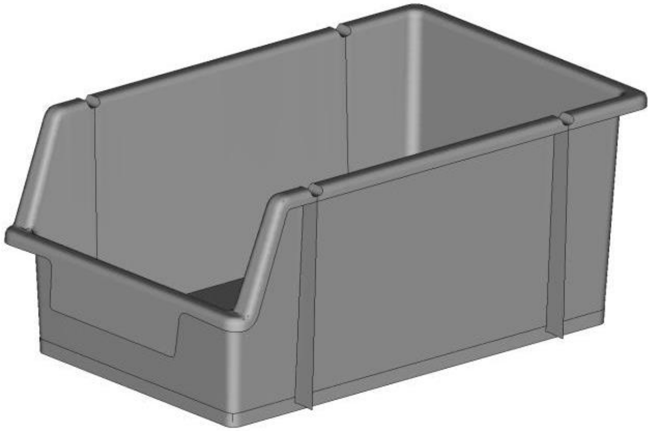
- Assumed material parameters:

E_f	ν_f	ρ_f	E_m	ν_m	ρ_m
74000.0 MPa	0.25	2.6 E-09 t/mm^3	5100.0 MPa	0.35	1.2E-09 t/mm^3

- Spherical impactor used (Ø 19.6 mm, v = 1.0 mm/ms, 5.2 kg)

- Homogenization using Halpin-Tsai equations

- Linear closure approximation

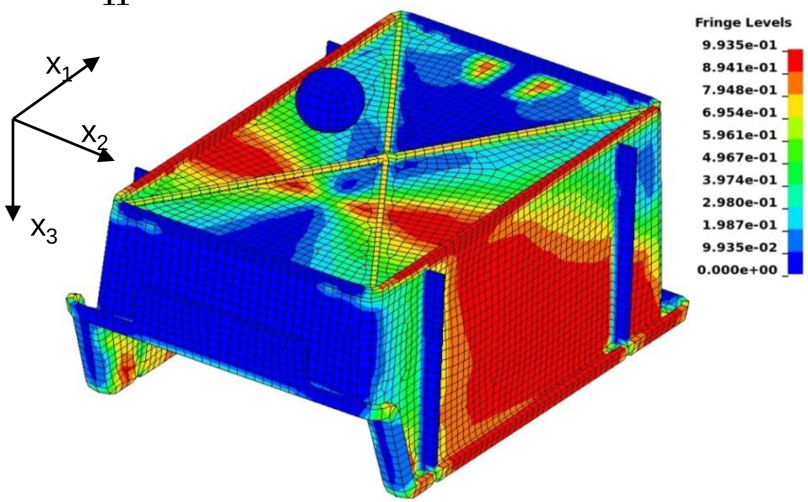


- For a first trial, Halpin-Tsai homogenization equations together with linear closure approximation show an acceptable accuracy compared to experiments

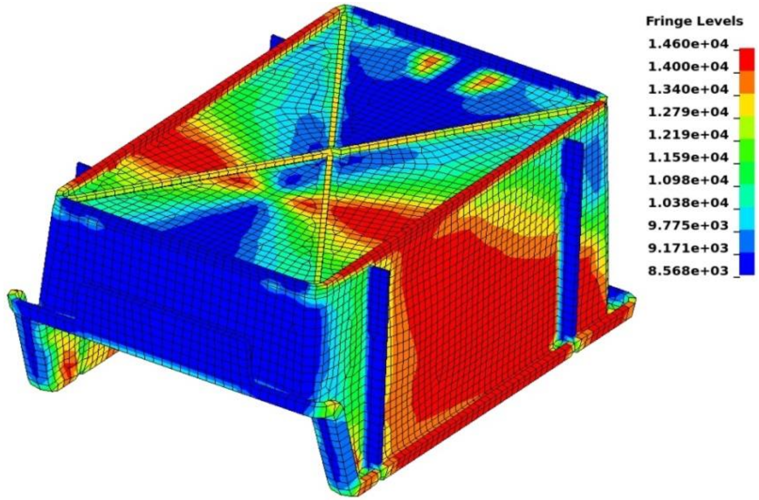
	E_1	$E_2 = E_3$
LS-DYNA, Halpin-Tsai, linear closure approximation	11600 MPa	7900 MPa
Dray et al. [2], experimental results	11800 MPa	7220 MPa

- Homogenization shows reasonable results for the components of the elasticity tensor compared to the fiber orientation tensor data

Fiber orientation tensor component a_{11}:



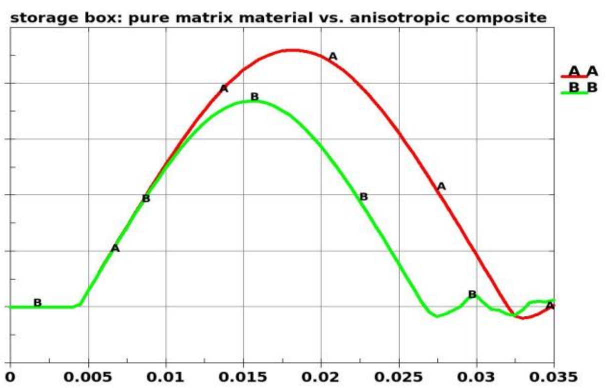
Elasticity tensor component $C_{1111}$of homogenized equivalent medium:



Numerical experiment (proof of concept)

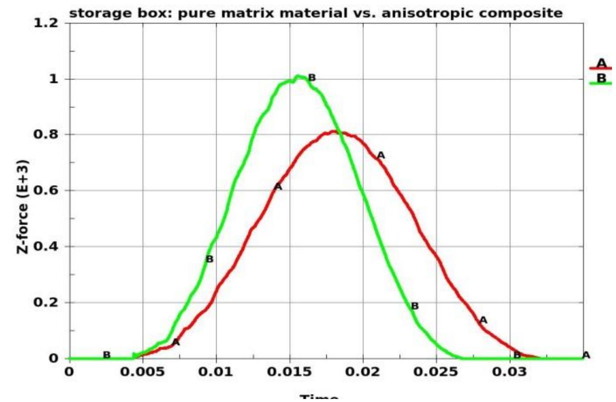
- Simulations using a pure matrix material compared to a consideration of the fiber inclusions show reasonable results as well.

z-displacement [mm] under impact loading



time [s]

z-contact force [N]

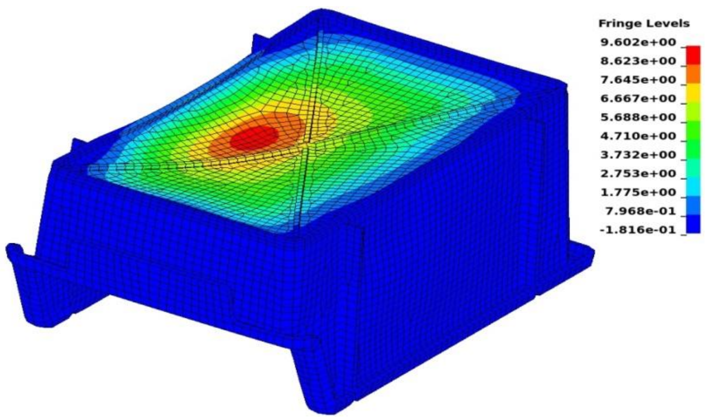


time [s]

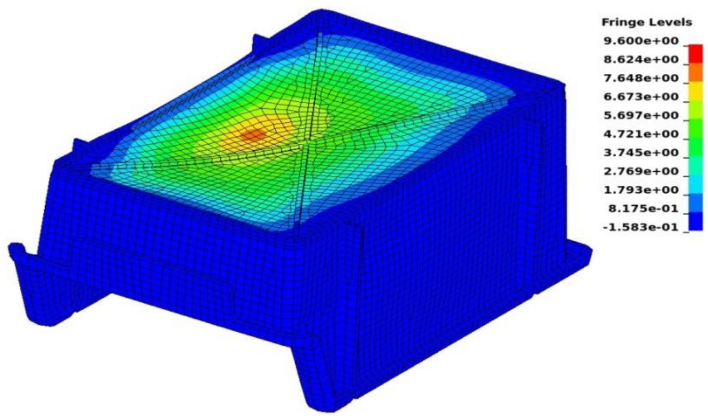
A: matrix material

B: composite

pure matrix material: u_z [mm], time=0.016 s :



Composite: u_z [mm], time=0.016 s:



Conclusion & Outlook

- A new material model for short fiber reinforced plastic materials is being introduced into LS-DYNA, considering various homogenization techniques and closure approximations

- Besides that, proper data mapping routines are being established in a new tool called DYNAmap (beta-status)

- So far, only a linear-elastic response is being implemented for solid elements only

- Material model will be extended for elasto-plastic behavior as well as thick shell and shell elements.

- Data mapping will be improved for a proper consideration of tensor direction and shape during averaging

- FIN -